IMPACT OF ORBIT PREDICTION ACCURACY ON LOW EARTH REMOTE SENSING FLIGHT DYNAMICS OPERATIONS

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ABSTRACT

This paper addresses the problem of orbit prediction and its impact on flight dynamics operations. In general, certain "knowledge" of the satellite's orbit is necessary to design and implement a ground-in-the-loop orbit control system. The operational constraints imposed by Low Earth Orbit (LEO) satellites and the stringent orbit control requirements driven by the use synthetic-aperture-radars (SAR) on board the satellites, give great importance to the orbit calculation chain.

After a brief discussion on how the orbit determination using on-board Global Positioning System (GPS) measurements is performed and then predicted ahead by precise force modelling, the paper focuses on the prediction accuracy evaluation. Appropriate on-going satellite missions are selected as test-beds, accurate GPS based Rapid Science Orbits (RSO) are used as references to evaluate the prediction errors.

Finally the results are applied to the German TerraSAR-X (TS-X) flight dynamics system design. If the long-term prediction accuracy (days) is shown to dictate the nominal maneuver planning and execution concept, the short-term prediction errors (hours) are expected to influence the orbit control performances and to pose critical limits to the minimum achievable interferometric baseline (IB) using a ground-based orbit control.

1. CASE OF STUDY: TERRASAR-X

1.1 The Mission

TS-X is an operational, advanced SAR-satellite system for scientific and commercial applications that will be realized in a public-private partnership between the German Aerospace Center (DLR) and Astrium GmbH. TS-X is a new generation, high resolution satellite operating in the X-band at 9.65 GHz. The launch of the 1-ton satellite into a 514 km sun-synchronous dusk-dawn orbit with an 11 day repeat period is planned on top of a Russian DNEPR-1 rocket for 2006. TS-X is to be operated for a period of at least 5 years and will therefore provide SAR-data on a long-term, operational basis.

1.2 Orbit Control Requirements

The TS-X orbit main characteristics are depicted in Table 1. SAR interferometry is the basic principle for the TS-X mission. This technique is based on the stereoscopic effect that is obtained by matching two SAR images obtained from two slightly different orbits, [1]. This off-set creates an interferometric baseline (IB). The TS-X IB is 500 m. For dedicated interferometric operations IB is reduced to 20 m, [2]. The relevant flight dynamics requirements are therefore to maintain the satellites osculating orbit within a maximum absolute distance of 250 m from a suitable target orbit, the so-called reference orbit. For dedicated operations this prescribed maximum distance is lowered to 10 m.

Table 1. TS-X orbit

Parameter	*Mission orbit*
Orbit type	Sun-synch. repeat orbit
Repeat period	11 days
Repeat cycle	167 orbits in the repeat
Orbits per day	15 + 2/11
Equatorial crossing time	18:00±0.25h ascending pass
Eccentricity	0.0011÷0.0012 frozen
Inclination	97.4438823
Argument of perigee	90°
Altitude at the equator	514.8 km
Semi-major axis	6892.9 km

1.3 Flight Dynamics System Concept

The design of the flight dynamics system faces two major constraints. From a space segment prospective, since a conservative mission approach with a state-of-the-art mono-propellant propulsion system is the baseline, an autonomous on-board orbit control is ruled out. From a ground segment prospective, the present routine operations scenario foresees only one ground station (Weilheim) with two contacts per day. As a result, an autonomous ground-in-the-loop orbit control system is under study at DLR/GSOC, [3]. The flight dynamics system must be able to perform automatically orbit determination (from GPS measurements), orbit prediction, maneuver planning and time-tagged command generation for the bang-bang orbit control.

The feasibility of the nominal orbit control and the high IB requirement are driven by the orbit prediction accuracy that is the subject of this paper.

2. ORBIT MODELING

The NAVLIB/NAVTOOLS software package provides a comprehensive model for the acceleration of an Earth orbiting spacecraft under the influence of gravitational and non-gravitational forces, [4] and [5]. It comprises

- the aspherical gravitational field of the Earth, the luni-solar third body gravitational perturbations, the luni-solar Earth tides among the mass forces,
- atmospheric drag and solar radiation pressure (SRP) among the surface forces.

