ABSTRACT

The growing interest in earth observation missions equipped with space-borne optical and synthetic aperture radar (SAR) sensors drives the accuracy requirements with respect to orbit determination and control. Especially SAR interferometry with its capability to resolve the velocity of on-ground objects (e.g. for traffic monitoring, ocean currents and glacier monitoring) and to determine highly precise digital elevation models is of significant interest for scientific applications. These goals may be achieved using along-track and repeat-pass interferometry with a satellite formation, based on the precise orbit control of one satellite with respect to the osculating trajectory of the second satellite. Such a control concept will be realized by the German TerraSAR-X mission, with an expected launch in 2006, using a virtual formation, where a single satellite will be controlled in a tight manner with respect to a predefined osculating reference trajectory. This is very challenging, since common orbit disturbances, like for close twin formations, do not cancel out in this scenario. The predefined trajectory in the TerraSAR-X case could also be the orbit of a second satellite. The paper describes the generation of such a virtual reference orbit, discusses the ground-in-the-loop control concept and presents results from a long-term simulation.

1. INTRODUCTION

The importance of space-borne synthetic aperture radar imaging evolved over the past two decades. More and more SAR mission are carried out and drive the requirements on the SAR processing, the orbit determination and the orbit control. Different SAR modes and technologies have been developed to generate high geometric and radiometric resolution images from the earth. The main research and application areas for SAR imaging are forestry, cartography, geology and risk assessment for fire, flood and storm damages, [1]. Especially SAR interferometry applications are of high interest. There, phase differences of SAR images of the same scene from two antennas, separated in across-track direction, are measured and processed to get information about the terrain elevation. Along-track interferometry in contrast is evaluating the time lag between two SAR instruments separated in along-track direction to receive velocity information of objects on ground. This can be done using a single satellite performing repeat pass interferometry (RPI), like Envisat and the European Remote Sensing Satellites (ERS). RPI allows velocity measurements from slow moving objects (e.g. glaciers), because the second SAR image is taken, when the satellite flies over the scanned region again, exact after one repeat cycle (typical repeat periods are in the order of 10 – 30 days). To receive instantaneous velocity measurements, a twin satellite formation is needed. A high velocity resolution demands along-track baselines smaller than one kilometer, [2]. These short baselines imply a precise orbit control concept. Commercial exploitation of SAR images often drives a conservative mission approach, canceling out autonomous orbit control concepts. To cope with this, the design of a ground-in-the-loop precise orbit control concept is analyzed and implemented in an operational flight dynamics environment.

2. STUDY CASE AND ORBIT CONTROL REQUIREMENTS

2.1 Mission Characteristics of the Study Case

The German TerraSAR-X mission comprises a high resolution X-band radar satellite. TerraSAR-X data products will be used for scientific and commercial funding. TerraSAR-X is funded as a Public Private Partnership between German Aerospace Center, EADS Astrium and Infoterra. The launch of TerraSAR-X is scheduled for April 2006. It will be launched in a sun-synchronous repeat orbit with the characteristics as shown in Table 1.

Table 1. TS-X orbit

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mission orbit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orbit type</td>
<td>Sun-synch. repeat orbit</td>
</tr>
<tr>
<td>Repeat period</td>
<td>11 days</td>
</tr>
<tr>
<td>Repeat cycle</td>
<td>167 orbits in the repeat</td>
</tr>
<tr>
<td>Orbits per day</td>
<td>15 + 2/11</td>
</tr>
<tr>
<td>Equatorial crossing time</td>
<td>18:00±0.25h ascending pass</td>
</tr>
<tr>
<td>Eccentricity</td>
<td>0.0011±0.0012 frozen</td>
</tr>
<tr>
<td>Inclination</td>
<td>97.4438823</td>
</tr>
<tr>
<td>Argument of perigee</td>
<td>90°</td>
</tr>
<tr>
<td>Altitude at the equator</td>
<td>514.8 km</td>
</tr>
<tr>
<td>Semi-major axis</td>
<td>6892.9 km</td>
</tr>
</tbody>
</table>
The wet mass of TerraSAR-X is 1238.0 kg with a cross section area of 3.2 m². The surface area for solar radiation pressure is 10 m². The satellite bus consists of GPS receivers (single and dual frequency), star sensors, coarse earth sun sensor (CESS), inertial measurement units (IMU) and magnetometers as sensors. The actuators are magnetorquers, reaction wheels and a mono-propellant propulsion system. The mission active lifetime is 5.5 years. TerraSAR-X is a single satellite flying tightly controlled to an osculating reference trajectory, which could also be the orbit of another satellite. The generation of this virtual reference orbit is described in the following paragraph.

