UNCONTROLLED ATTITUDE MOTION OF THE ORBITAL STATION MIR IN THE LAST YEARS OF ITS FLIGHT

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ABSTRACT

We analyze the real attitude motion of the orbital station Mir during long periods of its uncontrolled flight without a crew in 1999 – 2001. The motion was reconstructed by processing on board measurements of the Earth magnetic field strength. Every day, there was a run of measurements. The run duration was about an orbital revolution. The measurement data, obtained during a run, were processed jointly by means of the least squares method and integration of the station attitude motion equations. The processing resulted in the estimation of the initial conditions of the motion as well as parameters of the equations. Daily processing allowed to carry out experimental investigation of several types of regular uncontrolled attitude motion.

1. MONITORING OF UNCONTROLLED ATTITUDE MOTION OF THE STATION

Starting with 1999, the orbital station Mir had long periods of the flight without a crew. The problem arose for its operation during them with minimum fuel consumption and with acceptable output of the solar arrays. As a rule, the onboard computer was switched off during those periods. This fact restricted possibilities of controlling the station attitude motion. The problem was solved by the use of two modes of uncontrolled motion. One was a rotation of the station around its longitudinal axis with the angular rate of 0.2-0.3 deg/s, the axis oscillating near the normal to the orbital plane. The second mode began as the biaxial rotation of the station around its longitudinal and transverse axes with angular rates of 0.15 deg/s, the initial orientation of the station being arbitrary. The biaxial rotation was also carried out in the cases when the absolute value of the station angular rate exceeded a preset limit (0.4 deg/s) or when the energy output was too small.

The first mode of the motion was used rarely. Two conditions should be met if one uses it. First, the Sun should not be near the orbital plane. Otherwise this mode did not guarantee an acceptable energy output. Second, the realization of the mode used the onboard computer. Thus, the first mode could be used only at the beginning of the uncontrolled flight. The required initial conditions of the attitude motion were realized before switching off the computer. Usually the first mode existed at the beginning of the next prolonged period of the uncontrolled flight, when the Sun was far from the orbital plane. The second mode of motion was the main one, since biaxial rotation of the station was realized without the onboard computer.

The attitude motion of the station during uncontrolled flight was checked by the measurements of the Earth's magnetic field strength. These measurements were carried out by the onboard triaxial magnetometers. Usually a run of measurements was carried out every day. Its duration was about 90 min. The actual attitude motion of the station on the time interval containing the obtained data was reconstructed by standard methods (see for example [1-3]). The measurements of the Earth's magnetic field and their processing formed the basis of the monitoring of uncontrolled motion of the station. The monitoring allowed us to investigate experimentally the evolution of the station attitude motion on longed time intervals. The goal was to study the stability of the first above mode and to reveal the stable final modes, which finished motions starting with biaxial rotation. Bellow some results of this investigation are presented. More details can be found in [3, 4].

2. DETERMINATION OF ATTITUDE MOTION OF THE STATION BY MEASUREMENT DATA

The station is assumed to be a rigid body whose center of mass performs a Keplerian elliptic geocentric motion. The elements of this motion arose from the radio tracking of the station. Three right-hand Cartesian coordinate systems were used in order to describe the attitude motion of the station and measurement data.

The structural coordinate system $Oy_1y_2y_3$ is rigidly connected with the station. Point O is the station's center of mass. Axis Oy_1 is parallel to the longitudinal axis of the base unit and is directed to the *Kvant* unit. Axis Oy_2 is directed along the asymmetric solar battery of the base unit.

Principal central axes of inertia of the station form the coordinate system $Ox_1x_2x_3$. Axes Ox_i make small angles with axes Oy_i $(i = 1, 2, 3)$. Axes Ox_1 and Ox_2

correspond to the minimum and maximum moments of inertia, respectively. The moments of inertia with respect to axes Ox_2 and Ox_3 are close to each other.

In the *orbital coordinate system* $OX_1X_2X_3$ axis OX_3 is directed along the geocentric radius vector of point O. Axis OX_2 is the normal to the plane of the orbit.

