ORBIT VARIATION AND MANEUVER ABOUT WSO/UV

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ABSTRACT

WSO/UV (World Space Observatory/Ultra Violet) is a spacecraft that will be located near the collinear equilibrium point L_2 of Sun-Earth space revolving around the Sun, it's almost fixed with respect to the Sun and the Earth and its motion is period or quasi-period (i.e., the halo or Lissajous trajectory) around L_2.

For circular restricted three-body problem, the motion near L_2 can be conditionally stable; the motion of small mass object P is periodic or quasi-periodic in the case of linear problem. In fact, the orbit of the Earth move around the Sun is elliptical (the orbital eccentricity is $e = 0.016$), considering this factor, period or quasi-period trajectory does exist or not? What's more, there are some other dynamical mechanism, such as gravity of large planets (Venus, Mars and Jupiter et al) and radial pressure et al, periodic or quasi-periodic trajectory associated with initial disturbance will change or not? It is concerned the maintenance.

In this article, problems described above will be discussed, discipline and range of the variation of the orbit after the orbit insertion of WSO with the complete dynamical model, and give the amount of fuel needed for the orbit maintenance and the plan of control.

1. INTRODUCTION

WSO/UV will be located at the one of the Earth opposite to the Sun and will rotate synchronously with the Earth revolving around the Sun. There are two advantages: one is the reduced effect of the Sun, another is the facility of transmitting observation data to the Earth. It's necessary to maintain WSO/UV in the vicinity of a special point fixed in a rotating coordinate system. Such special points exist in the restricted three-body system of the Sun-Earth(Moon)-particle system and we call such special points collinear Lagrangian equilibrium points. In this paper the Earth(Moon) is referred to as the centre of mass of the Earth and the Moon. The reason why WSO/UV can be kept in the vicinity of an equilibrium point is concerned with the stability of these points.

2. COLLINEAR EQUILIBRIUM POINTS OF THE RESTRICTED THREE-BODY PROBLEM

Due to the negligible mass of WSO/UV relative to the mass of the Sun or the Earth, the problem of the motion of WSO/UV moving under the gravitational influence of the Sun-Earth(Moon) pair is restricted three-body problem. In this restricted three-body system the Sun and the Earth(Moon) are the two primary masses and WSO/UV is the small particle with negligible mass. If the two primary masses are moving in circular orbit, the distance between them is constant and they move about their common centre of mass at a constant angular velocity. In these circumstances it is natural to the introduction of a rotating coordinate system – synodic coordinate system C-xyz (see Fig. 1). The origin C is the centre of the two primary masses P_1, P_2 and x-y plane is the orbital plane of the two primary masses moving around C, and the two masses always lie along x axis. P_1,P_2 and P denote the Sun, the Earth(Moon) and the small particle (WSO/UV), respectively. The equations of motion of the small particle P are

$$\ddot{x} - 2\dot{y} = \partial\Omega/\partial x, \quad \ddot{y} + 2\dot{x} = \partial\Omega/\partial y, \quad \ddot{z} = \partial\Omega/\partial z \tag{1}$$

$$\Omega = \frac{1}{2}\left[\left(x^2 + y^2\right) + \mu\left(1-\mu\right)\right] + \frac{1-\mu}{r_1} + \frac{\mu}{r_2} \tag{2}$$

Here we choose the dimensionless unit system as follows

$$\begin{cases} [M] = M_1 + M_2 \\ [L] = AU\left(\text{astonomy unit}\right) \\ [T] = \left[AU^3/G\left(M_1 + M_2\right)\right]^{1/2} = 58^d.132352 \end{cases} \tag{3}$$

where M_1 is the mass of the Sun and M_2 is the mass of the Earth(Moon), and μ is defined by $\mu = M_2/(M_1 + M_2)$.

Eqn. 1 has three collinear equilibrium solutions and two triangular equilibrium solutions. For the Sun-Earth(Moon) system, $\mu = 3.0404 \times 10^{-6}$ and the coordinates of 3 collinear equilibrium points are:

$$\begin{cases} L_1(x_1, y_1, z_1) = (\text{-}0.9899859817,0,0) \\ L_2(x_2, y_2, z_2) = (\text{-}1.0100752006,0,0) \\ L_3(x_3, y_3, z_3) = (1.0000012668,0,0) \end{cases} \tag{4}$$