2.2 Reference Orbit Generation

The repeat orbit requirement poses tight constraints on the repeat cycle characteristics (exact and smooth ground-track repetition) and long term stability (passive eccentricity control) of the target trajectory. In the following, the steps needed to generate the reference orbit are shown:

1. Twoline element (TLE) orbit: based on the above mentioned orbit requirements and a given point of the ground track (e.g. separation point after launch), sets of TLEs are generated for the given point as well as for the subsequent ascending node. Using the TLEs of the ascending node, a set of osculating Keplerian elements is generated as initial guess for the next step.

2. Sunsynocronous orbit: The reference orbit has to be as realistic as possible, thus a state-of-the-art gravity model (Grace Gravity Model GGM01 of degree and order 120) is used to model the osculating trajectory. The initial estimated semi-major axis and inclination are varied to meet the same osculating longitude and latitude after one repeat cycle.

3. Frozen orbit: Mainly to reduce the number of maneuvers during the mission lifetime and to minimize the variation of altitude for successive passages over the same areas, a passive eccentricity control algorithm is used to get a “frozen” reference orbit.

4. Reference orbit: In order to meet exactly the same state vector in the Earth-fixed frame (Word Geodetic System WGS84) after one cycle, a non-linear optimization problem is defined and solved by the formulation of sequential linear least-squares subproblems. In particular a finite number of virtual maneuvers are spread over the reference orbit to decrease the stiffness, a cost function is defined as some measure of the velocity discontinuities and the optimization problem becomes the minimization (zeroing) of this function, subject to appropriate constraints. [3]. The optimized reference orbit is the main input for the flight dynamics orbit control system. It is stored and kept as an ephemeris. During the mission it can be recreated and updated with the actual TerraSAR-X orbit conditions.

2.3 Orbit Control Requirements

To fulfill the RPI requirements, TerraSAR-X has to be controlled in a tight manner with respect to the above mentioned reference orbit. For the control and the post check of the orbit control, a variable called space error \( \mathbf{E} \) was defined and introduced. The goal is to compare the reference and the real orbits in order to get a characteristic error variable and study a suitable control strategy. Figure 1 shows the conceptual representation of the space error.

Every orbital revolution of TerraSAR-X is divided in 36 equally spaced check points, at which the space error is evaluated. The space error \( \mathbf{E}_k \) for the check point \( k \) is defined as the vector difference between the position of the reference orbit and the real orbit at a time where the along-track component of the position difference is zero. It is not feasible to compare the check points from the reference orbit and the real orbit at the same absolute time, because small different orbital characteristics in along-track distance exist at this epoch. This component is cancelled out on board by a time correction algorithm using GPS data. To get the same conditions on ground, a search algorithm for the space error calculation has been developed, looking for the time \( t_{DT} \) at which this along-track difference is zero. At this point the space error is calculated.

The orbit control requirement for TerraSAR-X is to keep this space error within a deadband of ±250 m at every point in the osculating orbit. This guarantees the correct interferometric baseline for SAR imaging.
between the repeat passes. Mathematically the orbit control specification ($\pm 250$ m baseline) at check point $k$ can be expressed as $E_k \leq 250$, for any orbital revolution. The orbit control margin can be visualized as a tube defined around the reference orbit with a radius of 250 m. In Figure 2 two cross sections of the orbit tube are depicted for better comprehension. The magnitude of the space error must be smaller than 250 m during the entire mission.

![Fig. 2. Sections of the tube representing the orbit control requirements](image)

3. ORBIT PREDICTION ANALYSIS

A very important and influencing factor for a ground-in-the-loop orbit control concept is the orbit prediction accuracy. Due to the orbit conditions and the available ground station network there is no continuous upload and download capability to the spacecrafts. For the orbit control maneuver planning, the orbit has to be predicted from the time of the last ground station contact on, where the latest GPS measurements have been downloaded. The prediction error directly influences the space error and therefore the orbit control performance. To stay within the tight orbit tube with respect to the reference orbit, this prediction error has to be taken into account.