All models are supplemented by up-to-date Earth rotation parameters and solar/geomagnetic data. The force model briefly presented in the following is used in combination with a numerical integration of first order differential equations to predict the spacecraft orbit. The state vector of the spacecraft is defined as the 6-dimensional vector

$$\mathbf{x}(t) = \begin{Bmatrix} \mathbf{r}(t) \\ \mathbf{v}(t) \end{Bmatrix} \qquad (1)$$

containing the position $\mathbf{r}$ and velocity $\mathbf{v}$ at a given time t. It obeys the first order differential equation

$$\dot{\mathbf{x}} = \mathbf{f}(t, \mathbf{x}, \mathbf{p}) = \begin{Bmatrix} \mathbf{v}(t) \\ \ddot{\mathbf{r}} \end{Bmatrix} \qquad (2)$$

where

$$\ddot{\mathbf{r}} = \mathbf{a}(t, \mathbf{r}, \mathbf{v}, \mathbf{p}) \qquad (3)$$

denotes the spacecraft acceleration. Besides the time, the position and the velocity, the acceleration is assumed to explicitly depend on a set of force model parameters $\mathbf{p}$. Given the state vector $\mathbf{x}_0$ at an initial epoch t_0, the orbital motion is fully described by the initial value problem

$$\dot{\mathbf{x}} = \mathbf{f}(t, \mathbf{x}, \mathbf{p}) \qquad \mathbf{x}(t_0) = \mathbf{x}_0 \qquad (4)$$

The main parameters of the force model for TS-X and their current values are listed in Table 2.

Table 2. Parameters of the TS-X force model

Parameter	*Current value*
Satellite mass, m [kg]	1238.0
Earth gravity field degree	120
Earth gravity field order	120
Section for SRP, A_R [m^2]	10.00
Section for drag, A_D [m^2]	3.20
SRP coefficient, C_R	1.3
Drag coefficient, C_D	2.3

2.1 Mass Forces

Earth's static gravity field

The GRACE Gravity Model 01 (GGM01) was released on July 21, 2003. This model was estimated with 111 days of in-flight data (K-band, attitude and accelerometer data) gathered during the commissioning phase of the Gravity Recovery And Climate Experiment (GRACE) mission, which was launched on March 17, 2002. This model is between 10 to 50 times more accurate than all previous Earth gravity models at the long and medium wavelengths. This improvement has been possible by the measurement of the inter-satellite range-rate which is itself very sensitive to the Earth gravity field. In the resulting gravity model, GGM01S, much more detail is clearly evident in the Earth's geophysical features, [6]. The GGM01S field was estimated to degree and order 120, and is here implemented in order to model accurately LEO orbits.

Earth's time-varying gravity field

The gravitation of the Sun and Moon exerts a direct force on the body of the Earth and thus lead to a time-varying deformation of the Earth, [7]. The small periodic deformations of the solid body of the Earth are called solid Earth tides, while the oceans respond in a different way to lunisolar tidal perturbations, known as ocean tides. As a consequence, the Earth's gravity field is no longer static in nature, but exhibits small periodic variations, which also affect the motion of satellites. Here, the ocean tides are neglected because their amplitude is about one order of magnitude smaller than that of solid Earth tides. The perturbations of satellite orbits from the lunisolar solid Earth tides are derived by an expansion of the tidal-induced gravity potential using spherical harmonics in a similar way as for the static gravity field.

Third body gravitational perturbation

Since the forces exerted by the Sun and Moon are much smaller than the central attraction of the Earth, it is not necessary to know their coordinates to the highest precision when calculating the perturbing acceleration acting on a satellite. Here, the computation of Sun and Moon equatorial coordinates is based respectively on Chebyshev approximations of the Newcomb's and Brown's analytical theories, [7].

2.2 Surface Forces

Solar radiation pressure

The solar radiation pressure is due to the momentum transfer of photons emitted by the Sun and impinging the spacecraft surface. The acceleration of a satellite due to the solar radiation pressure is modeled as

$$\mathbf{a}_{rp} = \upsilon P_S \mathrm{AU}^2 C_R \frac{A_R}{m} \frac{\mathbf{r} - \mathbf{r}_S}{\|\mathbf{r} - \mathbf{r}_S\|^3} \tag{5}$$

where P_S is the force due to solar radiation at one astronomical unit (AU) acting on a unit area ($4.56 \cdot 10^{-6}$ N/m^2), A_R is the area exposed to the Sun rays, $\mathbf{r}_S$ is the Sun position vector in inertial coordinates. Equation (5) is commonly used in orbit determination programs with the option of estimating C_R as a free parameter. Orbital perturbations resulting from shadow transits are treated by the introduction of the shadow function $0 \leq \nu \leq 1$, that measures the degree of the Sun's occultation by a body like the Earth or the Moon, [7].

