LINEARIZING ASSUMPTIONS AND CONTROL DESIGN FOR SPACECRAFT FORMATION FLYING MANEUVERS

Kalyan C. Kappagantula[(1)], John L. Crassidis[(2)]

[(1)] *Iowa State University, Mechanical Engineering Department, Ames, Iowa 50011-2161, United States*
E-mail: kck@iastate.edu

[(2)] *University at Buffalo, Dept. of Mech. & Aero. Eng., Amherst, New York 14260-4400, United States*
E-mail: johnc@eng.buffalo.edu

ABSTRACT

In this paper the validity of neglecting the relative effect of the gravitational force of the Earth on a formation of spacecraft is studied. This relative effect is treated as an unknown disturbance acting on the system and all control laws are designed using a linear model that neglects this effect. A previously designed simple linear feedback controller is tested under different conditions using the linear model and the full nonlinear model that includes the gravitational force. All tests are carried out in the presence of saturation limits. The results show that the linear controller exhibits oscillations in the transient response and poor robustness under certain conditions. It also exhibits a high saturation tendency, thereby leading to increased fuel consumption. This controller also causes a high rise in the velocity errors at ordinary values of the gains. Based on the behavior of this controller, new controllers are proposed that overcome these drawbacks without any need for modifying the gains. The controllers, when tested under saturation limits exhibit high robustness characteristics due to their low saturation tendency and nearly eliminate oscillations in the transient response. Since these controllers operate under low control forces for a greater duration of the maneuver, they reduce the fuel required for the process. Simulation results are provided to show the effectiveness of these new controllers.

1. INTRODUCTION

Spacecraft formation flying is a key technology for space missions of the future. Relative navigation permits more flexibility in the mission goals of these formations. The reference point, which can be a physical satellite of the formation or a moving point in space about which the other satellites orbit, is called the chief while the orbiting satellites are known as the deputies. Formation flying of spacecraft comprises two kinds of problems. One involves rendezvous, docking, restructuring of or acquiring a formation, while the other involves maintenance of the relative orbit in the presence of disturbance forces. The former type involves steering a deputy satellite with respect to the chief from an initial state to a required final state. In these type of problems, the relative effect of the gravitational force of the Earth would be small compared to the control forces. This would make one suspect that when a reasonably robust controller is used, the relative effect of this force may be neglected for the duration of the control without affecting the system stability, thus reducing the nonlinear relative dynamics equations to simple linear ones.

The present work is based on [1], in which the authors assume that in a control dominated environment, the gravitational force of the Earth can be neglected. In [1] a simple feedback controller with some modifications, along with a saturation inequality, is proposed. Our present work addresses the various issues that arise under these assumptions. It should be noted that unlike the work in [1], the current work incorporates relative gravity models and also tests the consequences of neglecting the relative gravitational force when considering two satellites. Neglecting the relative effect of gravity in designing the control also means that the control must be robust in order to work in real-world situations. This fact is used as a basis to design controllers that are more robust to gravitational and disturbance inputs. A good way to impart robustness into a stable control design with saturation limits is to reduce the possibility of saturation, which is the approach taken here.

The paper is organized as follows. First, the simplified relative dynamics equation that neglects the effect of gravity is reviewed and the nominal control design of [1] is shown. Then, a new nonlinear controller is proposed, as well as another controller that helps to reduce saturation. Finally, simulation results are shown that compare all of the aforementioned control designs.

2. CONTROL DESIGN

2.1 Simplified Relative Dynamics Equations

We first present a simplified relative dynamics equations for the purpose of stability analysis. Since we are interested in the relative dynamics, all the vector quantities have been expressed in the Hill reference frame [2] of the chief satellite. It is assumed that there is no dichotomy in the orientations of the chief and the deputy and that the deputy maintains its orientation with respect to the chief during the maneuver.

The relative dynamics of the linearized model, which neglects the relative gravitational force of the Earth, can be written as [1]

$$D^2\mathbf{e} = D^2\mathbf{h} + \frac{\mathbf{u}_c}{M_c} - \frac{\mathbf{u}_d}{M_d} \tag{1}$$

where $\mathbf{e}$ is the position tracking error, $\mathbf{h}$ is the specified deviation that determines the relative orbit of the deputy with respect to the chief, $\mathbf{u}_c$ and $\mathbf{u}_d$ are the control inputs of the chief and deputy, respectively, and M_c and M_d are constants. The operator D^2 is defined as

$$D^2(.) = \ddot{(.)}_c + \boldsymbol{\alpha}_{cn} \times (.) + 2\boldsymbol{\omega}_{cn} \times \dot{(.)} + \boldsymbol{\omega}_{cn} \times (\boldsymbol{\omega}_{cn} \times (.)) \tag{2}$$

