SOLAR SAILS AS A TOOL FOR SPACECRAFT MOTION CONTROL NEAR SOLAR-TERRESTRIAL LIBRATION POINTS

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ABSTRACT

Two problems are considered connected with the use of solar sails on spacecraft (s/c) moving in the vicinity of solar-terrestrial libration points L1 and L2. First problem is related to the exploration of s/c trajectories modification under influence of solar radiation pressure in case with comparatively big solar sails are mounted on s/c. In this case trajectory is shifted further from the Earth and closer to the Sun along Sun-Earth line. The shift may reach one million km or more. Simultaneously its size and shape are also changed. The dependence of these variations upon the mass to area ratio is determined.

Such trajectories are interesting for solar wind shock waves investigations because it allows earlier detection of these waves as compared with use of "usual" motion of the s/c in L1 proximity.

The other problem explored in the paper is s/c motion control using variation of forces produced by solar radiation pressure.

The following approach for this variation is supposed: the sails include liquid crystal films with controllable transparency. When electrical voltage is applied film becomes transparent and solar pressure is minimal, in opposite case it is opaque and pressure increases. The other version of transparency variation use is two layer film: one layer is liquid crystal film; the other one is aluminium foil. With transparent liquid crystal film we have approximately two times higher solar radiation pressure than with opaque one. It gives the possibilities to control as orbital motion of s/c as its attitude.

It is shown in the paper that comparatively small solar sails not bigger than usual solar array panels may be used for orbital control of s/c in the vicinity Sun- Earth collinear libration points if above described technics is applied. The required sails area to s/c mass is determined for solving the task of keeping s/c in vicinity of libration point and for trajectory amplitude change maneuvers.

Thus the paper shows rather good perspective for the use of solar sails with controllable transparency for libration point missions.

1. SOLAR PRESSURE SHIFT OF LIBRATION POINTS

Well-known example of the Earth-Synchronous spacecraft (s/c) heliocentric orbital motion is the case when s/c is positioned in L1 or L2 solar-terrestrial libration points. In these cases s/c is moving around the Sun with the same period as the Earth being either 1.5 mln. km closer L1 or further L2 to the Sun than the Earth and situated on Sun-Earth line. It is possible because Earth in this case is decreasing (L1) or increasing (L2) the Sun gravity field acceleration to the extent required by necessity to keep orbital period of the s/c equal to Earth one.

Similar effect can be reached by use of solar sails. Using them in the vicinity of L1 libration point one can reach additional decreasing effect of solar gravity acceleration thus allowing to shift s/c Earth-Synchronous position closer to the Sun than L1 point.

Suppose we intend to shift s/c closer to the Sun than the Earth orbit of a radius by d distance keeping the same Earth heliocentric orbital angular rate ω.

For motion along this more close to the Sun circular orbit the required heliocentric acceleration is to be:

$$W_r = \omega^2 (a-d)\,. \tag{1}$$

If s/c is positioned on Sun-Earth line then the acceleration produced by the Earth gravity field force is:

$$W_E = -\frac{\mu_E}{d^2}\,, \tag{2}$$

the acceleration produced by Sun gravity field is:

$$W_S = \frac{\mu_S}{(a-d)^2}\,, \tag{3}$$

where μ_E, μ_S are the Earth and Sun gravitational constants respectively, the required acceleration to be produced by solar radiation pressure is

$$W_P = -F\frac{S}{m}\,, \tag{4}$$

where F is solar radiation pressure, S - s/c cross-section area, m - s/c mass.

Then the formula is valid:

$$\omega^2 (a-d) = \frac{\mu_S}{(a-d)^2} - \frac{\mu_E}{d^2} - F\frac{S}{m}\,. \tag{5}$$

Using the following values of parameters: μ_S = 132712517*10^3 km^3/s^2, μ_E = 398.6*10^3 km^3/s^2, a = 149597.81 tds. km, for full photons absorption:

$$F = 4.5 \cdot 10^{-6} \left(\frac{a}{a-d} \right)^2 \frac{N}{m^2} \;, \qquad (6)$$

and the above formula one can calculate the required solar radiation acceleration W_P and required m/S ratio for solar sails.

Some results of such calculations may be presented in Table 1.

Table 1.

d, tds km	W_P, m/s^2	$\frac{m}{S}$, kg/m^2
1500	0	—
2000	-0.0001333	0.03468
2500	-0.0002306	0.02018
3000	-0.0003118	0.01502
4000	-0.0004560	0.01042

The case given in the above table supposes that solar radiation is fully absorbed by the solar sails, i.e. the reflectivity coefficient is equal zero. From effectiveness point of view it is the worst case. The best case is full reflectivity of sail surface, when acceleration produced by solar radiation pressure is two times higher. Accordingly the maximum allowed mass to cross-section area ratio is two times higher than given in the table figures.