Atmospheric drag

Aerodynamic forces represent the largest non-gravitational perturbations acting on LEO satellites. However, accurate modeling of atmospheric forces is difficult because the physical properties of the atmosphere are not known very accurately. As shown in the sequel, the combination of large size and large uncertainties associated to the force modeling will have great influence on the orbit prediction error analysis. The satellite acceleration due to drag is modeled as

$$\mathbf{a}_d = -\frac{1}{2} \rho C_D \frac{A_D}{m} \mathrm{v}_r \mathbf{v}_r \tag{6}$$

where ρ is the atmospheric density at the location of the satellite, and $\mathbf{v}_r$ is the relative velocity vector with respect to the atmosphere, [7]. The C_D value is estimated as a free parameter in the orbit determination. The density of the upper atmosphere depends in a complex way on a variety of different parameters. The most evident dependencies, however, are

- its decrease with increasing altitude h,
- solar radiation effects: the diurnal *solar ultraviolet radiation heating*, the *extreme ultraviolet radiation of the Sun* on different time scales (27 days and 11-year spot cycle) and the *corpuscular solar wind*.

The Jacchia-Gill density model [8], computes the density from data on solar activity, $F_{10.7}$ (the 10.7 cm radiation index) and from the geomagnetic index, K_p (the three-hourly planetary geomagnetic index). It is basically a bi-polynomial approximation of the Jacchia 1971 standard density model that reduces the computing time by a factor of nine with a reasonable loss of accuracy (from 2 to 8 %).

2.3 Orbit Determination and Prediction

The operational ground-based orbit processing chain consists of two steps. First of all, on-board GPS measurements, extracted from satellite House-Keeping (HK) data, are combined with a dynamic force model by a so-called *orbit determination* (OD) process. Considering a nominal operational scenario for LEO satellites, the output of an OD is the best available knowledge of the orbit actually flown by the satellite in the past hours since the last HK dump. The OD's output represents the initial condition (IC) for a numerical integration of the equations of motions, the so-called *orbit prediction* (OP). The OP provides an orbit forecast over the next hours, until a ground station contact with a new HK dump takes place.

ODEM (Orbit Determination for Extended Maneuvers) is well established operational orbit determination and propagation program for Earth-orbiting satellites, [9].By the use of the above mentioned force models and the capability to operate with a variety of different tracking measurement types (like GPS, angel data, ranging data), the program is well suited for a broad regime of operational applications ranging from orbit determination of Low-Earth orbiting satellites to maneuver calibration of geostationary satellites. We make use of ODEM to perform daily OD's and OP's in a flight dynamics operational environment.

Although different measurement types are supported by the ODEM software, only GPS receiver solutions are of interest in this analysis. The OD is formulated as a sequential non-linear least-squares problem based on Givens rotations. For a detailed mathematical description of the estimation algorithm, the reader is referred to [10]. The OP is based on a standard numerical integration method for initial value problems. In particular an Adams-Bashforth-Moulton method for numerical integration of ordinary differential equations is adopted. This method employs variable order and step-size and is particularly suited for tasks like the prediction of satellite orbits.

3. ORBIT PREDICTION ACCURACY ANALYSIS

After an orbit determination and the prediction ahead in time, the future position of the satellite is not known exactly. The orbit prediction is affected by errors, caused by uncertainties in the dynamic modeling and initial conditions. The orbit prediction accuracy has been quantified by the implementation of a quasi-operational scenario as similar as possible to the TS-X baseline. Due to the fact that the atmospheric drag is the main source of uncertainties in the dynamic modeling, and considering that the same force is also the most important disturbance influencing the motion and the orbit of the satellite, it becomes visible how significant the influence of altitude and solar/geomagnetic activity on the prediction accuracy is.

The precise orbit trajectories to be used as a reference for the computation and evaluation of the prediction errors, should be at an altitude as close as possible to the TS-X altitude of 514 km. Furthermore the time span covered by the simulation should be representative with respect to the entire spectrum of solar flux/geomagnetic indices that TS-X is supposed to encounter during the mission active lifetime of 5.5 years. The on-going CHAMP (CHAllenging Minisatellite Payload) and GRACE (Gravity Recovery And Climate Experiment) missions represent suitable test-beds for the analysis. The mentioned satellites fly lower than TS-X, thus it would be possible either to use both orbits, deduce an altitude scale factor and estimate the prediction accuracy for the altitude of interest, or to consider only the satellite with altitude as close as possible to the value of interest and get a conservative estimation of prediction accuracy (the higher the altitude, the more accurate the orbit prediction is).