An orbit prediction accuracy analysis, [4] has been performed and the results are summarized in the following. As the prediction error mainly depends on the atmospheric density distribution, a satellite mission with an altitude close to the TerraSAR-X mission has been chosen. Precise orbits (position error of 5 cm 3d RMS 1 sigma) from the GRACE (Gravity Recovery and Climate Experiment) mission have been used as reference, to compare the predicted orbits with the real orbit and the prediction error was calculated with respect to the GRACE precise orbit. A data arc of 3.5 months from September 2003 to December 2003 has been considered, also covering the extreme high solar activity at the end of October 2003.

Every day of the 3.5 months data period an orbit determination over a 24 h GPS data arc is performed and the resulting state vector is propagated over an interval of three days. Natural orbit disturbances like geo-potential forces (gravity model up to degree and order 120), sun and moon, earth tides, atmospheric drag and solar radiation are considered by the state-of-the-art orbit prediction software [5], [6]. At each propagation day the predicted state vector is compared with the precise reference orbit at time intervals of one minute. The prediction error is the vector, defined by the difference between the real position of the reference orbit and the propagated position at the same absolute time. Table 2 shows the statistics of the maximum orbit prediction error splitted in the components of the orbital frame with the $z$-axis pointing towards the Earth center of mass (radial direction), the $y$-axis (cross-track direction) pointing in the direction of the orbital momentum vector and the $x$-axis (along-track direction) completing the right handed system. The maximum prediction errors over the one, two and three day prediction intervals are summarized within the reference data arc of 3.5 months and within a selected period of high solar activity (21st October/10th November 2003).

<table>
<thead>
<tr>
<th>All period</th>
<th>1 day Mean</th>
<th>1 σ</th>
<th>2 days Mean</th>
<th>1 σ</th>
<th>3 days Mean</th>
<th>1 σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radial, [m]</td>
<td>-0.2</td>
<td>11.1</td>
<td>-5.4</td>
<td>23.4</td>
<td>-10.4</td>
<td>40.7</td>
</tr>
<tr>
<td>Along, [m]</td>
<td>60</td>
<td>367</td>
<td>323</td>
<td>1835</td>
<td>1051</td>
<td>4287</td>
</tr>
<tr>
<td>Cross, [m]</td>
<td>-0.9</td>
<td>0.1</td>
<td>-1.3</td>
<td>1.5</td>
<td>-1.6</td>
<td>2.1</td>
</tr>
<tr>
<td>High-sol- activity</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Radial, [m]</td>
<td>-4.7</td>
<td>15.8</td>
<td>-14.1</td>
<td>32.9</td>
<td>-28.1</td>
<td>58.8</td>
</tr>
<tr>
<td>Along, [m]</td>
<td>160.4</td>
<td>528</td>
<td>948</td>
<td>2748</td>
<td>3210</td>
<td>6902</td>
</tr>
<tr>
<td>Cross, [m]</td>
<td>-0.9</td>
<td>1.0</td>
<td>-1.3</td>
<td>2.4</td>
<td>-1.4</td>
<td>3.3</td>
</tr>
</tbody>
</table>

The main prediction error contribution comes from the along-track component which is strongly influenced by the high uncertainties in the modeling of the aerodynamic drag. The nominal operational scenario for TerraSAR-X with a single ground station allows for two contacts per day. This implies that the orbit has to be propagated over a 24 h period and the predicted space error is calculated. If the predicted space error exceeds the control deadband, an orbit maneuver has to be planned. This concept is described in the next section. Within the propagation of the space error, the orbit prediction uncertainties have to be taken into account.

The behavior of the prediction error over the first 24 h after an orbit determination is studied. It is mainly a function of the propagation interval since the last orbit determination and the solar/geomagnetic activity. An error model has been generated in order to estimate in advance the prediction error depending on the solar/geomagnetic activity ($A_p$ and $F_{10.7}$ coefficients) and the time of propagation. For each component of the prediction error an appropriate polynomial
Approximation has been adopted and used to describe the error for each first day of prediction. Finally, a least-squares linear regression has been applied in order to correlate univocally the polynomial coefficients to $\Delta P$ and $F_{10.7}$. The polynomial coefficients are chosen during the mission, using up to date solar and geomagnetic data, and the prediction error is modeled and taken into account within the orbit control concept.