The same formula can be used for the case when s/c is put onto modified by solar radiation pressure L2 libration point: it is enough to change the sign of distance value d, and Earth gravitational constant μ_E onto negative one. Another quite obvious assumption here is that s/c moves on some trajectory near L2 out of Earth shadow.

2. APPLICATION OF LIQUID CRYSTAL TECHNOLOGY FOR SOLAR SAILS

In the contemporary space missions proposed projects for control the forces generated by solar radiation pressure, the change of sails attitude is supposed.

In our analysis the different approach is considered: sails include liquid crystal films with controllable transparency. By applying electrical voltage it is possible to change film from opaque to transparent with respective variation of solar radiation pressure from maximum to minimum value. Or for more effective application solar sails may be manufactured from two layers, one layer (on Sun side) is liquid crystal film and the other layer is mirror foil.

If liquid crystal film is transparent the solar radiation pressure is maximal because of mirror reflection of photons, if it is opaque then photons are absorbed and we have minimal pressure ideally two times less than in previous case.

I addition these two cases are different not only by value of solar radiation pressure but also by the direction of resulting forces: for mirror reflection case and flat surface force is directed orthogonally to the surface; for ideal absorption case this force is directed along Sun direction, what means that with opaque sails it is impossible to generate force orthogonal to Sun direction.

With the possibilities to change the ratio of mirror reflecting part of sails surface to the absorbing one we can change not only the value of resulting force but also the direction of this force without attitude variation.

These features of sails with liquid crystal film are described by formulae:

$$T_x = - T_0 (2 (1 - k) S \cos^2\varphi + k S \cos\varphi), \qquad (7)$$

$$T_y = - 2T_0 (1-k) S \cos\varphi \sin\varphi, \qquad (8)$$

where T_x, T_y - projection of s/c acceleration, produced by solar radiation pressure onto X, Y axes of solar-ecliptic coordinate system; $T_0 = F/m$; k - ratio of surface area with full absorption to the total area, it is supposed that the rest area has the ideal mirror reflectivity; φ - angle from X axis (Sun directed) to the vector orthogonal to the sail surface, it is supposed here that this vector lies in ecliptic plane.

Thus for executing maneuver with given acceleration components T_x, T_y it is enough to calculate k factor and φ angle by resolving the Eqns.7-8. As to the third acceleration component T_z the approach is similar, but should be mentioned as it follows from [1], the maneuver for motion control in Z direction is more effective to fulfill on the other parts of orbit than for motion in ecliptic plane. Also it should be reminded that for the nominally uncontrolled part of the trajectory the nominal value of T_x is supposed with nominal $T_y = 0$. So if consider ΔV maneuver as flight with some added to the nominal one acceleration, then for our case the maneuver acceleration along X is $\Delta T_x = T_x - 2T_0$.

3. SOLAR SAILED TRAJECTORIES IN L1 POINT VICINITY

For solar wind exploration the L1 point considered to be very convenient especially for early detection shock waves reaching the Earth. Taking into account some practical use of this early detection it is important to find the tools allowed to put s/c further from the Earth than L1 point. In the first chapter it was shown the solution of this problem by the use of solar sails. The estimation of necessary sails sizes were given for s/c positioned in libration points, shifted by sails.

Here the trajectories in vicinity of shifted L1 point are studied and also the transfer orbits from the low Earth parking orbit.

Transfer orbits have been chosen belonging to the one impulse trajectories manifold, i.e. the ones which allow s/c to orbit in vicinity of libration point by applying to it only one velocity impulse on low Earth parking orbit with fixed inclination with respect to equator equal 65^0.