where $\boldsymbol{\alpha}_{cn}$ denotes the rate of change of the angular velocity, $\boldsymbol{\omega}_{cn}$, of the chief with respect to the reference frame (see [3] for more details). For circular orbits, the constant angular velocity of the chief satellite is

$$\boldsymbol{\omega} \equiv \boldsymbol{\omega}_{cn} = \left[0,\ 0,\ \sqrt{\frac{\mu}{r_c^3}}\right]^T \tag{3}$$

where r_c is the radius of the circular orbit of the chief and the angular velocity is expressed in the chief's Hill frame. The desired relative distance between the chief and the deputy satellites can be assumed to be constant during the duration of the control. With these assumptions, the relative dynamics equation reduces to the very simple equation given by

$$\ddot{\mathbf{e}} + 2[\boldsymbol{\omega}\times]\dot{\mathbf{e}} + [\boldsymbol{\omega}\times]^2\mathbf{e} = \mathbf{w} - \frac{\mathbf{u}_d}{M_d} \tag{4}$$

where

$$\mathbf{w} = D^2\mathbf{h} + \frac{\mathbf{u}_c}{M_c} \tag{5}$$

and $[\boldsymbol{\omega}\times]$ denotes the cross product matrix [2], and all derivatives are taken with respect to the chief.

2.2 Linear and Nonlinear Controllers

We denote the simple feedback controller used in [1] as C1. It can be written as

$$\mathbf{u}_d = M_d(K_1\mathbf{e} + K_2\dot{\mathbf{e}} + \mathbf{w}) \tag{6}$$

where K_1 and K_2 are positive definite. It will be seen in the results that for the case of the saturated controller C1, since the initial position error is much higher than the velocity error, the correction of the position error leads to a high rise in the velocity error before saturation occurs in the opposite direction in order to reduce the velocity error. To prevent this phenomenon, a new controller is designed here by increasing the contribution from the velocity error terms. This controller is written as

$$\mathbf{u}_d = M_d[K_1\mathbf{e} + K_2\dot{\mathbf{e}} + K_3(\dot{\mathbf{e}}^T\dot{\mathbf{e}})\dot{\mathbf{e}} + \mathbf{w}] \tag{7}$$

where K_3 is positive definite. For simplicity, the feedback gains are chosen to be diagonal with $K_1 = k_1 I_{3\times3}$, $K_2 = k_2 I_{3\times3}$ and $K_3 = k_3 I_{3\times3}$, where k_1 k_2 and k_3 are positive. Substituting the controller defined by Eq. (7) into Eq. (4) and neglecting the higher powers of the components of the angular velocity $\boldsymbol{\omega}$ in relation to K_1, the relative dynamics of the system become

$$\ddot{\mathbf{e}} + \{K_2 + 2[\boldsymbol{\omega}\times]\}\dot{\mathbf{e}} + K_3(\dot{\mathbf{e}}^T\dot{\mathbf{e}})\dot{\mathbf{e}} + K_1\mathbf{e} = \mathbf{0} \tag{8}$$

A candidate Lyapunov function is chosen as

$$V_L = \frac{(\mathbf{e}^T K_1 \mathbf{e}) + (\dot{\mathbf{e}}^T\dot{\mathbf{e}})}{2} \tag{9}$$

where K_1 is a 3×3 matrix. Note that V_L is positive definite when K_1 is positive definite. Taking the time derivative of V_L we obtain

$$\dot{V}_L = \dot{\mathbf{e}}^T(K_1\mathbf{e} + \ddot{\mathbf{e}}) \tag{10}$$

The controller C2 leads to stability since the derivative of the candidate Lyapunov function, after substituting Eq. (8) into Eq. (10), becomes

$$\dot{V}_L = -\dot{\mathbf{e}}^T\{K_2 + 2[\boldsymbol{\omega}\times]\}\dot{\mathbf{e}} - \dot{\mathbf{e}}^T\{K_3(\dot{\mathbf{e}}^T\dot{\mathbf{e}})\}\dot{\mathbf{e}} \tag{11}$$

which is negative semi-definite when K_2 and K_3 are positive definite. Asymptotic stability follows from the global invariant set theorem [4].