Here the attention is focused on GRACE data, with brief references to CHAMP results. Precise orbit data from the GRACE-2 satellite have been considered within the period 1st September 2003 up to 15th December 2003. The chosen time span includes violent geomagnetic storms caused by high peaks of solar activity observed at the end of October and November 2003, see Figure 2. The GRACE satellites are orbiting at a mean altitude of 480 km during this period. The accurate reference orbits (ca. 5 cm, 3D, 1σ) used for orbit comparison have been gently provided by the GeoForschungsZentrum Potsdam (GFZ) and the Jet Propulsion Laboratory JPL respectively for CHAMP and GRACE. They are in SP3 format and sample the trajectory every 60 s.

3.1 Daily Orbit Determination

The current nominal TS-X mission baseline foresees the usage of only Weilheim (Germany) ground station for uplink, real-time telemetry and HK-Memory dump downlink (S-Band). Accordingly, the automated orbit processing executes daily OD's using as tracking data types the 24 hours GPS navigation solution. Constant solar flux and geomagnetic index values are used in order to get results normalized with respect to the solar activity. The adopted parameters are given in Table 3.

Table 3. Constant parameters for OD

Parameter	*Value*
Solar flux value, $F_{10.7}$	$160 \cdot 10^{-22}$ Ws/m^2
Geomagnetic index, K_p (a_p)	3 (15)

The drag coefficient is left as an estimation parameter and as a result, turns out to represent a fictitious quantity that contains the real solar/geomagnetic activity information, see Figure 1.

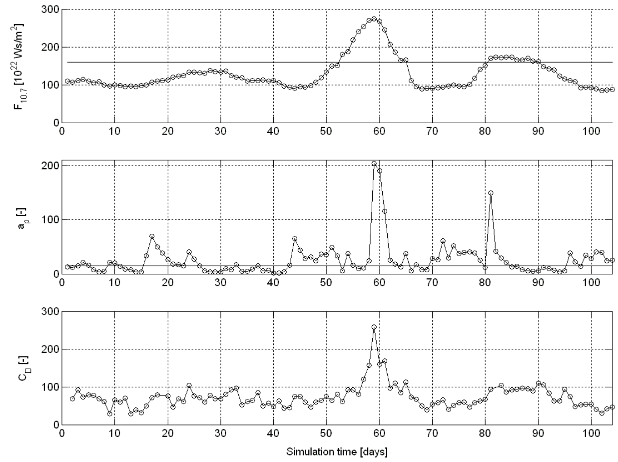

Fig. 1. Solar/geomagnetic activity and estimated drag coefficient during the simulation, since 01/09/2003.

The OD accuracy has been evaluated by subtracting the reference orbit position from the determined orbit position at the same absolute time. The resulting vector is mapped into a local horizontal frame aligned with the reference radial (R), along-track (T) and cross-track (N) directions. For each day of the simulation, mean and standard deviation of the prediction error components are calculated. Table 4 shows the maximum accuracy limits observed for GRACE-2 that can be expected also during high peaks of solar activity.

Table 4. OD accuracy. Gravity order&degree$^{20/120}$

Error components	*Mean*^{20}	$1\sigma^{20}$	*Mean*^{120}	$1\sigma^{120}$
Radial, [m]	< 2.0	< 7.0	< 0.5	< 2.0
Along-track, [m]	< 1.0	< 55	< 1.0	< 12
Cross-track, [m]	< 0.5	< 7.0	< 0.1	< 1.0

3.2 Subsequent Orbit Prediction

For each initial condition provided by the daily OD, an OP is performed over the subsequent days. The same assumptions are used for the solar/geomagnetic activity data in order to propagate the orbit. The OP accuracy is calculated as explained in the previous section. For the operational implementation of the TS-X orbit control we are interested in a

- long-term prediction accuracy evaluation (order of magnitude: days) in order to define the orbit control strategy and adapt the control dead-bands for the ±250 m interferometric baseline, and in a

- short-term prediction accuracy evaluation (order of magnitude: hours) in order to define the orbit control feasibility of the high accuracy (±10 m) differential interferometric baseline.