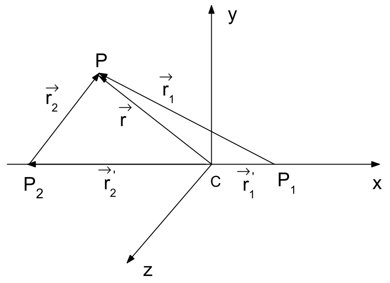

Fig. 1. Synodic coordinate system C-xyz

3. ORBITAL CHARACTER NEAR THE COLLINEAR EQUILIBRIUM POINTS

Since WSO/UV will be located near L_2, it's important to research the stability of the motion of small particle near three collinear equilibrium points with small position displacement. Given initial displacement $\Delta x = \xi, \Delta y = \eta, \quad \Delta z = \zeta$, then from Eqn. 1 we can obtain the equations of the motion with small displacement as follows:

$$\begin{cases} \ddot{\xi} - 2\dot{\eta} = \Omega_{xx}^0 \xi + \Omega_{xy}^0 \eta + \Omega_{xz}^0 \zeta + O(2) \\ \ddot{\eta} + 2\dot{\xi} = \Omega_{yx}^0 \xi + \Omega_{yy}^0 \eta + \Omega_{yz}^0 \zeta + O(2) \\ \ddot{\zeta} = \Omega_{zx}^0 \xi + \Omega_{zy}^0 \eta + \Omega_{zz}^0 \zeta + O(2) \end{cases}$$

(4)

where Ω_{xx}^0 et al. indicate that the 2nd order partial derivative of Ω with respect to positions are evaluated on the equilibrium points, $O(2)$ denotes 2nd order small disturbance. For three collinear equilibrium points, the linearized form of Eqn. 4 is

$$\ddot{\xi} - 2\dot{\eta} = \Omega_{xx}^0 \xi, \quad \ddot{\eta} + 2\dot{\xi} = \Omega_{yy}^0 \eta, \quad \ddot{\zeta} = \Omega_{zz}^0 \zeta \tag{5}$$

where

$$\Omega_{xx}^0 = 1 + 2C_0, \quad \Omega_{yy}^0 = 1 - C_0, \quad \Omega_{zz}^0 = -C_0 \tag{6}$$

$$C_0 = \frac{(1-\mu)}{r_1^3} + \frac{\mu}{r_2^3} \tag{7}$$

and r_1 (or r_2)denotes distance between the three equilibrium points and the Sun (or to the Earth(Moon)), also $r_1 = |x_i - \mu|$, $r_2 = |x_i + 1 - \mu|$. Solutions of Eqn. 5 are

$$\begin{cases} \xi = C_1 e^{d_1 t} + C_2 e^{-d_1 t} + C_3 \cos d_2 t + C_4 \sin d_2 t \\ \eta = \alpha_1 C_1 e^{d_1 t} - \alpha_1 C_2 e^{-d_1 t} - \alpha_2 C_3 \cos d_2 t + \alpha_2 C_4 \sin d_2 t \\ \zeta = C_5 \cos d_3 t + C_6 \sin d_3 t \end{cases} \tag{8}$$

where

$$\alpha_1 = \frac{1}{2}\left(d_1 - \Omega_{xx}^0 / d_1\right), \quad \alpha_2 = \frac{1}{2}\left(d_2 + \Omega_{xx}^0 / d_2\right) \tag{9}$$

$$\begin{cases} d_1 = \left(9C_0^2 - 8C_0\right)^{1/2} / 2 - \left(1 - C_0/2\right) \geq 0 \\ d_2 = \left(9C_0^2 - 8C_0\right)^{1/2} / 2 + \left(1 - C_0/2\right) \geq 0 \\ d_3 = \sqrt{C_0} \geq 0 \end{cases}$$

(10)

For the Sun-Earth(Moon) system, the value of d_1, d_2 and d_3 associated with L_1 、 L_2 and L_3 are

$$\begin{cases} L_1 : d_1 = 2.5326591755, d_2 = 2.0864535651, d_3 = 2.0152106639 \\ L_2 : d_1 = 2.4843167188, d_2 = 2.0570141899, d_3 = 1.9850748554 \\ L_3 : d_1 = 0.0028250833, d_2 = 1.0000026604, d_3 = 1.0000013302 \end{cases} \tag{11}$$

