DETERMINATION OF MOTION PARAMETERS OF ISS BY USE OF GPS MEASUREMENTS

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ANNOTATION

We estimate the accuracy of determining the weakly disturbed orbital motion of International Space Station by use of GPS-measurements as well as the accuracy of forecasting such a motion. The measurements are the station coordinates in Greenwich coordinate system. They are received from on board equipment ASN-2401. The mean square errors of approximation of the measurements do not exceed 30 m when the motion is determined in a time interval less than a day. Processing the GPS-measurements in three-hour interval allows making the forecast of the motion on 15 hours forward with an error in the station radius vector not exceeding 400 m at a right choice of ballistic coeffitient. Systematic errorss in measurements of the station velocity, carried out by equipment ASN-2401, are revealed.

DATA OF GPS-MEASUREMENTS AND PROCESSING PROCEDURE

Data of the ISS phase vector definition, obtained using the ACH-2401 equipment is the sequences of radius-vector and velocity values in the Greenwich frame (more exactly, in WGS84): $\mathbf{r} = (x, y, z)$, $\mathbf{v} = (v_x, v_y, v_z)$. Here $v_x = dx/dt$, $v_y = dy/dt$, $v_z = dz/dt$, t – time. Components of these vectors are specified in the common moments of time with the 1s interval during sessions of TMI direct transmission to the Earth and with interval of several minutes (≥ 4) during the TMI transmission from the onboard memory devices. Only the second-type data is used below. Because accuracy of the station velocity definition by the ASN equipment is not high (see below), the radius-vector measurements were smoothed only.

Used mathematical model of the station orbital motion was the differential equations of its CoM motion written in Greenwich frame with consideration for non-centrality of the Earth gravitation field and atmosphere drag. The field non-centrality was taken into account with accuracy of up to terms of (36,36) order inclusive in the Earth gravitational field expansion in series by solid spherical harmonics. Atmosphere was considered as fixed relative to the surface of. The atmosphere density was calculated according to model ГОСТ 25645.115-84 (revision of 1990.).

This model contains three parameters: diurnal mean F of solar activity index $F_{10.7}$, weighted mean F_{81} of index $F_{10.7}$ for previous 81 days and diurnal index of geomagnetic activity K_p. Values of these parameters with defined delay are referred a day for which the desired density is calculated. In general geomagnetic activity changes more fast than solar one, therefore in high-precision calculations the three-hour index k_p is used instead of index K_p.

For the GPS measurements smoothing two systems of the ISS motion equations were used differing by indices F, F_{81} and k_p representation in formulas of ГОСТ. Let's designate these systems as (I) and (II). In system (I) indices F, F_{81} and k_p were invariable over the whole interval of smoothing. In system (II) these indices dependence of time was taken into account, therewith functions $F = F(t)$, $F_{81} = F_{81}(t)$ and $k_p = k_p(t)$ were selected as piecewise constant ones. Intervals of $F(t)$ and $F_{81}(t)$ constancy were 1 day in long, intervals of function $k_p(t)$ constancy – 3 hours. Boundary points on constancy intervals were specified by the GOST conditions, values of functions were accepted as equaled to values of indices from Internet. When smoothing these measurements over the short intervals (less than 3 orbits) the station ballistic coefficient C was fixed. Over the long intervals (more than 3 orbits) this coefficient was used as a matching parameter.

Smoothing was performed as follows. By means of the station motion numerical integration the vectors $\mathbf{r}$ and $\mathbf{v}$ can be defined as a function of time. Solution of the motion equation with initial conditions $\mathbf{r}(t_0) = \mathbf{r}_0$, $\mathbf{v}(t_0) = \mathbf{v}_0$ will be designated as $\mathbf{r} = \varphi(t, \beta)$, $\mathbf{v} = \psi(t, \beta)$, $\beta = (\mathbf{r}_0, \mathbf{v}_0, c)$. Here the solution dependence of ballistic coefficient was taken into account. Let station coordinates are measured at instants t_k $(k = 1, 2, \ldots, N)$. Radius-vector composed of coordinates for moment t_k will be designated as $\mathbf{r}_k$. It is considered, that errors of the coordinate measurements are independent and have Gauss distribution with zero mean and the same but unknown standard deviation σ. Definition of vector β, specifying the motion equation solution smoothing these measurements (for definiteness the smoothing with ballistic coefficient refinement is considered) was performed by the least square method. Namely, minimization of the following functional was performed

$$\Phi(\beta) = \sum_{k=1}^{N} \left[\mathbf{r}_k - \varphi(t_k, \beta)\right]^2 .$$

