INTERPLANETARY FLIGHT CONTROL WITH ELECTRIC ENGINE IN VIEW OF THRUST ERRORS

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ABSTRACT

The optimal space flight to the planet with using electric propulsion system (EPS) is found on the assumption of nominal operating conditions of EPS as cruise engine. However on account of possible errors of program execution of engine thrust, the actual trajectory can deviate from the optimal nominal one and cannot provide performance of certain goal conditions of the space flight. The problem of definition of necessary correction of trajectory for optimal achievement by spacecraft (SC) of the final condition is investigated. Correction is realized by means of renewal of electric propulsion program during the space flight. Aprioristic estimations of possible schemes of electric propulsion program renewal are carried out. The following scheme of trajectory correction is offered. The trajectory is divided into equal time intervals. It is supposed that in the beginning of each time interval the position and velocity of spacecraft are known practically precisely. Errors of EPS operation are given as errors of value and direction of thrust. Because of these errors the spacecraft condition at the end of the time interval cannot coincide with nominal optimal one. It is assumed that the actual trajectory not strongly deviates from the optimal trajectory and the estimations of possible dispersion of spacecraft condition at the end of time interval are fulfilled in linear statement. Then the electric engine thrust program is changed in such manner that the spacecraft reaches the set final vector of condition at further optimal space flight without errors. The minimal duration of the interval is conditioned by following demands. During each of these intervals it is necessary: 1) to receive and process telemetric information about system operation for support current thrust program, 2) to carry out and process trajectory measuring, 3) to carry out ballistic computation for more precise definition of parameters of spacecraft motion and EPS operation, 4) to calculate new further thrust program of EPS, 5) to transmit new thrust program to spacecraft. The time interval of the thrust program renewal varies in admissible boundaries and for each of them the maximal mass expenses for the trajectory correction realization are estimated.

The numerical results are carried out for space flight to Mars within the framework of Russian “Phobos-sample-return” project. It is supposed to apply the Russian plasma engine SPT-140 to this flight with solar batteries as an energy source. It is assumed that the errors of thrust can reach 5 percent from nominal value and 1 degree on direction. Dependence of mass expenses on time interval value for thrust program renewal was found. This dependence was used for search of the optimal time interval and on the basis of this result the scheme of this trajectory correction was chosen and the maximal mass expenses were appreciated.

1. PRELIMINARY REMARKS

Forthcoming flight of spacecraft with EPS on interplanetary trajectory on time interval $T=[t_N, t_F]$ is investigated. EPS operation is carried out under the so-called program of thrust.

It is supposed, that nominal thrust program is realized from the moment of time t_N and provides (in case of the absolutely exact thrust program execution) the SC flight along the optimal nominal trajectory from the point of view of the SC mass expenses. The kinamatic parameters vector $\boldsymbol{X}(t)=(\boldsymbol{R}^T(t),\boldsymbol{V}^T(t))^T$ of this trajectory at the moments t_N and t_F coincides with the given vectors $\boldsymbol{X}_N=(\boldsymbol{R}_N{}^T,\boldsymbol{V}_N{}^T)^T$ and $\boldsymbol{X}_F=(\boldsymbol{R}_F{}^T,\boldsymbol{V}_F{}^T)^T$. The position $\boldsymbol{R}$ and velocity $\boldsymbol{V}$ vectors are determined in heliocentric non-rotating coordinate system *Oxyz* at J2000.0.

According to [1,2], it is supposed, that the value of the EPS thrust P depends on the electrical power supplied for electric engine, this power in turn depends on the SC position in the coordinate system *Oxyz*.