4. PRECISE ORBIT CONTROL

4.1 Ground-In-The-Loop Orbit Control System

The TerraSAR-X mission approach, the satellites Attitude and Orbit Control System (AOCSS) software and the actuator (thruster) hardware only allow for a ground-in-the-loop orbit control approach, instead of an autonomous orbit control system. There is a limited budget of thruster pulses for orbit control. Therefore the flight dynamics ground control system has also to be optimized to reduce the amount of thruster pulses over the mission active lifetime of 5.5 years. This cannot be done on board due to insufficient computational resources (e.g. one control planning run takes over 12 h on a 2.3 GHz Windows PC). Figure 3 shows the concept of the orbit control system. The space segment, mainly consisting of the TerraSAR-X satellite (depicted in red), provides the ground segment during ground station contacts with GPS data, like navigation solutions, code and carrier phase measurements, and AOCSS housekeeping data. The exact launch date defines the reference trajectory and is also an input for the flight dynamics system. Based on the GPS and housekeeping data an orbit determination is performed (orbit position accuracy of 2 m 3D RMS 1 sigma).

Inputs for the orbit determination and controller software are also external data, consisting of solar flux data, GPS auxiliary data and Earth rotation parameters from the International Earth Rotation Service (IERS). The controller checks the predicted space error over the next 24 h with the control deadband and calculates a time-tagged maneuver, which is uploaded in the next ground station contact. The TerraSAR-X flight dynamics system is based on a Linux PC cluster architecture with all software running autonomously, controlled by bash and Perl scripting. There is no manual interaction for the TerraSAR-X orbit control during the nominal mission phase.

4.2 Orbit Control Strategy

An analysis of the orbital variations induced by natural forces only, shows that the orbit will tend to diverge from its nominal position (the reference orbit): this produces a deviation, in relation to the target orbit, in certain mission-related physical parameters (known by convention as “operational” parameters) which are dependent on the orbital elements. Here, the operational parameters are the cross-track and the radial space errors and have been defined as the components $E_y$ and $E_z$ of the space error $E$. Orbit control may thus be defined as ensuring conformance with the nominal values of the operational parameters: this generally means that orbital corrections (orbital maneuvers) have to be generated several times during the satellite mission, by applying various forces to the satellite.

The evolution of the orbital parameters is studied. The cross-track space error $E_y$ mainly depends on the change of the orbital period due to atmospheric drag. The change of the semi-major axis and the Earth rotation contributes to a component in the cross direction of the space error $E_{y_0}$ at the ascending node (AN). Solar radiation pressure and sun/moon perturbations influence the inclination resulting in the change of $E_y$ at non-zero latitudes. It is important to note that for the TerraSAR-X orbit control, among the natural disturbance forces, the geo-potential is not playing an important role, because the satellite orbit and the target orbit models match almost perfectly (reference orbit optimization). The $E_y$-control is realized by in-plane maneuvers for changing $E_{y_0}$ and by out-of-plane maneuvers for the control of $E_y$.

The radial error contribution $E_z$ of the space error comes from changes in semi-major axis and eccentricity. There are no specific maneuvers planned for controlling $E_z$ because of the limited amount of thrust pulses. The radial error is controlled by distributing the $E_y$ in-plane control maneuvers over an optimized position within the orbit. Therefore the size of the in-plane maneuvers is driven by the $E_y$-control, whereas the location of the maneuvers is driven by the $E_z$-control.

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Fig. 3. Ground-in-the-loop orbit control scheme

The space error and the orbital elements deviations are calculated and handed over to the controller software.
In order to reduce the amount of thruster pulses, the time between consecutive in-plane maneuvers (maneuver cycle) has to be maximized. The approach for this optimization is explained in the following. If the predicted space error is going to exceed the control requirements, a maneuver with the corresponding needed velocity increment (delta-v) is calculated. The first guess of this in-plane delta-v is given by solving the Gauss equations, [7]. An iterative algorithm is refining this delta-v by a numerical propagation of the orbit with a Newton-search-approach. The target is to let the space error use the whole bandwidth of the control deadband limits. Emphasis within this process is given to the best possible modeling of the atmospheric conditions for the iteration interval. This is very important, because the size of the maneuver cycle significantly depends on the difference between the real atmosphere and the modeled one.

4.3 Orbit Control Implementation

The orbit control software outputs are mainly time-tagged maneuver commands, which are uploaded to the satellite. The control software can be subdivided in an out-of-plane maneuver (OOPM) planning branch and an in-plane maneuver (IPM) planning part.

For the OOPM a long term simulation over the mission active lifetime is triggered autonomous once per week. The orbit for this simulation is assumed to be drag free, which means, the satellite is drag controlled. A post-facto algorithm is used to plan the minimum necessary number of OOPM in order to keep the inclination difference between the real orbit and the reference orbit within the inclination deadband. The allowed inclination difference in the case of the TerraSAR-X mission is 0.0015°. With this inputs the number and size of the OOPMs are calculated and stored in a database.