The Φ minimization was performed by the Gauss-Newton method. With assumption accepted the estimate $\beta_* = \arg\min \Phi$ is a random vector with approximately Gauss distribution and value equaled to true value of β. Covariance matrix of this estimate and dispersion σ^2 estimate for these measurements of coordinates were calculated by formulas

$$K_\beta = \sigma_*^2 B^{-1}, \quad \sigma_*^2 = \frac{\Phi(\beta_*)}{3N-7}.$$

where B – calculated in point β_* matrix of standard equations system arising when Φ is minimized by the Gauss-Newton method, $2B \approx \partial^2 \Phi(\beta_*)/\partial \beta^2$. Approximation accuracy for these measurements obtained by the motion equation solution and accuracy of this solution definition will be characterized by the standard deviation σ_* of measurement errors, standard deviations $\sigma_i = \sqrt{K_{ii}}$ $(i = 1, 2, \ldots, 7)$ of vector β_* components and some statistical characteristics of remainders series

$$\delta\mathbf{r}_k = \mathbf{r}_k - \varphi(t_k, \beta_*),$$

$$\delta\mathbf{V}_k = \mathbf{v}_k - \psi(t_k, \beta_*) + \boldsymbol{\omega} \times \delta\mathbf{r}_k \quad (k = 1, 2, \ldots, N).$$

where $\boldsymbol{\omega}$ – Earth angular velocity. Remainders $\delta\mathbf{V}_k$ characterize discrepancies of the station absolute geocentric velocity approximation.

When minimizing $\Phi(\beta)$, dimensionless ration $\kappa_c = c/c_0$ is convenient to vary but not ballistic coefficient c. Below everywhere σ_7 is the standard deviation of parameter κ_c.

Remainders $\delta\mathbf{r}_k$ and $\delta\mathbf{V}_k$ are calculated in the Greenwich frame therefore it is difficult to correlate them with the station orbit. To correlate these remainders with the orbit by natural manner we will consider their components in special local orthonormal bases. Let $\mathbf{r}(t)$, $\mathbf{V}(t)$ are the station geocentric radius-vector and its absolute geocentric velocity in approximating solution $\mathbf{V}(t) = \mathbf{v}(t) + \boldsymbol{\omega} \times \mathbf{r}(t)$. Let introduce orthonormal vectors

$$\mathbf{e}_1(t) = \frac{\mathbf{V}(t)}{|\mathbf{V}(t)|}, \quad \mathbf{e}_2(t) = \frac{\mathbf{r}(t) \times \mathbf{V}(t)}{|\mathbf{r}(t) \times \mathbf{V}(t)|},$$

$$\mathbf{e}_3(t) = \mathbf{e}_1(t) \times \mathbf{e}_2(t) \approx \frac{\mathbf{r}(t)}{|\mathbf{r}(t)|}.$$

and each pair of remainders $\delta\mathbf{r}_k$, $\delta\mathbf{V}_k$ will be considered in its own basis with basis vectors $\mathbf{e}_1(t_k)$, $\mathbf{e}_2(t_k)$ $\mathbf{e}_3(t_k)$. Components of vectors $\delta\mathbf{r}_k$ and $\delta\mathbf{V}_k$ in this basis will be designated as ξ_{1k}, ξ_{2k}, ξ_{3k} and ξ_{4k}, ξ_{5k}, ξ_{6k} respectively. Series ξ_{ik} $(k = 1, 2, \ldots, N)$ will be characterized by its mean square root S_i. Median of appropriate series of module $|\xi_{ik}|$ will be designated as m_i.