Generally speaking, the program thrust can be executed with errors. Therefore flight should be planned in view of aprioristic errors of the thrust program performance and opportunity to change the thrust program at some fixed moments $t_i \in T$ for i = 2,3,…,k-1, k>2. The moments of time t_1 and t_k are equal to t_N and t_k respectively. The thrust program starting from the moment t_i and the trajectory corresponding to this program on interval $[t_i, t_k]$ have number i, i=1,2,…,k-1. It is obvious, that the first trajectory corresponds to the nominal thrust program:

$$X(t_1) = X_N \,, \quad (1)$$

$$X(t_k) = X_F \,. \quad (2)$$

The estimation of additional expenses ΔM of the SC mass for realization of target flight at the account possible of deviation δP and α from the magnitude P and direction $\boldsymbol{e}$ of thrust at each current time has practical significance. The deviation δP is characterized by relative value $\nu = \delta P / P$. The random values ν and α can take on values, absolute values of which are not surpassing the given small values ν_l and α_l (for "Phobos-Ground" project $\nu_l < 0.06$, $\alpha_l < 1^\circ$):

$$|\nu| \le \nu_l \,, \; |\alpha| \le \alpha_l \,. \quad (3)$$

The values ν_l and α_l in domain $Oxyz$ for each current moment determine domain Ω of the possibility increment $\delta\boldsymbol{P}$ of the thrust vector $\boldsymbol{P}=P\boldsymbol{e}$.

The additional expenses ΔM calculation with known values of the SC initial mass M_N, parameters of EPS ,ν_l and α_l is carried out at following assumptions:

- The SC flight on all interval T is realized in small vicinity of the nominal (first) trajectory;

- Each program of thrust in case of EPS ideal operation provides performance of a terminal condition with minimal losses of the SC mass;

- At each moment the random vector$\delta\boldsymbol{P}$ has the uniformly distribution low [2] in the three-dimensional domain Ω;

- The SC trajectory kinematic parameters at the initial moment t_N is known with neglected errors;

- Deviations of kinematic parameters of the SC trajectory take place only because of the mentioned above errors of the thrust program execution.

2. ALGORITHM OF THE PROBLEM DECISION

Before to state algorithm of calculation of additional expenditure of the mass necessary for performance of a target task of flight, it is necessary to present mathematical model of the SC motion on the optimum trajectory.

2.1 Mathematical model of the SC motion

The SC interplanetary flight with EPS on the optimum trajectory is described in coordinates system Oxyz by fourteen differential equations system:

$$\begin{cases} \dfrac{d\boldsymbol{R}}{dt} = \boldsymbol{V} \\ \dfrac{d\boldsymbol{V}}{dt} = \boldsymbol{g} + \delta \dfrac{P}{M}\boldsymbol{e} \\ \dfrac{dM}{dt} = -\delta \cdot q \\ \dfrac{d\boldsymbol{\Psi}_{\boldsymbol{R}}}{dt} = -[\dfrac{\partial \boldsymbol{g}}{\partial \boldsymbol{R}}]^T \boldsymbol{\Psi}_{\boldsymbol{V}} - \\ \delta[\dfrac{\partial P}{\partial \boldsymbol{R}}]^T \dfrac{(\boldsymbol{\Psi}_V, \boldsymbol{e})}{M} + \delta[\dfrac{\partial q}{\partial \boldsymbol{R}}]^T \Psi_M \\ \dfrac{d\boldsymbol{\Psi}_V}{dt} = -\boldsymbol{\Psi}_R \\ \dfrac{d\Psi_M}{dt} = \delta \dfrac{P(\boldsymbol{\Psi}_V, \boldsymbol{e})}{M^2} \end{cases} \,, \quad (4)$$

where

$$\boldsymbol{g} = -\mu_s(1-\chi)\frac{\boldsymbol{R}}{|\boldsymbol{R}|} - \sum_{n=1}^{10} \mu_n \Big(\frac{\boldsymbol{R}-\boldsymbol{R}_n}{|\boldsymbol{R}-\boldsymbol{R}_n|^3} + \frac{\boldsymbol{R}_n}{|\boldsymbol{R}_n|^3}$$

is total vector of perturbation accelerations, μ_S is Sun gravitation parameter, χ is coefficient describing solar light influence on the SC, μ_S and $\boldsymbol{R}_n$ are gravitation parameters and position vectors of Solar system planets and the Moon in current moment t;

M is the current SC mass;

$q(t)$ is defined function of the mass flow rate;

δ (t) is function accepting values 0 or 1 and determining intervals of the EPS operating time;

$\boldsymbol{\Psi}_R$, $\boldsymbol{\Psi}_V$, are adjoint vectors [4] of the SC position and velocity vectors **R**, **V** respectively;

Ψ_M is adjont variable of mass.