We specify the position of the system $Ox_1x_2x_3$ with respect to the orbital one by two sets of angles. The first set is formed by the angles γ, δ and β [1-3], introduced in the following manner. The system $OX_1X_2X_3$ can be transformed into the system $Ox_1x_2x_3$ by three successive rotations, (1) through angle $\delta+\pi/2$ about the OX_2 axis, (2) through angle β about new OX_3 axis, and (3) through angle γ about the new OX_1 axis coinciding with the Ox_1 axis. The angles γ and β specify the direction of the OX_2 axis in the system $Ox_1x_2x_3$, and the angle δ specifies the rotation of the station about this axis with respect to the orbital system. The second set is formed by the angles ψ, θ and φ determined in the following manner [3]. The system $OX_1X_2X_3$ can be transformed into the system $Ox_1x_2x_3$ by three consequent rotations, (1) through angle ψ about the OX_3 axis; (2) through angle θ about the new OX_2 axis; (3) through angle φ about the new OX_1 axis coinciding with the Ox_1 axis. The angles ψ and θ specify the direction of the Ox_1 axis in the orbital coordinate system and the angle φ specifies the rotation of the station about this axis.

We specify the position of the system $Ox_1x_2x_3$ with respect to the structural system by the angles α_c, β_c and γ_c. The structural system should be successively rotated through these angles about the axes Oy_2, Oy_3 and Oy_1 in order to be transformed into the system $Ox_1x_2x_3$.

We designate the matrix of transition from the $Ox_1x_2x_3$ system to the orbital system as $\| a_{ij} \|_{i,\,j=1}^{3}$. Here a_{ij} is the cosine of the angle between the axes OX_i and Ox_j. The elements of the matrix are expressed as functions of the angles ψ, θ, φ or the angles γ, δ β. We also designate the matrix of transition from the $Ox_1x_2x_3$ system to the structural system as $\| b_{ij} \|_{i,\,j=1}^{3}$. The matrix $\| b_{ij} \|$ was close to the unit one [3]. Its elements are expressed as functions of the angles α_c, β_c, γ_c.

The gravitational and restoring aerodynamic torques were taken into account in the equations of the station attitude motion. The equations had the form [3]:

$$\begin{gathered}
\dot\omega_1 = \mu(\omega_2\omega_3 - \eta a_{32}a_{33}) + \kappa(v_2p_3 - v_3p_2) \\
\dot\omega_2 = \frac{1-\lambda}{1+\lambda\mu}(\omega_1\omega_3 - \eta a_{31}a_{33}) + \frac{\lambda\kappa}{1+\lambda\mu}(v_3p_1 - v_1p_3) \\
\dot\omega_3 = -(1-\lambda+\lambda\mu)(\omega_1\omega_2 - \eta a_{31}a_{32}) + \lambda\kappa(v_1p_2 - v_2p_1) \\
\dot a_{11} = a_{12}\omega_3 - a_{13}\omega_2 - \omega_0 a_{31} \\
\dot a_{12} = a_{13}\omega_1 - a_{11}\omega_3 - \omega_0 a_{32} \\
\dot a_{13} = a_{11}\omega_2 - a_{12}\omega_1 - \omega_0 a_{33} \qquad (1) \\
\dot a_{31} = a_{32}\omega_3 - a_{33}\omega_2 + \omega_0 a_{11} \\
\dot a_{32} = a_{33}\omega_1 - a_{31}\omega_3 + \omega_0 a_{12} \\
\dot a_{33} = a_{31}\omega_2 - a_{32}\omega_1 + \omega_0 a_{13} \\
\lambda = \frac{I_1}{I_2}, \quad \mu = \frac{I_2 - I_3}{I_1}, \quad \eta = \frac{3\mu_e}{r^3} \\
\kappa = E\rho\sqrt{v_1^2 + v_2^2 + v_3^2}
\end{gathered}$$

Here, dots over letters denote differentiation on time t, ω_i and v_i $(i = 1, 2, 3)$ are the components of the absolute angular rate of the station and of its velocity relative to the Earth's surface in the $Ox_1x_2x_3$ system. Parameters p_i characterize the aerodynamic torque acting upon the station, ω_0 is the module of the absolute angular rate of the orbital coordinate system, I_i are the moments of inertia of the station with respect to the Ox_i axes, μ_e is the gravitational parameter of the Earth, r is the geocentric distance of the point O, ρ is the atmosphere density at this point, and E is a scale factor. When Eqs. (1) are numerically integrated, time is measured in terms of 1000 s, length – in 1000 km, velocity is expressed in km/s, and the unit of measuring the angular rate is 0.001 s^{-1}. The atmosphere density is calculated in kg/m^3 according to model [5], and $E = 10^{10}$. Elements a_{2i} are expressed through a_{1i} and a_{3i} by formulas $a_{21} = a_{32}a_{13} - a_{33}a_{12}$, etc.