The IPM planning calculates the predicted space error based on the latest orbit determination by comparing the predicted orbit and the target trajectory. The control tube requirements are checked and the control deadband is adapted based on the short-term space error prediction model (see section 3), taking actual solar and geomagnetic data into account. If the allowed control limits are reached an IPM is planned. The location of the maneuver is chosen to control the eccentricity in an optimal way, [8]. The corresponding delta-v is optimized with the concept as stated above. The OOPM are read in from the database and are considered in the IPM planning. Operational constraints like no-maneuver windows or special times and events can be included in the IPM process.

5. SIMULATION RESULTS

5.1 Simulation Environment and Setup

The operational orbit control software is coded in Fortran 90 and is implemented on cluster Linux platforms with a processing speed of 2.6 GHz. The different software modules are steered by scripting. The various outputs are analyzed using a Matlab/Simulink environment. A simulation run covers the mission active lifetime of 5.5 years and usually takes over 12 hours to be completed. The orbit propagation software is the same as mentioned above in the orbit prediction section. The numerical integration of the equations of motion is performed with a six-elements (position and velocity) state vector using a first-order differential equation. The integration method is an Adam-Bashforth-Moulton method for ordinary differential equations. The initial conditions are set to a state vector with an epoch in April 2006.

5.2 Numerical Results

The software is able to control the satellite over the mission active lifetime within the mission requirements for the orbit tube. Figure 4 shows the space error E for a 5 years simulation. The maximum excursions of the space error are plotted over the number of repeat cycles. It can be seen that E stays within the requirement of 250 m. Only some parts of the osculating orbit are out of the deadband. The orbit of TerraSAR-X is controlled within the deadband for 99.7 percent of all check points, required are only 67.5 percent of all orbit check points. Figure 5 is showing the operational parameters $E_y$ and $E_z$, the main components of the space error. The blue points are check points at non-zero latitudes, whereas the green points mark the errors at the ascending nodes.

![Fig. 4. Controlled space error within TerraSAR-X mission active lifetime](image)
The maneuver cycles for the first year of the simulation are represented in Figure 6. At the beginning the typical cycle interval for the IPM is in the order of 10 days, for the OOPM the cycle time is around 80 days. At the end of the mission the maneuver cycle for the IPM reduces to one maneuver per day, due to the increasing solar activity in the next solar cycle.

The total number of thrust pulses for IPM is 319 and for OOPM 27. The allowed IPM budget is 320±120 pulses (depending on the behavior of the real atmosphere) and 30 pulses for OOPM. The delta-v range for the IPM is from 1 cm/s at the beginning of the mission up to 6 cm/s at the end of the mission. The delta-v for the OOPM varies from 10 cm/s up to 35 cm/s.

6. SUMMARY AND CONCLUSION

A precise orbit control concept to keep a satellite in a tight manner with respect to an osculating target trajectory has been developed and studied. This allows for an accurate effective baseline recreation, which is needed in SAR formations to optimize interferometric image processing. The orbit control requirements for the TerraSAR-X have been presented and the impact on the flight dynamics system has been derived. For optimal SAR repeat pass interferometry the TerraSAR-X satellite has to be controlled within an orbit tube of 250m radius around a virtual reference trajectory, which also can be the orbit of a second satellite. The space error \( \mathbf{E} \), the major control variable for the orbit system, has been defined and explained. The results of an orbit prediction analysis, an important influence factor for the orbit control strategy, has been shown and discussed. The orbit prediction accuracy has a significant impact on the space error and drives the performance of the orbit control. Models for the prediction error components have been developed and implemented in the control concept. The ground-in-the-loop orbit system of TerraSAR-X has been depicted and the various system components have been explained and the orbit control strategy has been detailed. Numerical results from a long term simulation have been presented and show that the precise orbit control concept allows to keep a satellite tightly controlled to a target trajectory. Special emphasis has been given to the modeling of the real atmospheric conditions, influencing the aerodynamic drag acting on the satellite. Future work and knowledge have to be invested in the analysis of the control behavior in different atmospheric conditions, considering the whole range of solar activities. Additional strategies to cover this significant performance driver have to be developed to guarantee safe ground operations of close satellite formations performing optimum SAR interferometric imaging.

7. REFERENCES

5. NAVLIB/F90 - A Package Based Fortran 90 Library for Flight Dynamics Applications; FDS-GEN-0010; Issue 2.2; DLR-GSOC (2004).