For optimum trajectory and for any considered trajectory with number i, $i=1,2,\ldots,k-1$, the formula take place

$$\boldsymbol{e} = \frac{\boldsymbol{\Psi}_V}{|\boldsymbol{\Psi}_V|} \,, \quad (5)$$

value δ depends on sign of the so-called switching function f^*, $f^* = \dfrac{P \cdot |\boldsymbol{\Psi}_V|}{M} - \Psi_M q$:

if $f^* > 0$ then $\delta = 1$, and if $f^* < 0$ then $\delta = 0$. (6)

2.2 Algorithm description

Estimation of mass ΔM is made in the assumption that parameters of the nominal trajectory (1-st trajectory) are earlier received, i.e. the values of adjoint variables at the moment t_1 are determined. Calculation of these adjoint variables is an independent labour-consuming problem that in the present paper is not considered.

Besides at the moment of time t_i for $i=2,3,...,k-1$ the six-measured ellipsoid of concentration E_i of kinematic parameters $\boldsymbol{X}_i = \boldsymbol{X}(t_i)$ of the SC trajectory is supposed to be known. The ellipsoid E_i corresponds to errors of program execution of engine thrust on interval $[t_{i-1}, t_i]$ of the SC flight. The covariance matrix K_i determining size and axis direction of the ellipsoid of concentration E_i is calculated by a special technique. This technique has independence importance and consequently its description is presented in separate paragraph of present paper (see below).

The SC motion on the trajectory with number i is considered, $i=1,2,...,k-1$. Because of errors of the thrust program execution on interval of flight $[t_i, t_{i+1}]$, the kinematic parameter vector of the SC trajectory in moment t_{i+1} can have any perturbation in limits of the ellipsoid E_{i+1}. Therefore, pursuing the purpose as mass ΔM to have the majorizing estimation of additional expenses of the SC mass on realization of its flight on all the interval T, the following is necessary: a) the mass increment for the flight interval $[t_i, t_{i+1}]$ is equal:

$$\Delta M_{i+1}^{i+1} = -\nu_1 (M_i^i - M_{i+1}^i) , \qquad (7)$$

where M_i^i, M_{i+1}^i are the SC masses in moments t_i t_{i+1} for the SC trajectory with number i; b) the mass increment for the flight interval $[t_i, t_{i+1}]$ is equal to the minimal increment on set of all possible trajectories.

In view of stated, procedure of calculation of the mass ΔM is basically reduced to the following computing cycle for $i=2,3,...,k-1$.

1. According to the Eqn. 7 the mass ΔM_i^i is calculated.

2. The increment ΔM_k^i of the SC mass in moment t_k is minimized by a choise of the allowable trajectory with number i. The increment ΔM_k^i is determined as

$$\Delta M_k^i = \frac{\partial M_k}{\partial \boldsymbol{X}_i} \Delta \boldsymbol{X}_i + \frac{\partial M_k}{\partial \boldsymbol{\Psi}_i} \Delta \boldsymbol{\Psi}_i + \frac{\partial M_k}{\partial M_i} \Delta M_i^i , \qquad (8)$$

where $\Delta \boldsymbol{X}_i = \boldsymbol{X}_i(t_i) - \boldsymbol{X}_{i-1}(t_i)$, $\Delta \boldsymbol{\Psi}_i = \boldsymbol{\Psi}_i(t_i) - \boldsymbol{\Psi}_{i-1}(t_i)$, $\boldsymbol{X}_i = (\boldsymbol{R}^T, \boldsymbol{V}^T)^T$, $\boldsymbol{\Psi}_i = (\boldsymbol{\Psi}_R{}^T, \boldsymbol{\Psi}_V{}^T)^T$, vectors $\boldsymbol{X}_i$ and $\boldsymbol{\Psi}_i$ correspond to the trajectory with number i. On the allowable trajectory the terminal condition

$$\frac{\partial \boldsymbol{X}_k}{\partial \boldsymbol{X}_i} \Delta \boldsymbol{X}_i + \frac{\partial \boldsymbol{X}_k}{\partial \boldsymbol{\Psi}_i} \Delta \boldsymbol{\Psi}_i + \frac{\partial \boldsymbol{X}_k}{\partial M_i} \Delta M_i^i = 0 \qquad (9)$$

is satisfied.