The method set forth for measurement data smoothing and estimation of constructing accuracy for smoothing solutions relates to the idealized model of errors contained in measurements. But among real measurements there are some number of rough spikes. To eliminate them from the smoothing procedure and to increase accuracy of β_* definition this procedure was divided into several steps.

At the first step with the ballistic coefficient extracted from the orbit radio monitoring data, functional Φ was minimized by initial conditions $\mathbf{r}_0, \mathbf{v}_0$. For obtained solution the median m of remainders modulus $|\delta\mathbf{r}_k|$ was calculated. Those points $(t_k, \mathbf{r}_k)$ were eliminated from the processing, for which $|\delta\mathbf{r}_k| > 3m$. With above assumptions on measurement errors, $m \approx 1.54\sigma$ and probability of the remainder with specified value of modulus is less than 0.001 (remainders modulus have the Maxwell distribution with parameter σ). At the step the Φ minimization by $\mathbf{r}_0$ and $\mathbf{v}_0$ was repeated for modified set of measurements. Obtained estimates of initial conditions were considered as final if measurement interval was short. In the case of long interval 2 additional steps are executed. The third step consisted of the β_* calculation by measurements allowed for the second step. Then the measurements rejection described above was performed, therewith the earlier rejected points were checked too. For revised set of values $(t_k, \mathbf{r}_k)$ new estimate β_* was defined which was considered as final.

RESULTS OF GPS-MEASUREMENTS SMOOTHING

Below are the smoothing results for measurement data obtained mainly during a period from 25.V to 30.V.2002. Figure 1 shows curves of indices F, F_{81}, k_p and a_p at this interval. In system (I) for specified interval it was taken up that $F = F_{81} = 171$, $k_p = 3$ ($a_p = 15$). This is the data used in MCC. In both systems it was taken up that $c_0 = 0.003773$ m^2/kg. This value of ballistic coefficient was used in MCC too.

Data selected for smoothing was divided into 10 time intervals, their main characteristics are presented in Table 1. The first 9 intervals belong to the interval indicated above, the tenth one is more later. At this interval in system (I) $F_{10.7} = F_{81} = 150$, $k_p = 3-$ ($a_p = 12$), $c_0 = 0.003365$ m^2/kg. All these intervals were considered as long ones and processed in 4 steps. In Table: N –a number of points included into processing at the first step; N' – number of points included into the processing at the final step; t_1 and t_N – initial and end points of the initial measurement interval (Moscow decree time DMT is used here). For convenience in all cases the initial conditions were specified at the moment $t_0 = t_1$. Point t_0 did not change at all intervals of processing in spite of the occasional rejection of the $\mathbf{r}_1$ measurement. In some cells of there are two values for parameter N'. The first one corresponds to system (I), the second – system (II). When these values coincide for both systems, the Table contains only one value. Intervals 6, 7, 8 and 9 are filled with measurements from the previous intervals of Table, the part of measurements considered as incorrect is eliminated.

Table 1. Processed intervals

Interval	t_1, date, time (h:min)	t_N, date, time (h:min)	$t_N - t_1$, 10^3 c	N	N'
1	25.V.02 03:10:40	25.V.13 30:56	37.22	75	67
2	26.V.02 03:41:16	26.V.02 14:23:02	38.51	102	86
3	28.V.02 02:27:51	28.V.02 12:56:54	37.74	97	91, 92
4	29.V.02 02:46:31	29.V.02 17:53:56	54.45	118	113
5	30.V.02 01:53:01	30.V.02 12:15:01	37.32	107	100
6 (1∨2)	25.V.02 03:47:13	26.V.02 14:23:02	124.55	160	157, 156
7 (3∨4)	28.V.02 02:27:51	29.V.02 17:53:56	141.97	206	205
8 (4∨5)	29.V.02 02:46:31	30.V.02 12:15:01	120.51	214	214
9 (3∨4∨5)	28.V.02 02:27:51	30.V.02 12:15:01	208.03	307	300, 282
10	22.VII.03 21:51:24	23.VII.03 23:24:32	91.99	80	80