2.3 Controller to Reduce Saturation

In this section we design a controller with the intention of reducing the possibility of saturation. As seen in the previous section, when the gain matrices K_1, K_2 and K_3 are constants and positive definite, the structure of the controller in Eq. (7) gives rise to asymptotic stability. Using that as our basis, the basic structure of the controller C2 is retained but the constant feedback gain matrices are replaced by variable ones. The following controller is proposed:

$$\mathbf{u}_d = M_d\left(\frac{K_1}{1+(\mathbf{e}^T\mathbf{e})^n}\,\mathbf{e} + \frac{K_2 + K_3(\dot{\mathbf{e}}^T\dot{\mathbf{e}})}{1+(\dot{\mathbf{e}}^T\dot{\mathbf{e}})^m}\,\dot{\mathbf{e}} + \mathbf{w}\right) \tag{12}$$

where n and m are positive real numbers less than 1 and all the gain matrices are positive definite. Note that in Eq. (12), when the tracking error is large, the denominator is large, which balances out one or more of the force components in the numerator, thereby reducing the possibility of saturation. The constant 1 is added to avoid division by zero when the tracking error is zero. The reason for choosing the second term in the control structure goes along similar lines. With this control structure, the relative dynamics equation, after substituting Eq. (12) into Eq. (4), can be written as

$$\ddot{\mathbf{e}} + \frac{K_2 + K_3(\dot{\mathbf{e}}^T\dot{\mathbf{e}})}{1+(\dot{\mathbf{e}}^T\dot{\mathbf{e}})^m}\,\dot{\mathbf{e}} + 2[\omega\times]\dot{\mathbf{e}} + \frac{K_1}{1+(\mathbf{e}^T\mathbf{e})^n}\,\mathbf{e} = \mathbf{0} \tag{13}$$

2.3.1 Stability Proof

To prove the stability of controller C3, the following candidate Lyapunov function is considered:

$$V_L = \int_{\infty}^{t} \frac{\dot{\mathbf{e}}^T K_1 \mathbf{e}}{1 + (\mathbf{e}^T \mathbf{e})^n}\, dt + \frac{1}{2}\dot{\mathbf{e}}^T \dot{\mathbf{e}} \tag{14}$$

With $K_1 = k_1 I_{3\times3}$, where k_1 is positive, we can write

$$V_L = k_1 \int_{\infty}^{t} \frac{\dot{\mathbf{e}}^T \mathbf{e}}{1 + (\mathbf{e}^T \mathbf{e})^n}\, dt + \frac{1}{2}\dot{\mathbf{e}}^T \dot{\mathbf{e}} \tag{15}$$

V_L must first be shown to be positive definite. Defining $s \equiv \mathbf{e}^T \mathbf{e}$, we can write

$$\frac{1}{2}\frac{ds}{dt} = \dot{\mathbf{e}}^T \mathbf{e} \tag{16}$$

The second term in Eq. (15) is zero if and only if $\dot{\mathbf{e}}$ is zero. To show the positive definiteness of the candidate Lyapunov function, it is sufficient to show that a necessary condition for the first term in Eq. (15) to vanish is $\mathbf{e} = \mathbf{0}$. Denote the first term by β:

$$\beta \equiv k_1 \int_{\infty}^{t} \frac{\dot{\mathbf{e}}^T \mathbf{e}}{1 + (\mathbf{e}^T \mathbf{e})^n}\, dt = \frac{k_1}{2}\int_0^s \frac{ds}{1 + s^n} \tag{17}$$

Since the integrand in Eq. (17) is always greater than zero, the integral β is also positive, the exception being the case when $s = 0$. The necessary and sufficient condition for $s = 0$ is $\mathbf{e} = \mathbf{0}$, which is therefore also the necessary condition for $\beta = 0$. Thus the positive definiteness of the candidate Lyapunov function V_L is proved.

Taking the time derivative of V_L gives

$$\begin{aligned} \dot{V}_L &= \frac{\dot{\mathbf{e}}^T K_1 \mathbf{e}}{1 + (\mathbf{e}^T \mathbf{e})^n} + \dot{\mathbf{e}}^T \ddot{\mathbf{e}} \\ &= \dot{\mathbf{e}}^T \left(\frac{K_1 \mathbf{e}}{1 + (\mathbf{e}^T \mathbf{e})^n} + \ddot{\mathbf{e}} \right) \end{aligned} \tag{18}$$

Substituting Eq. (4) into Eq. (18) the time derivative of V_L can be written as

$$\dot{V}_L = -\dot{\mathbf{e}}^T \left(\frac{K_2 + K_3(\dot{\mathbf{e}}^T \dot{\mathbf{e}})}{1 + (\dot{\mathbf{e}}^T \dot{\mathbf{e}})^m} + 2[\boldsymbol{\omega}\times] \right) \dot{\mathbf{e}} \tag{19}$$

In Eq. (19), $[\boldsymbol{\omega}\times]$ is a constant skew-symmetric matrix and the first term is always positive definite whenever K_1 and K_2 are positive definite. Thus, the term in the brackets is always positive definite. Asymptotic stability follows directly from the global invariant set theorem.