Included in Eqns. 8, 9 and other, the partial derivatives is calculated on the nominal trajectory by differential method. For this the Cauchy problem repeatedly is solved for system of Eqn. 4 and taking into account Eqns. 5, 6. Minimization of ΔM_k^i with the defined value of ΔM_i^i is easily reduced to minimization of the linear form of a kind $\Delta M_k^i = (\boldsymbol{C}, \boldsymbol{X}_i) + \alpha$ on set of the vectors $\Delta \boldsymbol{X}_i$ belonging to an ellipsoid E_i or, in other words, satisfying equation $\Delta \boldsymbol{X}_i{}^T K_i \Delta \boldsymbol{X}_i \le 1$. Minimization of the linear form on an ellipsoid is carried out by means of analytical formulas.

To not enter new designations, we shall consider: the deviation $\Delta \boldsymbol{X}_i$ minimizes the functional, Eqn. 8, and the mass ΔM_k^i is the minimal value of the functional.

3. Taking into account Eqn. 9 , deviation $\Delta \boldsymbol{\Psi}_i$ of adjoint vector is calculated.

4. The mass increment ΔM_j^i and the mass M_j^i for time t_j ($j=i+1, i+2,..., k-1$) are calculated by means of formulas:

$$\Delta M_j^i = \frac{\partial M_j}{\partial \boldsymbol{X}_i} \Delta \boldsymbol{X}_i + \frac{\partial M_j}{\partial \boldsymbol{\Psi}_i} \Delta \boldsymbol{\Psi}_i + \frac{\partial M_j}{\partial M_i} \Delta M_i^i , \qquad (10)$$

$$M_j^i = M_j^{i-1} + \Delta M_j^i . \qquad (11)$$

After output from this cycle the increment ΔM_k^k is calculated:

$$\Delta M_k^k = -\nu_1 (M_{k-1}^{k-1} - M_k^{k-1}) \qquad (12)$$

and, at last, the required mass is

$$\Delta M = \left| \sum_{i=2}^{k} \Delta M_k^i \right| . \qquad (13)$$

3. CALCILATION TECHNIQUE OF THE COVARIANCE MATRIX

The SC flight with EPS during some interval $[t_i, t_{i+1}]$ is considered. The thrust program of EPS is given. It is required to define the covariance matrix describing a dispersion of the kinematic parameters of the SC trajectory at the moment t_{i+1} because of the errors of the thrust program execution. For this purpose it is necessary to express dispersion of the random increment $\delta \boldsymbol{P}$ of the thrust vector $\boldsymbol{P}=P\boldsymbol{e}$ at the current time t in convenient form. And, pursuing the purpose to receive a guaranteed estimation of additional expenses ΔM of the SC mass M, it is possible to expand initial domain of the allowable dispersion of a random vector $\delta \boldsymbol{P}$ only.

3.1 Covariance matrix of the current thrust increment

The point defined by the random increment $\delta\boldsymbol{P}$ at the current moment t (from the interval $[t_{i-1},t_i]$) belongs to domain $\Omega(t)$, the sizes and position of which in the coordinates system $Oxyz$ are defined by constants ν_l, α_l, and also nominal module and direction of the EPS thrust. The domain $\Omega(t)$ belongs to the right cylinder $\Omega_C(t)$, the size of which is $2\nu_l P$ and the base radius is $P\sin\alpha_l$. As values ν_l, α_l are small, domains $\Omega(t)$ and $\Omega_C(t)$ practically coincide with each other. Therefore the cylinder $\Omega_C(t)$ is accepted as domain of possible changes of the random vector $\delta\boldsymbol{P}$. The Cartesian frame $s(t)$ with the centre in the point of the current SC position and orientation determined by unit vectors $\boldsymbol{e}(t)$, $\boldsymbol{e}_1(t)$, $\boldsymbol{e}_2(t)$ is entered in consideration. Vectors $\boldsymbol{e}_1(t)$, $\boldsymbol{e}_2(t)$ are orthogonal each other and each of them is orthogonal to vector $\boldsymbol{e}(t)$. In this coordinate system it is easily to define the axes of an ellipsoid of rotation $E(t)$ (around of the axis $\boldsymbol{e}(t)$) which contains all points of the cylinder $\Omega_C(t)$ and has volume differed minimally from the cylinder volume. The semi axes of this ellipsoid lay along axes of coordinate system $s(t)$ and are accordingly equal to $P\nu_l\sqrt{3}$, $P\sqrt{\frac{3}{2}}\sin\alpha_l$, $P\sqrt{\frac{3}{2}}\sin\alpha_l$.