Long-term analysis

For each simulation's day a three-days-long OP starting from the last GPS data point in the HK memory has been generated. For safety reasons only the maximum prediction errors are considered. In particular, the maximum along-track, cross-track and radial error calculated over the first, second and third day of prediction are stored for the entire period of interest. Results show a strict correlation between maximum prediction errors and solar/geomagnetic activity. Table 5 presents the statistics of the maximum observed prediction errors over the entire simulation and over a selected period of high solar activity (21st October - 10th November 2003).

Table 5. OP accuracy over 1, 2 and 3 days of prediction. Statistics of the maximum errors.

All period	*1 day*		*2 days*		*3 days*	
	Mean	*1σ*	*Mean*	*1σ*	*Mean*	*1σ*
Radial, [m]	-2.0	11.1	-5.4	23.4	-10.4	40.7
Along, [m]	60	367	323	1835	1051	4287
Cross, [m]	-0.9	1.0	-1.3	1.5	-1.6	2.1
High-sol-activity	*1 day*		*2 days*		*3 days*	
	Mean	*1σ*	*Mean*	*1σ*	*Mean*	*1σ*
Radial, [m]	-4.7	15.8	-14.1	32.9	-28.1	58.8
Along, [m]	160.4	528	948	2748	3210	6092
Cross, [m]	-0.9	1.0	-1.3	2.4	-1.4	3.3

Mean and standard deviation of the maximum prediction errors increase similarly with time and solar/geomagnetic parameters. The along-track error is the most time-dependent component and increases by a factor of 4÷5 times per day. Following in magnitude are the radial component, with an increase of 2÷3 times per day, and the cross-track component with an increase of 1÷2 times per day. The high solar/geomagnetic activity drives a severe increase of the along-track and radial error components, especially during the first day of prediction, while the cross-track error is almost unaffected by this..

Short-term analysis

The first impact of the OP accuracy analysis is given by the large along-track error encountered within the first 24 hours of orbit-prediction from ground. An on-board data-take (D.T.) time correction turns out to be always necessary for SAR interferometry, in particular the ground-commanded D.T. time must be corrected on-board based on the latest GPS measurements. The ground-based orbit control must take into account the fact that when a SAR D.T. occurs, the along-track separation between spacecraft and target orbit ideally vanishes. For an operational implementation, one is not interested in the prediction error defined so far, but in a more realistic reproduction of the D.T. configuration. As a consequence the so-called space error (**E**) is introduced and used as control variable for the ground-in-the-loop flight dynamics system, [3]. **E** is a two-dimensional vector defined as the difference between predicted and target orbit at the relative time of along-track error zeroing.

Table 6 shows the maximum expected **E**'s uncertainty as a function of the time of propagation since the last valid GPS data point. The space prediction error, **DE** is estimated assuming that the OP error components reach their maximum values simultaneously at the equator (worst case scenario).

Table 6. Maximum space prediction error over 24 hours

Hours	*1*	*2*	*3*	*4*	*5*	*6*
DE_{max} [m]	7.3	9.9	11.4	12.1	14.7	16.1
	7	*8*	*9*	*10*	*11*	*12*
	17.9	20.8	22.1	26.1	29	30.6
	13	*14*	*15*	*16*	*17*	*18*
	34.8	38.6	40.3	45.7	49.5	51.9
	19	*20*	*21*	*22*	*23*	*24*
	58	61.7	70.7	82.6	92.3	102

In general DE is larger than the vector formed by the sole radial and cross-track components of the OP error defined in Table 5. This is due to the fact that the residual along-track error (ΔT), combined with the Earth's angular velocity ($v_\oplus \approx 500$ m/s at TS-X altitude), gives an additional out-of-plane contribution (ΔC) to the calculation of **DE** (e.g. $\Delta C = \Delta T \cdot (v_\oplus / v_{TS\text{-}X}) \approx \Delta T/15$ at the equator).

4. ORBIT PREDICTION ERROR MODEL

An error model has been generated in order to estimate in advance the OP errors as a function of solar/geomagnetic activity (a_p and $F_{10.7}$ coefficients) and time of propagation (t < 24 h). An appropriate polynomial approximation has been adopted to describe the three OP error components for the duration of each first day of prediction. Finally a least-squares linear regression has been applied in order to correlate univocally the polynomial coefficients to a_p and $F_{10.7}$.