$C_1, C_2, C_3, C_4, C_5, C_6$ are integral constant. From Eqn. 8 we find the motion of small particle P with small disturbance near L_1 is unstable and P will be away from L_1 fast. On the other hand, selecting suitable initial conditions to make $C_1 = C_2 = 0$ can make the motion to be conditionally stable. The initial conditions are

$$\dot{\xi}_0 = \left(\frac{d_2^2}{\alpha_2 d_2}\right)\eta_0, \quad \dot{\eta}_0 = -\left(\alpha_2 d_2\right)\xi_0 \tag{12}$$

and Eqn. 8 deforms to

$$\begin{cases} \xi = \xi_0 \cos d_2 t + \eta_0 \sin d_2 t / \alpha_2 \\ \eta = -\alpha_2 \xi_0 \sin d_2 t + \eta_0 \cos d_2 t \\ \zeta = C_5 \cos d_3 t + C_6 \sin d_3 t \end{cases} \tag{13}$$

Apparently, motion described by Eqn. 13 is restricted to be near L_i and its trajectory is quasi-periodic. Eqn. 12 is the maneuver condition to maintain the WSO/UV near L_2 point.

4. PLOTS OF THE MOTION NEAR COLLINEAR EQUILIBRIUM POINTS IN SYNODIC COORDINATES SYSTEM

4.1 Motion near L_2

Only initial manoeuvre is implemented according to Eqn. 12 and the initial condition is $\xi_0 = \eta_0 = \zeta_0 = \dot{\xi}_0 = 0.0001$, the whole integrating interval is 126 days. Results (see Fig. 2 and Fig. 3) demonstrate a disperse trajectory. Hence, we can not maintain WSO/UV near the L_2 point with only initial manoeuvre is implemented.

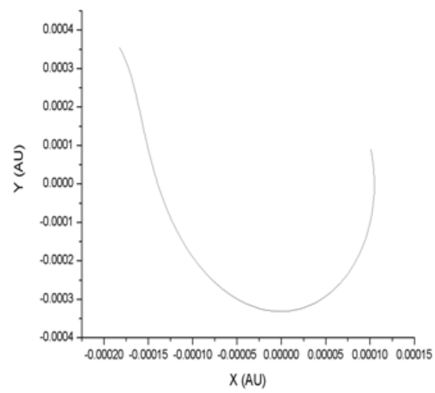

Fig. 2.

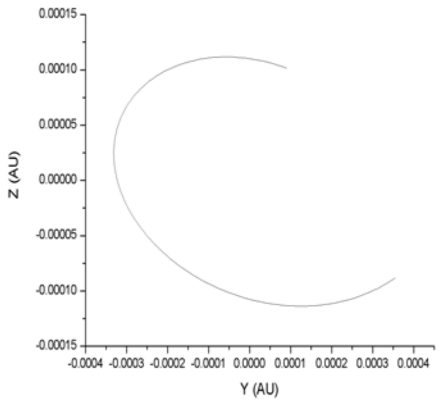

Fig. 3

4.2 Motion near L_3

Only initial manoeuvre is implemented, the specific initial condition is $\xi_0 = \eta_0 = \zeta_0 = \dot{\xi}_0 = 0.0001$, the whole integrating interval is 10000 days. Results (see Fig. 4 and Fig. 5) demonstrate better stability than that near L_2 and WSO/UV will remain near L_3 with only initial manoeuvre is implemented.