The dispersion of random vector $\delta\boldsymbol{P}$ is permitted within the limits of the ellipsoid $E(t)$ and the corresponding covariance matrix in coordinate system $s(t)$ is

$$\tilde{K}_P(t) = P^2(t)K\ , \tag{14}$$

where

$$K = \begin{pmatrix} 3\nu_l^{\,2} & 0 & 0 \\ 0 & \frac{3}{2}\sin^2\alpha_l & 0 \\ 0 & 0 & \frac{3}{2}\sin^2\alpha_l \end{pmatrix}. \tag{15}$$

Using transition matrix $B(t)$ from system $s(t)$ to system $Oxyz$:

$$B(t) = (\boldsymbol{e},\boldsymbol{e}_1,\boldsymbol{e}_2)\ , \tag{16}$$

the covariance matrix of errors of the thrust $\boldsymbol{P}$ is

$$K_P(t) = B(t)\cdot\tilde{K}_P(t)\cdot B^T(t)\ . \tag{17}$$

3.2 Formula for covariance matrix calculation

Let $F(t_i,\tau)$ be matrix of partial derivatives:

$$F(t_i,\tau) = \frac{\partial\boldsymbol{X}(t_i)}{\partial\boldsymbol{V}(\tau)} \tag{18}$$

corresponding to influence of the SC velocity vector at moment $\tau\in[t_{i-1},t_i]$ on the kinematic parameters of SC trajectory at the moment t_i.

The unit interval of time $[\tau,\tau+1]$ is considered. It is assumed that during this entire unit interval the module and direction of thrust received at the moment τ, the SC mass M, the transition matrix $B(t)$ and matrix $F(t_i,\tau)$ are constant. This unit interval of time is broken into equal small intervals. Then, in this case assuming that errors of thrust execution take place only on the pointed unit interval, the covariance matrix at the moment t_i is

$$\delta K_n(\tau) = \sum_{j=1}^{n}\frac{jF(t_i,\tau)B(\tau)\tilde{K}_P(\tau)B^T(\tau)F^T(t_i,\tau)}{n^2M^2(\tau)}\ . \tag{19}$$

The value $\lim_{n\to\infty}\delta K_n(\tau)$ can approximately be accepted as derivative $\frac{\partial K_i}{\partial\tau}$. Thus the required formula is

$$K_i = \frac{1}{2}\int_{t_{i-1}}^{t_i} F(t_i,\tau)\frac{P^2(\tau)}{M^2(\tau)}B(\tau)KB^T(\tau)F^T(t_i,\tau)d\tau\ . \tag{20}$$

Calculation of matrixes $F(t_i,\tau)$ is realized by the numerical method using repeated numerical integration of system of the Eqns. 4. The diagonal matrix included in Eqn. 20 does not depend on time and depends only on values ν_l and α_l.

4. NUMERICAL RESULTS

The numerical results were obtained for the following values of parameters. The nominal optimal trajectory and characteristics of EPS are given from paper [2]. The time interval of the thrust program renewal is 2 weeks. It is assumed that $\nu_l = 0.05$ и $\alpha_l = 1^0$.

As a result of calculations the estimation of the mass expenses necessary to parry the errors of the thrust performance was received. These expenses are 19 kg that makes about 5 percent from the mass expenses for realization of the nominal trajectory.

5. CONCLUSION

The scheme of control of flight with taking into account errors of the thrust execution by means of change of the thrust program in the fixed moments was considered. The estimation of control is value of the additional expenses ΔM of the mass necessary for performance of a target task. The algorithm and the corresponding software for calculation of the mass ΔM have been developed. According to the obtained results of calculations for the Phobos-Ground project, mass expenses make about 5 percent from the expenses of the SC mass necessary for realization of nominal flight.

6. REFERENCES

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