The variables a_{1i} and a_{3i} are not independent, they are constrained by the relations of orthogonality of the matrix $\| a_{ij} \|$. For this reason, we expressed the initial conditions for a_{1i} and a_{3i} through the angles γ, δ and β. Thus, the initial conditions of a solution of Eqs. (1) were expressed by 6 quantities.

The parameters λ and μ in Eqs. (1), as well as the angles α_c, β_c and γ_c were known: $\lambda^\circ = 0.686$,

$\mu° = 0.168$, $\alpha_c° = -0.147$, $\beta_c° = 0.025$, $\gamma_c° = 0.011$ (the angles are expressed in radians). These values were used at the initial stage of processing the measurement data. Only the parameters p_i and initial conditions of the station motion were determined at this stage. At the second stage, we refined also the quantities λ, μ, α_c, β_c and γ_c. Below, we describe the results of the second stage.

The magnetometer measurements were processed in the following way. On the basis of the data obtained during a run, functions $h_i(t)$ $(i = 1, 2, 3)$ were constructed, specifying the components of the local strength $\mathbf{H}(t)$ of the Earth's magnetic field in the structural system and in the proper time interval $t_0 \le t \le t_1$. Using the Keplerian approximation of the station orbital motion and the analytic model of the Earth's magnetic field, components $H_i(t)$ $(i = 1, 2, 3)$ of $\mathbf{H}(t)$ in the orbital systems were calculated. The sets of functions $h_i(t)$ and $H_i(t)$ should be linked by certain relations. This condition allows to formulate a criterion for finding the solution of Eqs. (1), which approximates the actual station motion on the interval $t_0 \le t \le t_1$. Namely, the solution, which minimize the functional

$$\Phi = \sum_{i=1}^{3} \left\{ \sum_{n=0}^{N} [h_i(\tau_n) - \hat{h}_i(\tau_n)]^2 - (N+1)\Delta_i^2 \right\} +$$
$$\varepsilon_1 \left(p_1^2 + p_2^2 + p_3^2 \right) + \varepsilon_2 [(\lambda - \lambda°)^2 + (\mu - \mu°)^2 +$$
$$(\alpha_c - \alpha_c°)^2 + (\beta_c - \beta_c°)^2 + (\gamma_c - \gamma_c°)^2] \quad (2)$$
$$\Delta_i = \frac{1}{N+1} \sum_{n=0}^{N} [h_i(\tau_n) - \hat{h}_i(\tau_n)]$$
$$\hat{h}_i(t) = \sum_{j,k=1}^{3} H_j(t)\, a_{jk}(t)\, b_{ik}, \quad \tau_n = \frac{(N-n)t_0 + nt_1}{N}$$

is a sought approximation. Here, Δ_i are the constant shifts in the measurement data, $\varepsilon_1 = 0.001$, $\varepsilon_2 = 0.01$. The parameters ε_1, ε_2 are expressed in measurement units of physical quantities used in integration of Eqs. (1). These parameters were used for taking into account a priori values of the station parametrs. As a rule, there was $N = 60$. Functional (2) was considered as a function of 14 variables: 6 initial conditions at the instant t_0 and 8 model parametrs. Minimization of Φ was carried out by Levenberg- Marquardt method. The maximal error in determining the station attitude in described way did not exceed $4°$. The check of accuracy was carried out by using star sensors measurements.

Sometimes, neighboring measurement runs were separated by a few orbit revolutions. Measurement data of those runs could be smoothed by the same solution of Eqs. (1). Different magnetometers were used usually in such runs, therefore the method of processing were modified. The functions $h_i(t)$ were constracted and the shifts Δ_i were determined for each interval separately.