3. SIMULATION RESULTS

This section presents the results obtained by using the controllers C1, C2 and C3 for the saturated and unsaturated cases. The response using the nonlinear model, which takes into account the relative gravitational force, is compared with the linear model, which neglects this force. A low Earth orbit of 6700 km is chosen. For the purpose of simulations, an inclination difference of 0.001 is chosen between the desired and the actual positions of the deputy satellite with respect to the chief satellite. The values of the gains k_1, k_2 and k_3 (k_3 for C2 and C3 only) are chosen to be 0.1, 1.5 and 1, respectively.

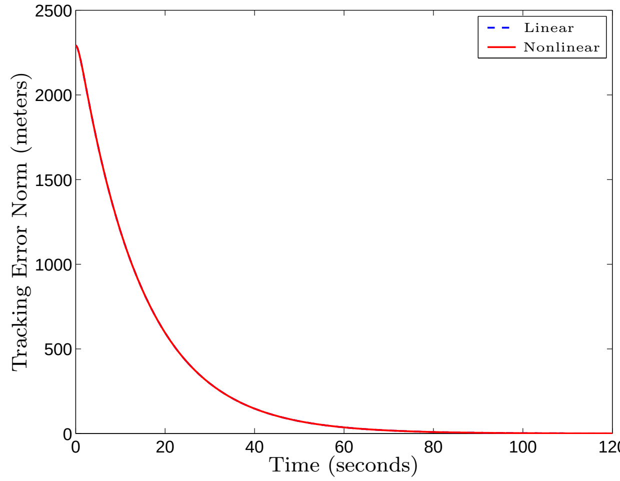

Fig. 1. Unsaturated Controller C1

Figs. 1 and 2; 3 and 4; 5 and 6 compare the responses of the unsaturated and the saturated controllers for C1, C2 and C3, respectively. It can be seen from these figures that the saturated and unsaturated controllers exhibit a great difference for controller C1, while they are almost the same for C2 and C3. This is because C2 and C3 operate under low control forces, due to the increased emphasis on the velocity terms. To understand this we first investigate the transient responses of the velocity errors for the three controllers. Figs. 7, 8 and 9 show that the transient response involves high peaks in the error velocities for C1 but not for C2 and C3. This fact can be explained as follows: Since the initial position error is high compared to the velocity error, a low emphasis on the latter terms leads to saturation for a greater duration of time and a consequent rise in the velocity errors, before these increasing velocity error terms can diminish the effect of the high (though decreasing) position errors leading to either a loss of saturation or saturation in the opposite direction. But when the contribution from the velocity terms is high, the effect of the position errors gets diminished much earlier, thereby reducing the saturation tendency. As will be shown later, an increased emphasis on the velocity terms within the limits of stability leads to low control forces.

Figs. 10, 11 and 12 show the transient response at a low control saturation limit ($F_{sat} = 0.1$N) for controllers C1, C2 and C3, respectively. Since C2 and C3 operate under low control forces for a greater duration