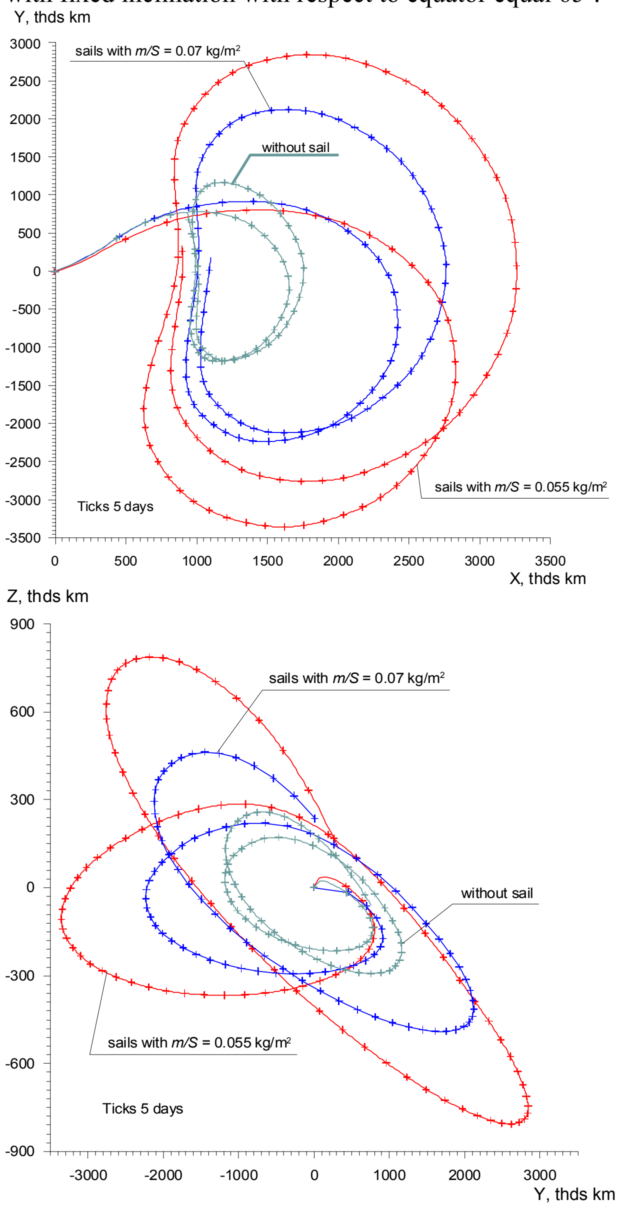

Fig.1. Solar sails size impact on near L1 orbits

As a tool for analysis numerical integration method for differential equation of s/c motion has been used (with nonrotating reference coordinate system connected to the Earth center):

$$\ddot{\bar{r}} = -\frac{\mu_E}{|\bar{r}|^3}\cdot\bar{r} - \frac{\mu_S}{|(\bar{r}-\bar{r}_S)|^3}\cdot(\bar{r}-\bar{r}_S) - \frac{\mu_S}{|(\bar{r}-\bar{r}_M)|^3}\cdot(\bar{r}-\bar{r}_M) + \frac{\mu_S+\mu_E}{|\bar{r}_S|^3}\cdot\bar{r}_S + \frac{\mu_M+\mu_E}{|\bar{r}_M|^3}\bar{r}_M + \bar{T}_{fe} + \bar{T} \qquad (9)$$

,where $\bar{r}$ - s/c vector with respect to Earth center; $\bar{r}_S$ - Sun vector; $\bar{r}_M$ - Moon vector; $\bar{T}_{fe}$ - acceleration of s/c due to Earth flattening; $\bar{T}$ - acceleration produced by solar radiation pressure.

$$\bar{T} = 2\cdot\frac{F\cdot S}{m}\cdot r_{sm}^2\cdot\frac{\bar{r}-\bar{r}_S}{|\bar{r}-\bar{r}_S|^3}, \qquad (10)$$

where r_{sm} is mean distance from Earth to Sun taken as reference value for calculating solar radiation.
Using Eqn.10 it is possible to include produced by sails acceleration in second term of Eqn.9 modifying μ_S to μ_{S1} by extraction from it the term taking into account solar radiation pressure:

$$\mu_{S1} = \mu_S - 2\cdot\frac{F\cdot S}{m}\cdot r_{sm}^2. \qquad (11)$$

After that the Eqn.9 looks as the one without solar radiation pressure. But it should be mentioned that this transformation is not fully equivalent to the change one Sun onto another with smaller mass, because the term taking into account the inertia forces generated by Sun on reference system (connected with Earth) are kept the same, i.e. with previous gravitational constant. Nevertheless the similarity of equations gives grounds to apply for solar sailed trajectories design the same methods as for "usual" libration points orbits, described in [1]. The one impulse manifold of libration point trajectories are characterized by maximum amplitude along Y axis. It is possible to present these manifolds by the sets of osculating orbital parameters at the point of injection of s/c onto transfer orbit from low Earth orbit.
The envelope of the mentioned parameters may be broad enough as it was shown by some examples in [3]. So being inside envelope it is necessary to determine only one parameter for transfer orbit, semimajor axis for example, what was done for the case illustrated by Fig.1. The following parameters have been taken after injection: eccentricity e=0.99, inclination towards equator $i=65^0$, perigee argument $\omega=10^0$, latitude argument $u=10^0$, date 7 August 2005. With the use of methods described in [1] the semimajor axis was calculated a=674810 km for $\mu_{S1} = 1298\cdot 10^8$ km^3/s^2 what corresponds mass to area ratio m/S = $0.07 kg/m^2$ and ideal mirror reflection of flat sails. It was supposed that after injection onto transfer trajectory s/c flies with folded sails until it reaches 1300000 km from the Earth center, after this the sails are deployed.
Moving along transfer trajectory s/c comes to solar-ecliptic XZ plane after 196 days of flight since injection, including some loop preceeding libration point orbit (LPO) itself. The X coordinate at this arrival to LPO is 1006000 km, again s/c returns to XZ plane with X=2761700 km, what happens on day 327 since

injection. Full LPO is completed with s/c in *XZ* plane on day 460, i.e. motion period in ecliptic plane is about 264 days what is almost 1.5 times longer than for LPO without sails.

Amplitude of orbit in *Y* direction is about 2123000 km. Also on Fig.1 the LPO for larger sails is given with $\mu_{S1} = 1280 \cdot 10^8$ km^3/s^2 (m/S = 0.0549 kg/m^2). For this case it is needed more time to reach LPO: 222 days, with X=860000 km, and X_{max}=3260000 km at day 371, the period of motion in ecliptic plane is 294 days, the *Y* amplitude is up to 2840000 km. Thus with larger solar sails LPO becomes also larger with its center moving further from the Earth.

4. SOLAR SAILS AS A TOOL FOR ORBITAL MANUEVERS

S/c motion on LPO is to be controlled for the following list of purposes (obviously not full):

- to keep s/c on nonescape trajectory in vicinity of libration point;
- to avoid too close angular approach to the Sun in order to exclude unexceptable level of Sun influence on radiolink;
- to decrease the amplitude of the orbit to the limits demanded by experiments onboard s/c.

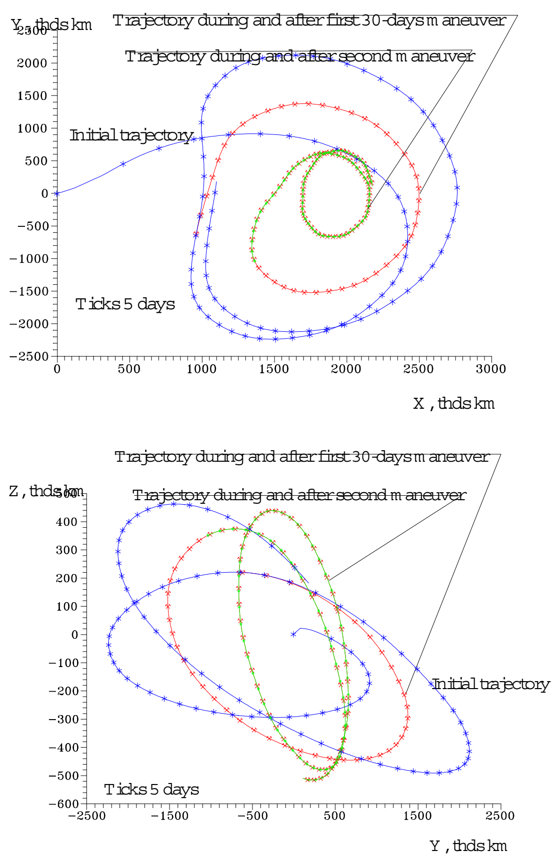

Fig.2. Solar sailed maneuvers in vicinity of L1 point

The last two points differ only by magnitude of necessary maneuvers; both ones are to satisfy the condition to apply acceleration in such a way which does not cause leaving LPO. According to [1] there is a plane of maneuver directions satisfying this condition for the case without sails.

Numerical experiments have shown that it is true for the sailed maneuvers also. To illustrate this Fig.2 is presented.

It was supposed that first maneuver begins at 185.5 day after injection onto transfer orbit for $\mu_{S1} = 1298 \cdot 10^8$ km^3/s^2 and lasts 30 days with solar sails generating total additional to the nominal one acceleration ΔT = 0.00008096 m/s^2 lying in ecliptic plane and directed under constant angle towards Sun direction. This angle was calculated from the condition to receive nonescaping L1 vicinity orbit.