3. ROTATION ABOUT THE AXIS OF THE MINIMAL MOMENT OF INERTIA

Let the station orbit be circular, axis Ox_1, be the axis of material symmetry of the station, and the atmosphere be absent. Then the station attitude motion can be described by Eqs. (1), in which $\omega_0 = \text{const}$, $\eta = 3\omega_0^2$, $\mu = p_1 = p_2 = p_3 = 0$. These equations admit a family of partial solutions which, in terms of the angles ψ, θ, and φ, can be written in the form

$$\psi = \pm\frac{\pi}{2}, \quad \theta = 0, \quad \varphi = (c_1 \mp \omega_0)t + c_2 \qquad (3)$$
$$\omega_1 = c_1, \quad \omega_2 = \omega_3 = 0.$$

Here, c_1 and c_2 are arbitrary constants. Family (3) is called a cylindrical precession. It describes the station stationary rotatations around the axis Ox_1 directed along the axis OX_2, i.g. the normal to the orbit plane.

The shortcoming of a cylindrical precession consists in its instability for safficiently small $|c_1|$. The smaller λ, the greater the module of border (with respect to stability) values of c_1. This fact does family (3) useless for prolonged satellites with $\lambda << 1$. For the station Mir the border values were acceptable. For $\lambda = 0.69$, $\psi = \pi/2$, solutions (3) are instable exponentially at $-0.13\ \text{deg/s} < c_1 < 0.18\ \text{deg/s}$ [6].

The mode, based on family (3), was used four times; September 7 – 24, 1999, February 10 – 19 and June 17 – 22, 2000, January 28 – February 2, 2001.The examples of reconstruction of the station motion in this mode are presented in Figs. 1 and 2. Here and below it was assumed that $t_0 = 0$; the actual values of t_0 are shown in the figure captions. Figs. 1 and 2 are naturally separated in three parts, the left, middle, and right ones. The right parts illustrate the concordance of the functions $h_i(t)$ and $H_i(t)$ by the extremal of functional (2). The solid lines are the plots of functions $\hat{h}_i(t)$, the markers correspond to the points $(\tau_n, h_i(\tau_n) - \Delta_i)$ $(n = 0, 1, \ldots, N)$. Quantitatively the fit of these functions is characterized by the standard deviation $\sigma = \sqrt{\Phi_{\min}/(3N-14)}$, where $\Phi_{\min}$ is the value of functional (2) at the extremal. The values of σ are shown in the figure captions. The middle parts of the figures present the plots of time behavior of the components ω_i of the station's absolute angular rate, and the left parts of the figures present the plots of time behavior of the angles θ, ψ and of the angle $\Lambda = \arccos|\cos\theta\sin\psi|$ between the Ox_1 and OX_2 axes. Both figures were obtained as a result of the joint processing of two neighboring measurement runs.

Fig. 1 presents the segment of the first use of the mode considered, Fig. 2 – the segment in the final part of the last use. Large maximum values of Λ in Fig. 2 can be explained by a larger atmospheric drag, because at that time the orbit of the station was considerably lower than in 1999. Maximum values of Λ in the initial part of the last use did not exceed $28°$ [4].

Aerodynamic torque had appreciable influence on the evolution of the quantities

$$\varpi_1 = \frac{1}{t_1 - t_0}\int_{t_0}^{t_1}\omega_1 dt\,, \quad \Lambda_m = \max\Lambda(t)\ (t_0 \le t \le t_1)\,.$$

This evalution during the first three applications of the mode was described in [3]. The first and third applications were the most interesting. The first application was the most prolonged. The evolution of ϖ_1 during it was very fluent, parabolic. At first, this quantity increased from the initial value -0.244 deg/s (1999.09.07) to the value -0.188 deg/s (1999.09.13), then it decreased to the value -0.346 deg/s (1999.09.24). The quantity Λ_m decreased (not monotonically) from the ininial value $20°$ (1999.09.07) to the value $13°$ (1999.09.17) and became stabilize near this value.
During the third application, the quantity ϖ_1 increased monotonically from the value -0.143 deg/s (2000.06.17) to the value -0.107 deg/s (2000.06.22). The quantity Λ_m decreased from the value $31°$ (2000.06.17) to the value $19°$ (2000.06.20, in that measurement interval $\varpi_1 = -0.127$ deg/s). Than it increased to $53°$ (2000.06.22). To all appearances, the border (with respect to stability) value of ω_1 was passed (2000.06.20).