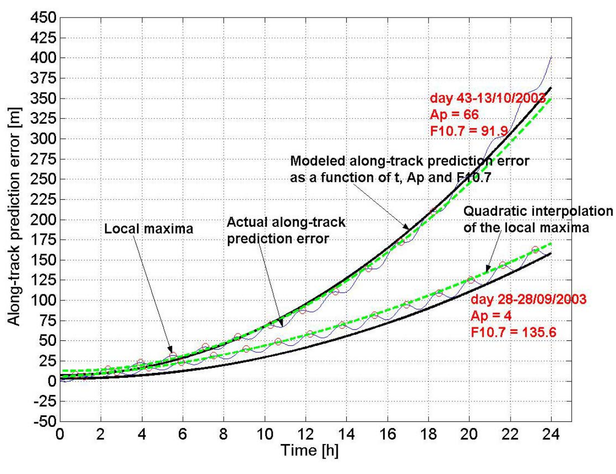

Fig. 2. Along-track error model estimation

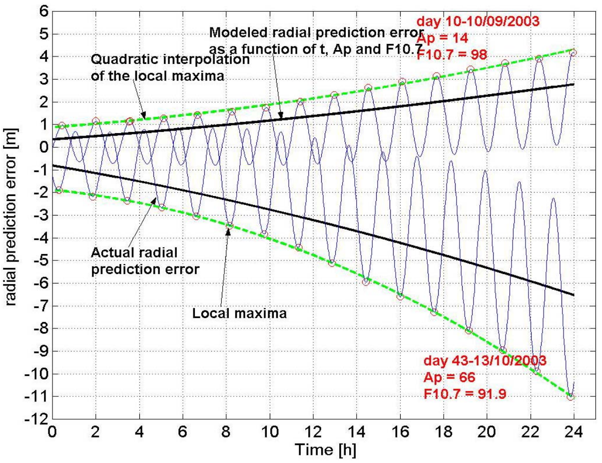

Fig. 3. Radial error model estimation

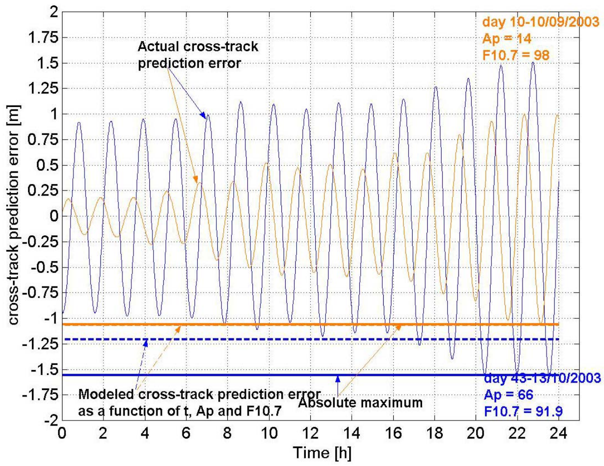

Fig. 4. Cross-track error model estimation

As an example two days are depicted in Figure 2, with the two associated along-track error profiles (blue line). First of all a quadratic interpolation (green line) of the local maxima (red circles) is carried out for each simulation's day. The along-track error quadratic approximation is then given by a linear regression of the coefficients population (black line). The same approach has been adopted for the radial component, whereas the cross-track error is better approximated by a constant function, see Figures 3 and 4.

5. IMPACT ON FLIGHT DYNAMICS OPERATIONS AND CONCLUSIONS

5.1 TS-X Nominal Baseline (±250 m)

The long-term prediction accuracy analysis shows that the maximum space prediction error amounts to 33m (3D, 1σ) after 1 day, 150m after 2 days, 364m after 3 days. The limited amount of available thruster pulses and the fact that the TS-X trajectory shall be confined within a tube with a 250m-radius defined w.r.t. the target orbit impose a 24 hours baseline for the ground-in-the-loop orbit control. The flight dynamics tasks (OD, OP and maneuver planning) must be a daily process. Two ground-contacts per day are necessary. One of them will be used for an HK-dump, the other one for the upload of a maneuver command, if necessary. The presented OP error model will be tuned during the mission and will be adopted in order to adapt the orbit control dead-bands, depending on the expected solar activity.

5.2 TS-X High Accuracy Baseline (±10 m)

The short-term prediction accuracy analysis shows that the maximum space prediction error amounts to 10m (3D, absolute) at 2 hours after an orbit determination, 20m at 8 hours, 30m at 12 hours. The differential interferometric baseline can only be achieved time to time as a single event. The minimum achievable baseline is driven by the OP error expected at the time of the accurate SAR data-take. With the present operational scenario the minimum realistic interferometric baseline is approximately 35 m.

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