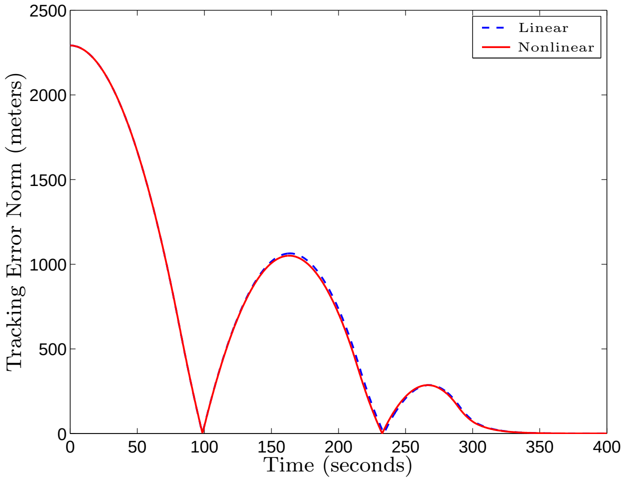

Fig. 2. Saturated Controller C1, F_{sat} 5N

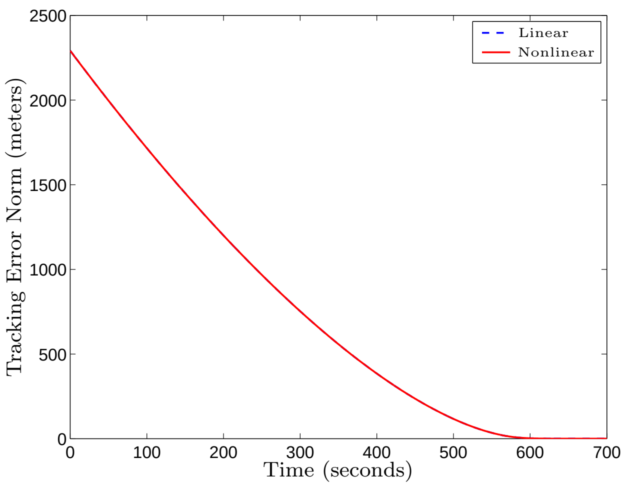

Fig. 3. Unsaturated Controller C2

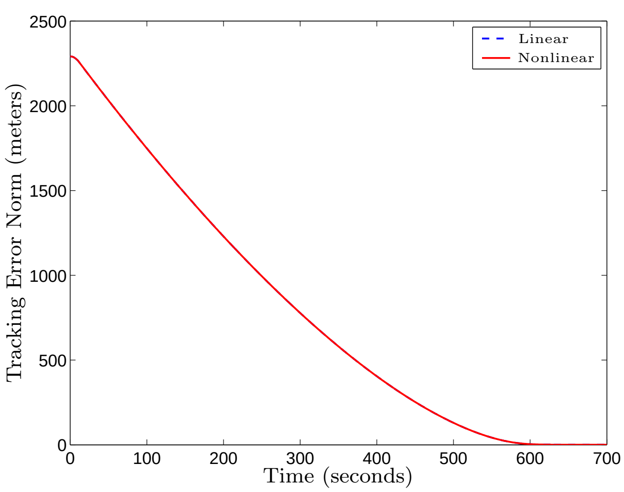

Fig. 4. Saturated Controller C2, F_{sat} 5N

of time, their transient responses are less susceptible to changes in the saturation limit than C1. Note that the increased oscillations in C1 lead to a large disagreement between the linear and the nonlinear model responses. But, from Fig. 12, we see that controller C3

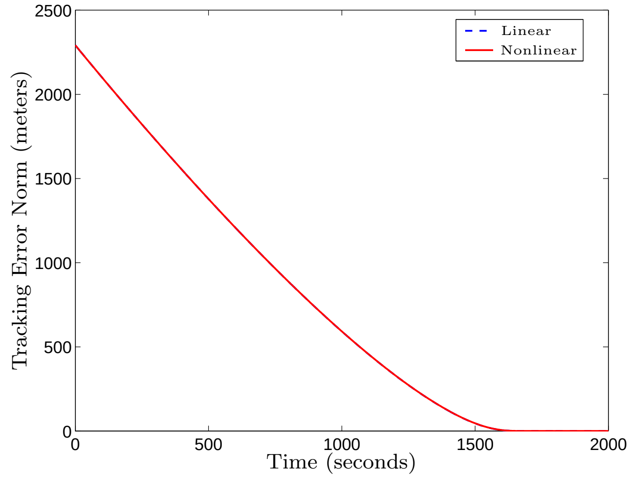

Fig. 5. Unsaturated Controller C3

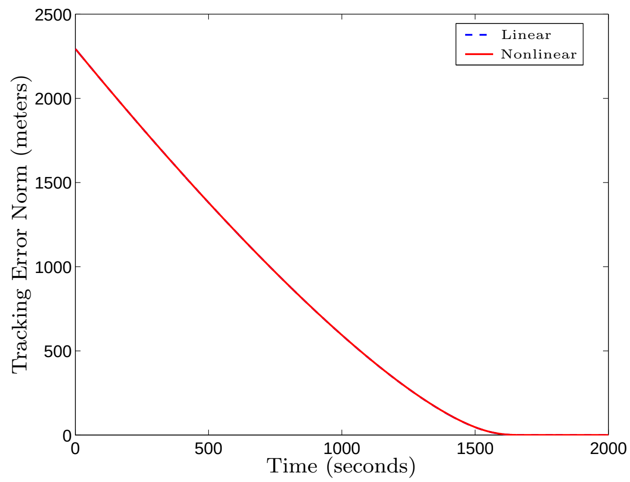

Fig. 6. Saturated Controller C3, F_{sat} 5N

Table 1. Unsaturated vs. Saturated Controllers

Controller	F_{sat}	Fuel (Linear)	Fuel (Nonlinear)
C1	∞	2752N	2749N
C1	0.1N	1552N	1340N
C2	∞	867.3N	857.2N
C2	0.1N	147.7N	131.4N
C3	∞	65.9N	71.3N
C3	0.1N	38.3N	41.1N

has equivalent linear and nonlinear model responses, which indicates that it is the most robust controller of the three. The z direction, which is the dominant axis, control inputs for the linear and nonlinear model responses are shown in Figs. 13 and 14, respectively. Table 1 shows the required fuel for all controllers. Clearly, the amount of control effort is significantly reduced using controllers C2 and C3 as compared to using C1, while also providing a faster decay time.