Smoothing results for measurement data at intervals of Table 1 by solutions of systems (I) and (II) are presented in Tables 2 – 7. Tables 2 – 4 were obtained using system (I), Tables 5 – 7 – using system (II). Tables 2 and 5 contain values of parameter κ_c, standard deviation of errors in initial data and standard deviations of smoothing solutions defined parameters – initial conditions and κ_c. Tables 3, 4, 6, 7 contain indicated above characteristics of remainders series ξ_{ik} $(k = 1, 2, \ldots, N)$ for $i = 1, 2, \ldots, 6$. The obtained results relate to intervals of the station low-disturbed motion. At the present time such intervals are short. In particular, there an interval with large disturbances between intervals 2 and 3 of Table 1. Due to short intervals of low-disturbed motion the GPS-measurements cannot be used in full scope to verify models of atmosphere drag influence on the station motion.

Analysis of processing results for intervals 1 – 9 confirms conclusion [1] that system (II) has no advantages over system (I) at relatively short (less than 3 days) intervals. In this case system (I) was slightly more exact too. But such conclusion is true “as average” – with no significant geomagnetic activity. In this case this activity was small . In addition, index F was stable in the period considered. As a consequence, the values of parameter κ_c have not much changed with each interval. (see Tables 2, 5).

Table 2. Parameter κ_c, standard deviations of measurement data errors and of smoothing solutions of system (I).

Interval	κ_c	σ_*	σ_1	σ_2	σ_3	σ_4	σ_5	σ_6	σ_7
		m				mm/s			
1	0.908	13.8	3.0	5.3	4.8	4.5	5.1	4.1	0.012
2	1.000	16.4	4.2	2.2	5.0	2.6	6.2	2.5	0.011
3	1.001	18.1	4.1	1.7	4.5	2.6	5.5	2.2	0.014
4	1.019	21.2	2.4	7.0	2.3	4.9	1.9	6.2	0.006
5	1.022	17.4	2.8	6.0	2.6	4.0	3.1	5.3	0.012
6 (1∨2)	0.926	21.1	3.5	1.2	4.1	2.5	4.9	2.1	0.0012
7 (3∨4)	0.980	33.9	4.9	1.9	5.4	3.2	6.3	2.8	0.0012
8 (4∨5)	0.984	22.8	1.9	5.4	1.8	3.8	1.5	4.8	0.0011
9(3∨4∨5)	0.987	31.4	3.8	1.7	4.2	2.4	4.9	2.1	0.0004
10	0.669	16.9	3.8	2.8	2.0	1.6	3.1	3.5	0.0004

Table 3. Mean square values of series for remainders of system (I) smoothing solutions

Interval	s_1	s_2	s_3	s_4	s_5	s_6
	m			mm/s		
1	9.9	16.9	15.5	44.7	592	8029
2	9.3	20.6	20.6	76.3	600	8000
3	11.8	21.0	20.9	50.5	614	8034
4	20.1	24.1	20.1	58.2	617	8033
5	12.4	21.6	18.0	77.6	641	8039
6 (1∨2)	21.4	21.8	20.3	64.3	597	8013
7 (3∨4)	44.2	33.2	19.3	53.6	614	8033
8 (4∨5)	22.1	25.9	19.7	69.3	629	8036
9(3∨4∨5)	35.8	36.6	19.4	64.0	620	8035
10	7.7	24.2	13.9	25.9	588	8058

Accuracy of station motion definition by data of GPS-measurements corresponds to passport accuracy of coordinate measurement by equipment ACH-2401 (50 m). Accuracy characteristics for coordinate measurement data approximation - values σ_*, s_1, s_2 and s_3 – do not exceed the station geometry dimensions. At the same time the approximation error has a little increase with the smoothing interval increasing that shows some inadequacy of motion equation used.