The value of this angle is - 32.464^0. As a result of maneuver *Y* amplitude has decreased from initial 2123000 km to 1522000 km with corresponding decrease of *X* amplitude. The second maneuver begins at 399.5 days after injection and has the same parameters as previous maneuver except the direction. The angle with Sun direction is here -50.3485^0. The *Y* amplitude has become equal 664000 km, *X* oscillates between 1652000 km and 2153000 km. Period of orbit is also decreased to 227 days from initial 264.

For execution of described maneuver using solar sails it is enough to calculate the required φ angle of the sail turn (angle between *X* axis and normal to the sails surface) and *k* factor, included as unknown in Eqns.7-8.

The values of φ and *k* as it is follows from resolution of Eqns.7-8 are the next: 42.27^0, 0.3444 and 38.1^0, 0.03465 for the first and second maneuvers respectively.

Orbit amplitude along *Z* can be changed by applying acceleration component along *Z* with *X*,*Y* components keeping s/c on nonescape trajectory.

The easiest case for solar sails implementation is its use for keeping s/c on nonescape trajectory. For this usage rather small sails can fulfill the task and besides, with applying liquid crystal films the attitude of s/c can be constant with respect to Sun with normal to sails surface following the Sun. Control force in this case can be varied simply by changing the ratio of mirror reflecting and absorbing parts of sails. Rather small area of the sails is expected to be sufficient for these purposes of correction maneuvers. For example for 100 kg mass of s/c 10 m^2 sails allow to apply ± 3.5 m/s ΔV per year what satisfies demands for required correction maneuver estimated in [1].

5. MOTION OF S/C WITH SOLAR SAILS IN VICINITY OF L2 POINT

Modification of the near L2 point orbits under influence of solar radiation pressure is opposite to the one observed in case of L1 orbits: orbit moves close to the Earth and becomes smaller.

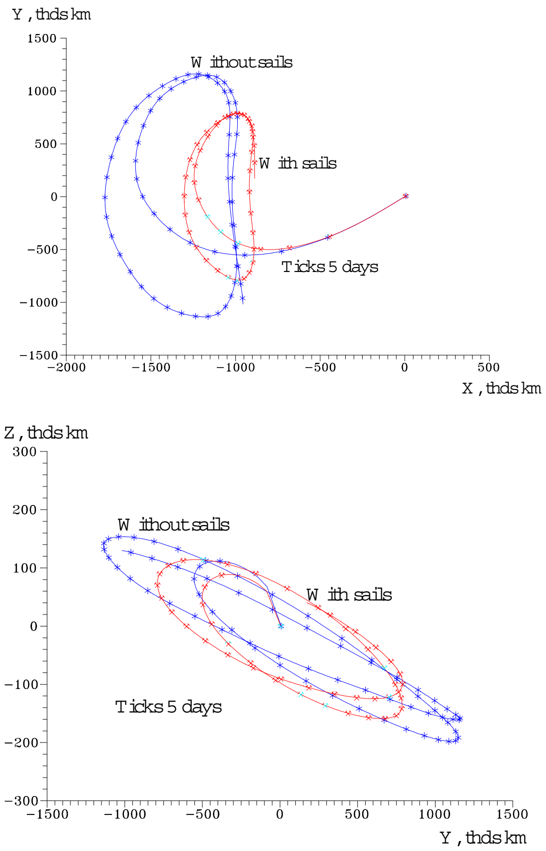

Fig.3. Comparison of L2 vicinity trajectories with and without solar sails.

It is illustrated by Fig.3 where two orbits are presented: for the flight without sails (injection to transfer orbit at 7 August 2005, e=0.985, i=65^0, Ω=180^0, ω=u=0^0, a=660087 km) and with sails (a=527583.76 km).

The sails are deployed at distance 900000 km from the Earth center.

For the usual trajectory X coordinate oscillates between -1020000 km and 1771000 km with Y amplitude about 1000000 km; with sails X coordinate lies between -916000 km and 1300000 km, Y amplitude is 790000 km, period is decreased from 185 to 138 days.

6. CONCLUSIONS

Thus the use of solar sails allows to shift orbits in vicinity of solar-terrestrial libration points significantly: for L1 point orbit moves from the Earth and for L2 - to the Earth. Simultaneously the sizes of these orbits for one impulse injection case are increased for L1 case and decreased for L2. The possibilities to control the LPO parameters using sails were shown by examples of s/c equipped by sails with liquid crystal films. Namely the sizes of the orbit can be changed. Comparatively small sails may be used for orbit correction maneuvers in order to keep s/c in vicinity of libration points. For this task the sizes of sails are not significantly larger than standard solar panels.

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