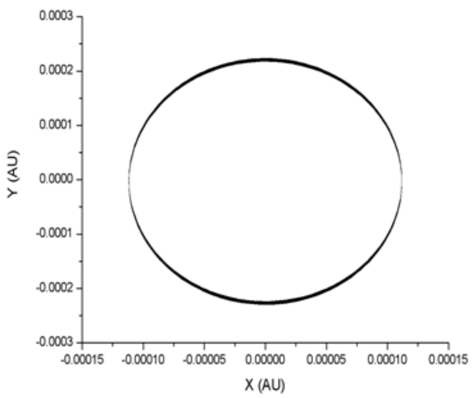

Fig. 4

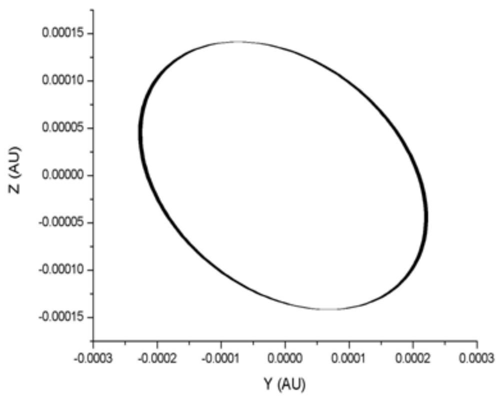

Fig. 5

4.3 Compare of the conditional stability near L_1, L_2 and L_3

From numerical results we can see that the conditional stability near L_3 is much better than that near L_1 or L_2. One possible explanation is given as follow: a small particle near L_3 will make up a restricted two-body system with the Sun, then the Earth(Moon) is just a small disturbing source and the quantity of the disturbance is very small (about 7.5×10^{-7} in dimensionless), so the motion of small particle could be stable near L_3; on the other hand, when a small particle is near L_1 or L_2, distance to the Earth(Moon) is much smaller and then the Earth(Moon) is not a small disturbing source any more, so the small particle will be away from the stable region fast. We also could explain the phenomenon from the following truth: the quantity of d_1 associated with L_3 is much smaller than that associated with L_1 or L_2 (see Eqn. 11), from Eqn. 8 we know that bigger quantity of d_1 makes less stability, thus we can understand the reason why small particle near L_3 is more stable than particle near L_1 or L_2.

5. ORBIT MAINTENANCE OF WSO/UV

Orbit manoeuvre according to Eqn. 12 can maintain WSO/UV near L_2.

Specific initial conditions according to Eqn. 12 are $\xi_0 = 1\times10^{-4}$, $\eta_0 = 1\times10^{-4}$, $\zeta_0 = 1\times10^{-3}$, $\dot{\zeta}_0 = 0$. To restrict the maximum velocity change ΔV is less than 1m/s, the interval is chosen as 4 days and the total manoeuvre is 912 times, thus WSO/UV will be stable near L_2 for 10 years and the main science objects will be satisfied. If the maximum ΔV is set to 2m/s, the interval can be much longer. Results have been shown in Fig. 6 and Fig. 7.

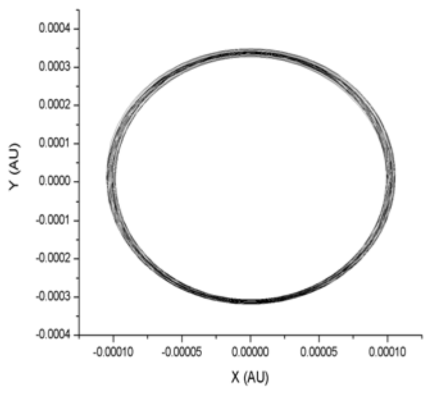

Fig. 6

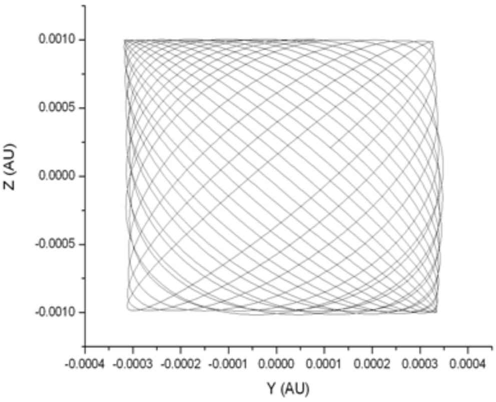

Fig. 7

6. ORBIT MAINTAIN IN REAL FORCE MODEL

There are two primary difference between the real force model and the simplified force model (circular restricted three-body model): firstly, the orbital eccentricity of the Earth(Moon) with respect to the Sun is about 0.017, that's to say the orbit of the Earth(Moon) is a time-variable ellipse; secondly, other solar planets' gravity has influence to the motion of WSO/UV. Since the quantity of the influence above is small enough, dynamical characters given by simplified force model are hold although there are some quantitive variation. It's still possible to remain WSO/UV near L_2 though limited orbit manoeuvre implemented.

6.1 Equations of motion in real force model

Apparently, it's suitable to choose the mean ecliptic coordinate system with respect to solar barycentre of J2000, orbit elements and ephemeris of solar planets including the Earth(Moon) are given in this system, in fact the mean orbit elements used in our calculation are from [3]. If $\vec{R}, \vec{R}_E$, and R_p denote the position vectors of WSO/UV, the Earth(Moon) and the planet, respectively. The equations of motion of WSO/UV is

$$\ddot{\vec{R}} = -(1-\mu)\frac{\vec{R}}{R^3} - \mu\left(\frac{\vec{\Delta}_E}{\Delta_E^3} + \frac{\vec{R}_E}{R_E^3}\right) - \mu_P\left(\frac{\vec{\Delta}_P}{\Delta_P^3} + \frac{\vec{R}_P}{R_P^3}\right) \quad (14)$$

where μ_p denotes the dimensionless mass of the planet, $\vec{\Delta}_E$ and $\vec{\Delta}_p$ are position vectors of WSO/UV with respect to the Earth(Moon) and the planet, they are defined by

$$\vec{\Delta}_E = \vec{R} - \vec{R}_E, \vec{\Delta}_P = \vec{R} - \vec{R}_P$$

(15)

6.2 Obtaining the conditions for manoeuvre

To maintain WSO/UV near L_2, conditions derived from Eqn. 12 are still available in the real force model, just notice that the parameters used in Eqn. 12 should be associated with the temporal value derived from the time-variable elements of the Earth(Moon).