4. ROTATION ABOUT THE AXIS OF THE MAXIMAL MOMENT OF INERTIA

As it was mentioned above, when the output from the solar arrays of the station in its uncontrolled flight became small enough, the biaxial rotation about its longitudinal and transverse axes was applied. Both angular rates was equal 0.15 deg/s, the orientation of the station was not taken into account. In particular, all cases of the use the mode described in the previous Section ended in this way. Starting with the biaxial rotation, the attitude motion of the station changed spontaneously during several weeks, but sometimes regularity was revealed in this variation. An example of a regular evolution of the attitude motion of the station is presented in [3]. The final motion was the rotation of the station around its axis of the maximal moment of inertia, namely, the Ox_2 axis, which oscillated near the normal to the orbital plane.

That segment of uncontrolled motion was lasted from 1999.09.24 to 1999.12.07. The motion at the end of it is shown in Fig.3. The figure is similar to Figs. 1, 2, but its left plots present time behavior of the angles γ, β, and of the angle $\Psi = \arccos|\cos\gamma\cos\beta|$ between the axes Ox_2 and OX_2. The angular rate ω_2 did not reveal any evolution during the last two weeks of that segment. The discribed final motion was not observed always (it is possible, the time interval was not sufficiently long for its realization), but it existed before the station's deorbiting. Fig. 4 shows the station's motion in the last days of its uncontrolled flight. The station angular rate was very large at that time – certain limitations were canceled in view of the end of the flight. Five days before that time, 2001.03.16, the motion was similar: extremal values of ω_1 and ω_3 were practically the same, extremal values of γ and β were a few degrees greater, but the quantity ω_2 was less. It oscillated within the limits 0.750 – 0.964 deg/s. Twisting of the station was caused by the aerodynamic torque.

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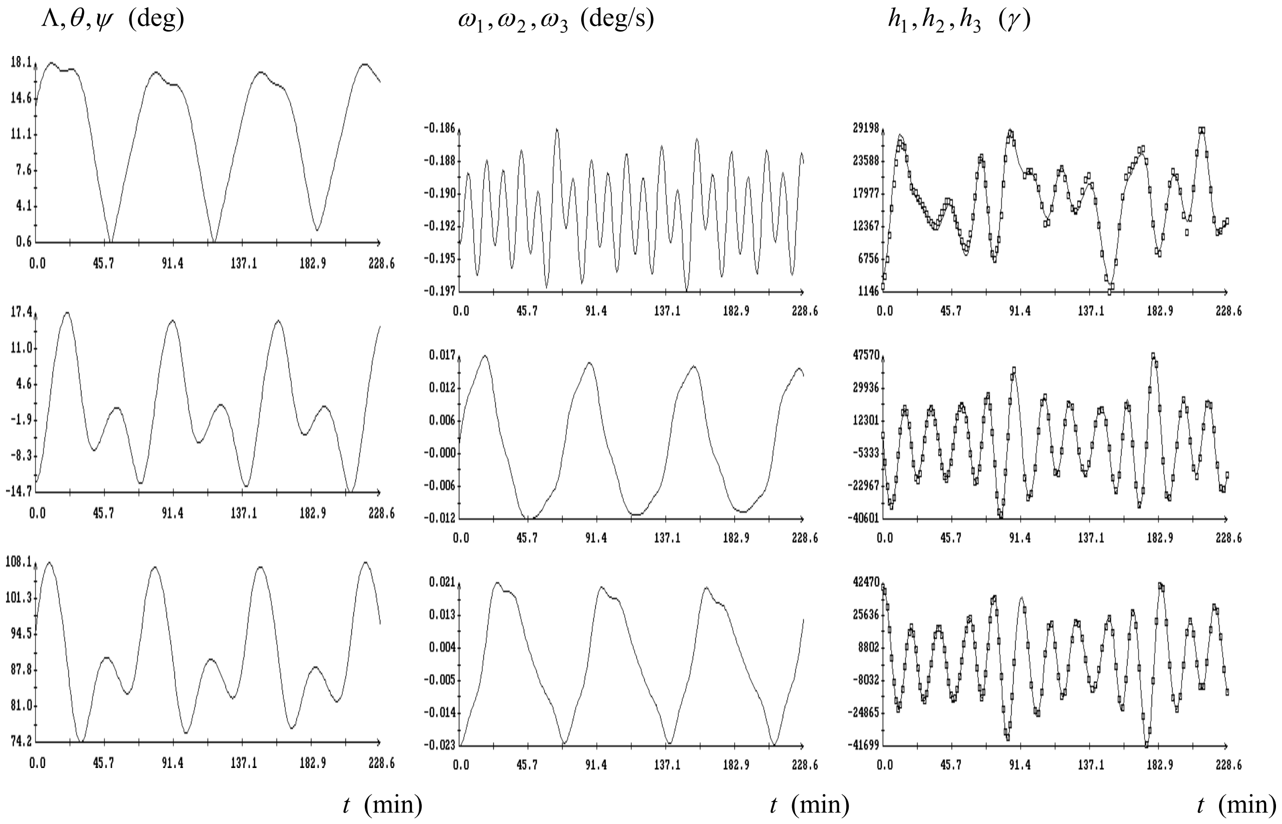