Our analysis in terms of position and velocity errors

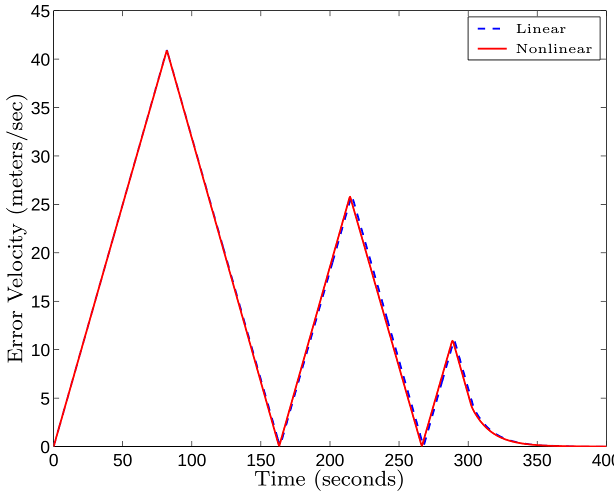

Fig. 7. Saturated Controller C1, F_{sat} 5N

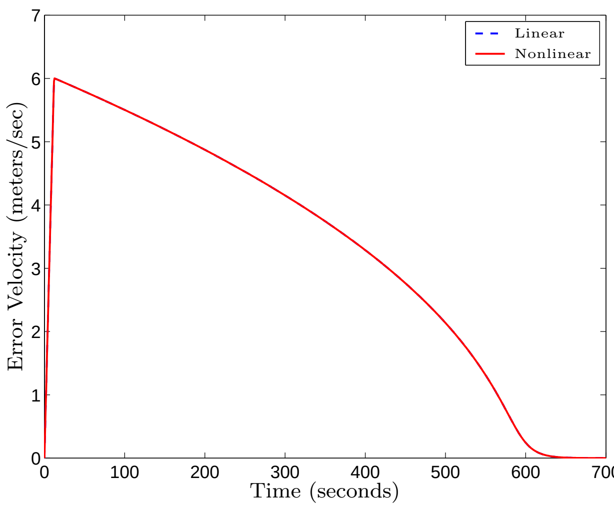

Fig. 8. Saturated Controller C2, F_{sat} 5N

can also be applied to test the system behavior when the control parameters are altered. Since increased emphasis on the velocity (position) error terms leads to lower (higher) control forces, any parameter change that causes more contribution from the velocity (position) error terms must increase (decrease) the time duration of the maneuver. This analysis is applicable only within the limits of stability and when there are no oscillations. (A high increase in the position term contribution tends to create oscillations in the system.) An example illustrating this for controller C2 is shown in Fig. 15.

We have seen how controllers can be designed intelligently in order to reduce the saturation tendency. Control saturation occurs when the contribution from the position and the velocity terms are not properly matched so that one of them dominates over the other, as in the case of controller C1. Apart from promoting control saturation, this domination also leads to a high rise in the value of the dominated variable, since the control is saturated in one direction for a greater