A transformation of coordinate system is necessary in the calculation of conditions for manoeuvre. In the temporal synodic coordinate system, the barycentre of the Sun and the Earth(Moon) are still constant

$$\vec{r}_1' = (\mu, 0, 0)^T, \quad \vec{r}_2' = (\mu - 1, 0, 0)^T$$

(16)

the location of L_2 does not change in the dimensionless coordinate system and its coordinate is $x_2 = -1.010075$.

If $\vec{r}(x,y,z)$ and $\dot{\vec{r}}(x,y,z,t)$ denote the position and velocity in the temporal synodic system C-xyz, $\vec{R}$ and $\dot{\vec{R}}$ denote that in the J2000 mean ecliptic coordinate system, transformation from $(\vec{R}, \dot{\vec{R}})$ to $(\vec{r}, \dot{\vec{r}})$ is

$$\begin{cases} \vec{r} = R_z(f)(M)\vec{R} + \vec{r}_1' \\ \dot{\vec{r}} = R_z(f)(M)\dot{\vec{R}} + \dot{R}_z(f)(M)\vec{R} \end{cases}$$

(17)

$$\begin{cases} \vec{R} = (M)^T R_z(-f)(\vec{r} - \vec{r}_1') \\ \dot{\vec{R}} = (M)^T \left[R_z(-f)\dot{\vec{r}} + \dot{R}_z(-f)(\vec{r} - \vec{r}')\right] \end{cases} \quad (18)$$

$$\dot{R}_z(f) = \begin{pmatrix} -\sin f & \cos f & 0 \\ -\cos f & -\sin f & 0 \\ 0 & 0 & 1 \end{pmatrix} \dot{f}, \quad \dot{f} = 1 \quad (19)$$

$$(M) = R_z(\pi + \omega) R_x(i) R_z(\Omega)$$

(20)

where f, ω, i, Ω are elements of the Earth(Moon) with respect to the Sun and they are derived from mean orbit[3], $R_x(\theta)$ and $R_z(\theta)$ are defined by

$$\begin{cases} R_x(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix} \\ R_z(\theta) = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{cases} \quad (23)$$

6.3 Some results from numerical simulation for manoeuvre in real force model

The initial epoch is $MJD = 52544.5$ and the interval of maintenance is 10 years.

Initial conditions according to Eqn. 13 are $\xi_0 = 1\times10^{-4}$, $\eta_0 = 1\times10^{-4}, \zeta_0 = 1\times10^{-4}, \dot{\zeta}_0 = 1\times10^{-4}$, the manoeuvre interval is 4 days and the change of velocity ΔV is less than 1m/s, total manoeuvre is 912 times. Fig. 8 and Fig. 9 demonstrate the results.

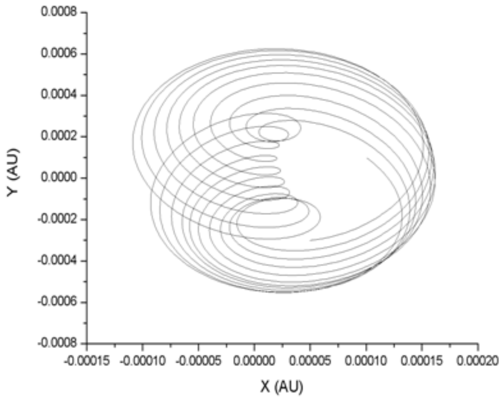

Fig. 8

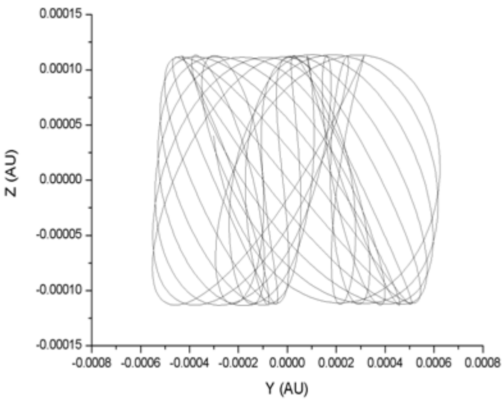

Fig. 9

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