Fig. 1. The instant $t = 0$ on the plots corresponds to 15:38:17 Moscow Legal Time 12.09.1999, $\sigma = 856\gamma$.

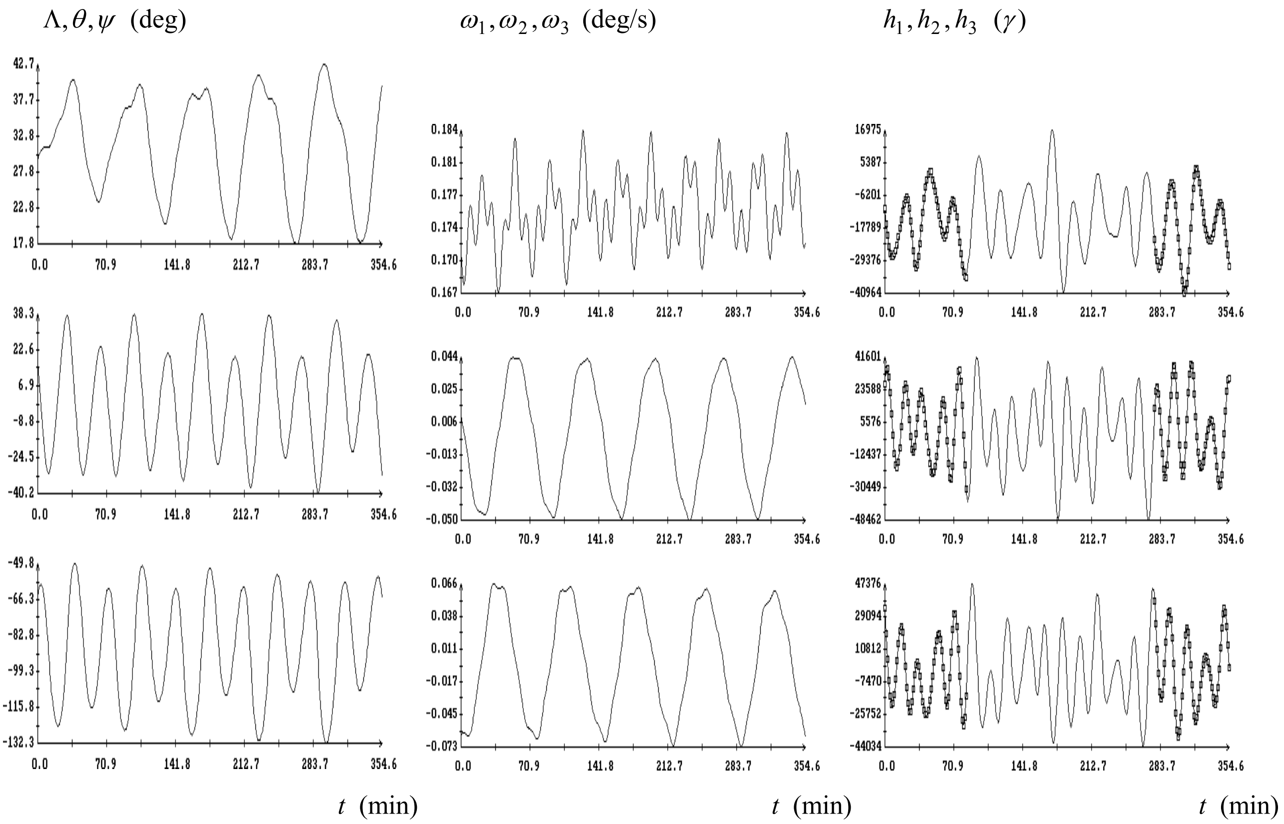

Fig. 2. The instant $t = 0$ on the plots corresponds to 