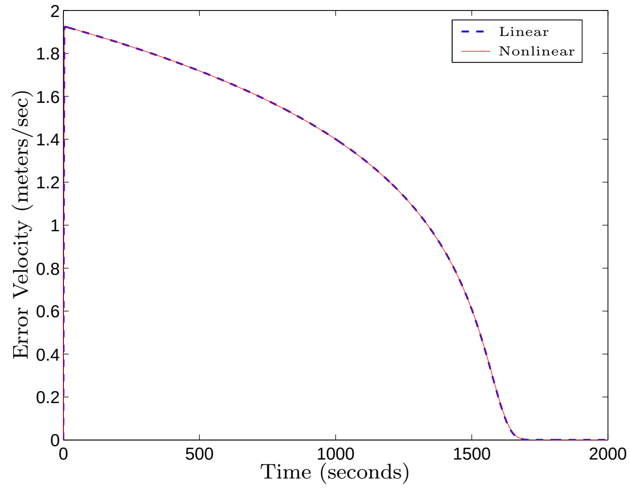

Fig. 9. Saturated Controller C3, F_{sat} 5N

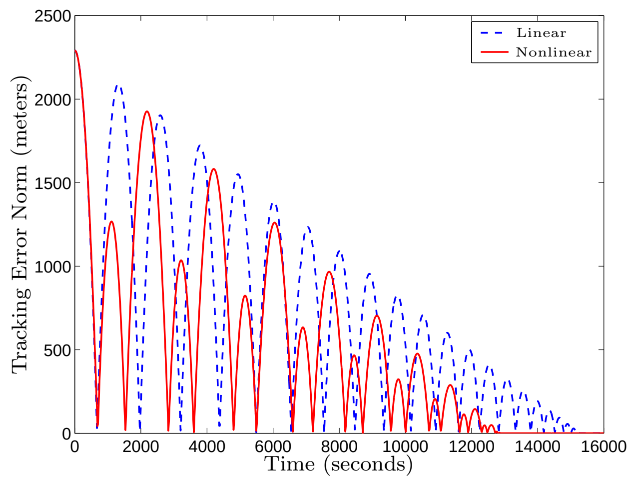

Fig. 10. Saturated Controller C1, F_{sat} 0.1N

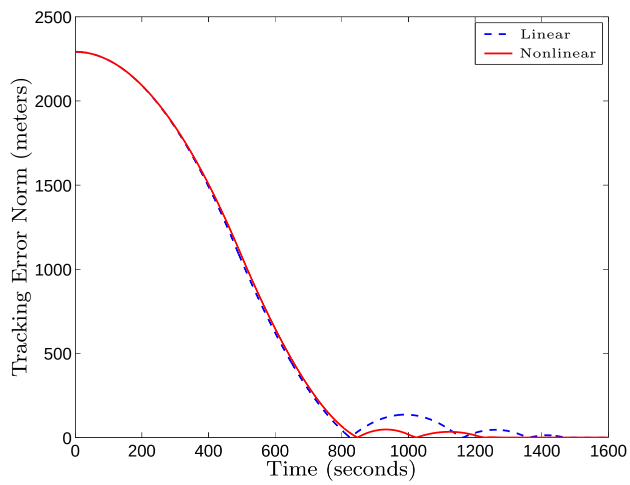

Fig. 11. Saturated Controller C2, F_{sat} 0.1N

duration of time. Further, control switching may also take place, which can give rise to oscillations. The following type of controllers is proposed:

$$\mathbf{u}_d = M_d[K_1\mathbf{e} + K_2\dot{\mathbf{e}} + \mathbf{f}(\dot{\mathbf{e}}) + \mathbf{w}] \tag{20}$$