01:32:33 Moscow Legal Time 02.02.2001, $\sigma = 806\gamma$.

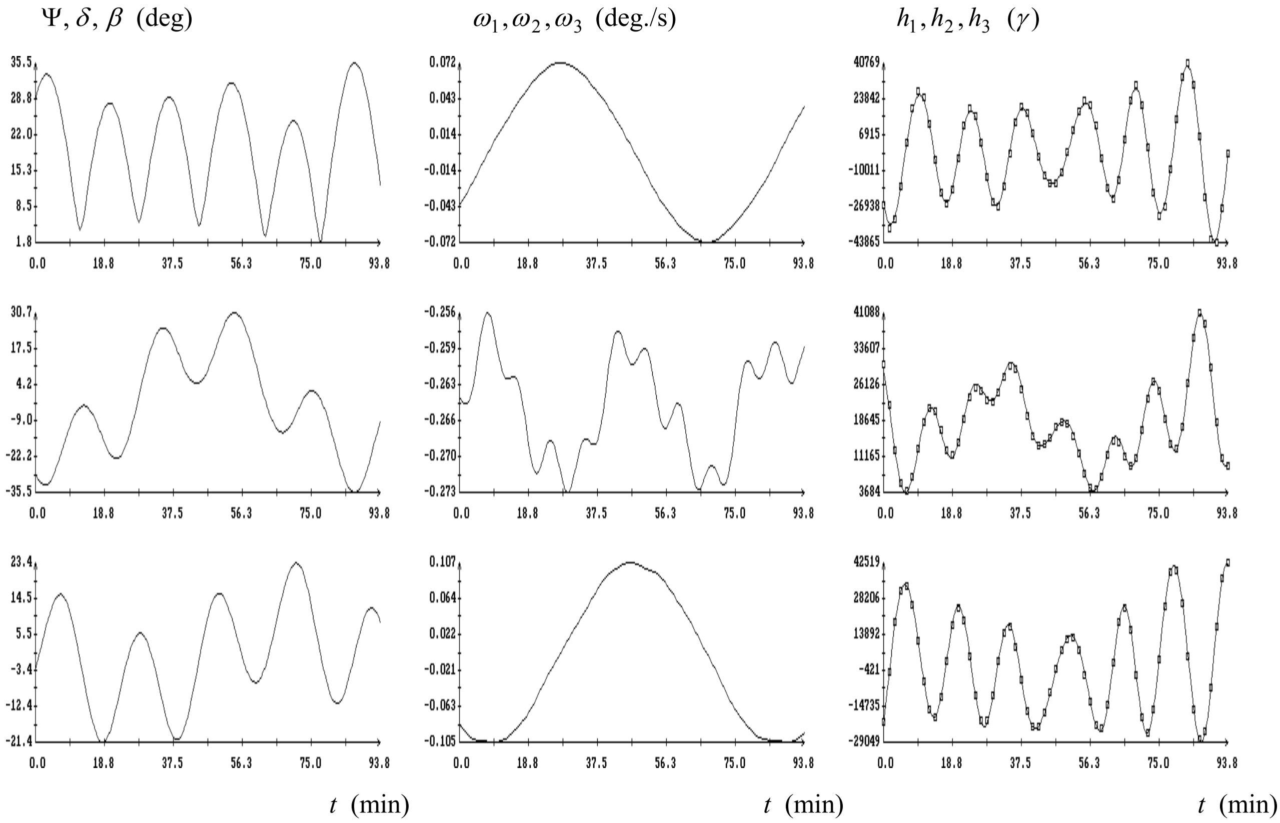

Fig. 3. The instant $t = 0$ on the plots corresponds to 07:54:02 Moscow Legal Time 07.12.1999, $\sigma = 813\gamma$.