where the function $\mathbf{f}(\dot{\mathbf{e}})$ is chosen in such a way that

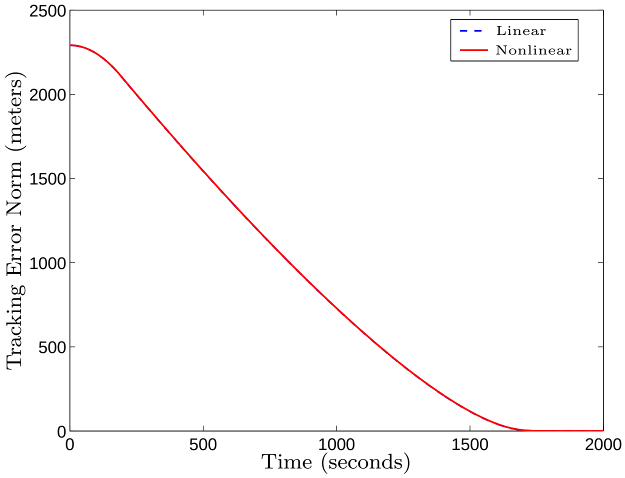

Fig. 12. Saturated Controller C3, F_{sat} 0.1N

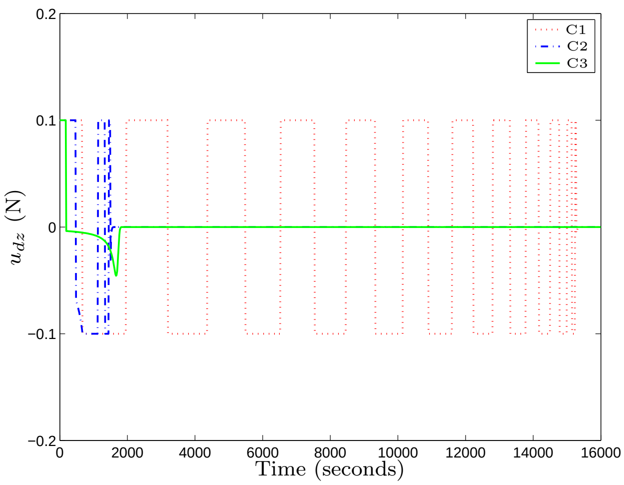

Fig. 13. Linear Control for C1, C2 and C3

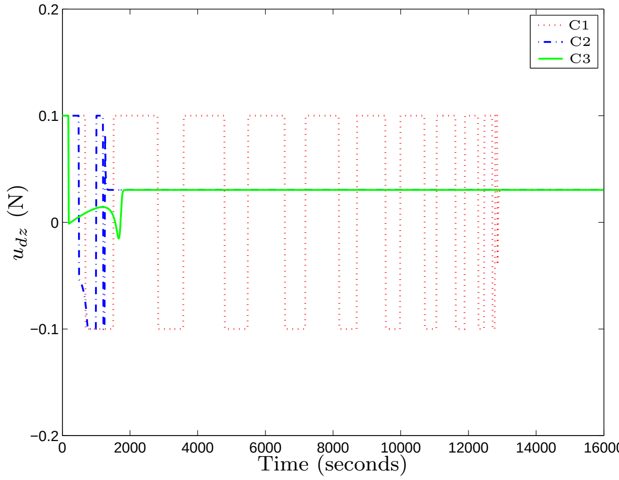

Fig. 14. Nonlinear Control for C1, C2 and C3

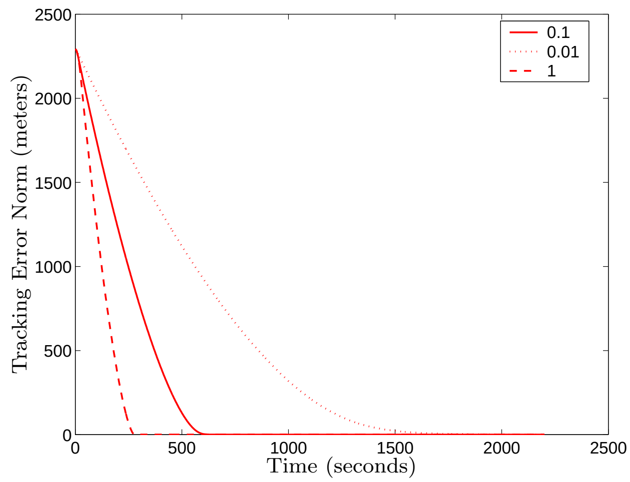

Fig. 15. Variation of k_1 for Controller C2

the controller would asymptotically stabilize the system apart from preventing a high rise in the velocity. The latter condition can be satisfied by using higher powers of the velocity errors in the control. For testing this type of controller, the function $\mathbf{f}$ is chosen to be $\mathbf{f} = K_3(\dot{\mathbf{e}}^T\dot{\mathbf{e}})^2\dot{\mathbf{e}}$, where K_3 is a positive definite matrix. Results indicate that this controller provides even better characteristics than controllers C2 and C3 (see [3] for more details).

4. CONCLUSIONS

The assumption of neglecting gravity in the control design of formation flying spacecraft has been studied. Two controllers, C2 and C3, have been designed by taking a careful look at the limitations of the original saturated simple feedback controller C1. For the unsaturated and 5N saturated cases, all controllers had similar responses for their respective nonlinear and linear model uses. Though controllers C2 and C3 take a longer time for the maneuver, the fuel consumption itself is reduced, a fact which is due to the low control forces. The real power of these new controllers was evident when the saturation limit was small (0.1N), which should that both the C2 and C3 controllers had better transient response characteristics over C1 and consumed much less fuel.

5. REFERENCES

1. Wang, P. K. C., Hadaegh, F. Y., and Mokuno, M., "Formation Flying of Multiple Spacecraft with Automatic Rendezvous and Docking Capability," *AIAA Guidance, Navigation, and Control Conference and Exhibit*, 11-14 August 2003, Austin, Texas, AIAA 2003-5363.

2. Junkins, J. L. and Schaub, H., *Analytical Mechanics of Space Systems*, AIAA Education Series, AIAA, Reston, VA, 2003.

3. Kappagantula, K. C., *Linearizing Assumptions and Control Design for Spacecraft Formation Flying Maneuvers*, Master's thesis, University at Buffalo, State University of New York, 2004.

4. Slotine, J. J. E. and Li, W., *Applied Nonlinear Control*, Prentice Hall, NJ, 1990.