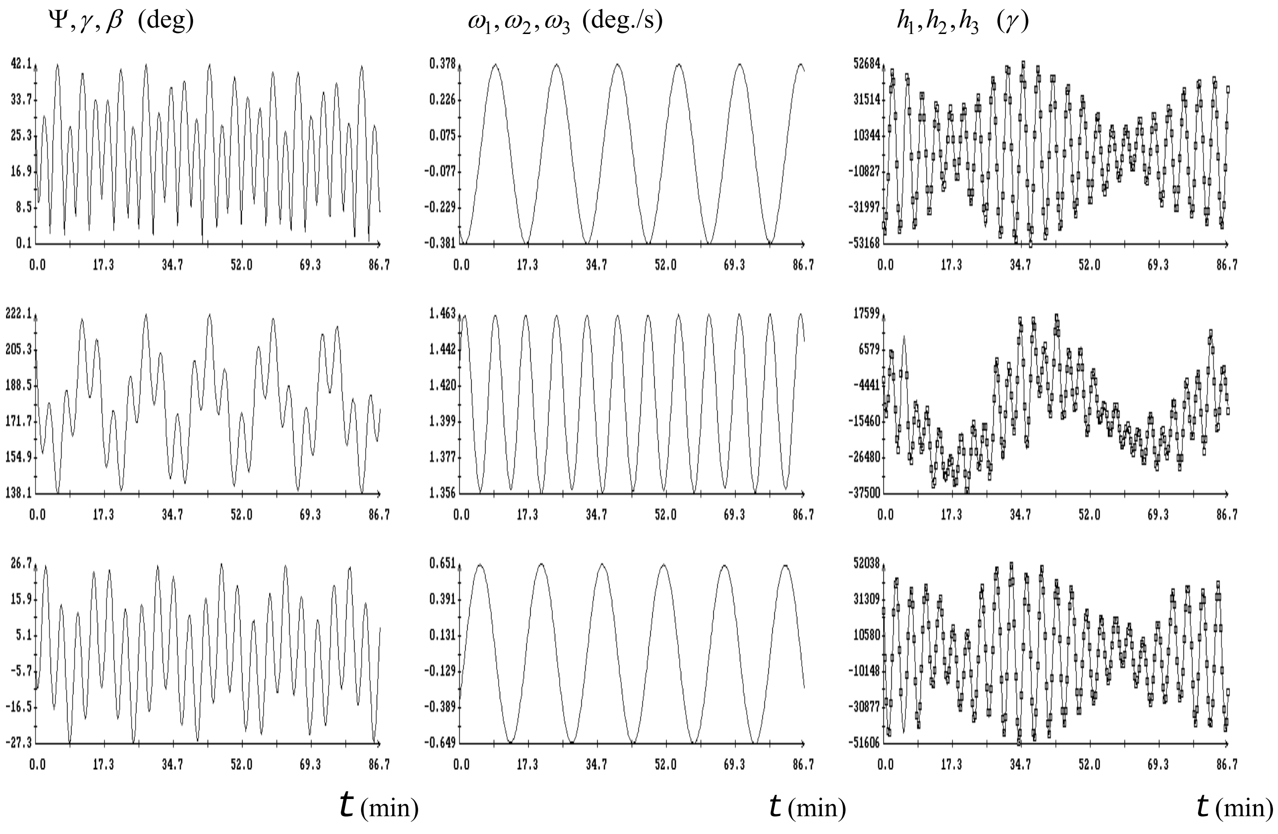

Fig. 4. The instant $t = 0$ on the plots corresponds to 12:01:12 Moscow Legal Time 21.03.2001, $\sigma = 943\,\gamma$.