Table 4. Medians of series of system (I) smoothing solutions remainders modulus.

Interval	m_1	m_2	m_3	m_4	m_5	m_6
	m			mm/s		
1	6.4	11.1	10.9	29.0	531	8033
2	6.6	11.7	10.9	29.9	593	8017
3	7.0	16.9	12.4	33.3	637	8035
4	15.1	16.8	11.9	32.9	614	8027
5	8.7	14.7	12.4	31.6	649	8027
6 (1∨2)	14.8	15.6	10.0	29.7	556	8023
7 (3∨4)	30.9	25.6	11.7	32.9	631	8026
8 (4∨5)	15.8	15.6	11.9	34.4	637	8027
9(3∨4∨5)	23.1	25.1	11.2	33.2	630	8027
10	3.4	22.9	11.4	18.4	578	8059

Table 5. Parameter κ_c , standard deviations of measurement data errors and parameters of smoothing solutions of system (II).

Interval	κ_c	σ_*	σ_1	σ_2	σ_3	σ_4	σ_5	σ_6	σ_7
		m				mm/s			
1	0.974	13.8	3.0	5.3	4.7	4.5	5.1	4.1	0.013
2	0.977	16.4	4.2	2.2	5.0	2.6	6.2	2.5	0.011
3	0.841	18.8	4.3	1.8	4.8	2.7	5.8	2.3	0.012
4	0.945	21.2	2.4	7.0	2.3	4.9	1.9	6.2	0.006
5	0.987	17.2	2.8	6.0	2.6	4.0	3.1	5.3	0.012
6 (1∨2)	0.948	18.4	3.1	1.1	3.6	2.2	4.3	1.8	0.0011
7 (3∨4)	0.897	45.0	6.5	2.5	7.3	4.2	8.4	3.7	0.0015
8 (4∨5)	0.926	23.8	1.9	5.6	1.9	4.0	1.6	5.0	0.0011
9(3∨4∨5)	0.918	35.6	4.7	1.6	5.3	2.9	6.2	2.6	0.0004

Note the stable type of systematic errors in station velocity measurements. The remainders ξ_{6k} (deficiencies of the absolute velocity component approximation by radius-vector) have a constant component of about 8 m/s. In remainders ξ_{5k} (deficiencies on normal to orbit plane) the periodic component dominates with orbital period and amplitude of about 8 m/s. If these systematic components will be deleted, than modified remainders ξ_{5k} and ξ_{6k}, will look alike the remainders ξ_{4k} (deficiencies along the absolute velocity vector).

Table 6. Mean square values of series for remainders of system (II) smoothing solutions

Interval	s_1	s_2	s_3	s_4	s_5	s_6
	m			mm/s		
1	9.5	16.9	15.5	44.7	592	8029
2	9.3	20.6	20.6	76.3	600	8000
3	13.7	21.2	21.3	49.5	616	8037
4	19.9	24.1	20.1	58.2	617	8033
5	12.1	21.6	17.9	77.5	641	8039
6 (1∨2)	12.8	21.5	20.0	64.3	598	8014
7 (3∨4)	67.7	33.2	18.9	53.1	614	8033
8 (4∨5)	25.2	25.9	19.2	69.0	629	8036
9(3∨4∨5)	50.3	34.8	18.9	64.6	620	8036

Table 7. Medians of series of system (II) smoothing solutions remainders modulus.

Interval	m_1	m_2	m_3	m_4	m_5	m_6
	m			mm/s		
1	6.1	11.1	10.9	29.0	531	8033
2	6.3	11.7	11.1	29.7	593	8017
3	5.5	16.7	12.8	31.6	639	8038
4	14.4	16.8	11.9	33.0	614	8026
5	7.4	14.8	12.5	31.8	649	8026
6 (1∨2)	9.1	15.1	9.8	29.2	564	8024
7 (3∨4)	50.2	25.6	11.1	32.5	631	8027
8 (4∨5)	19.9	15.6	11.1	33.3	637	8029
9(3∨4∨5)	24.1	23.8	11.0	31.3	623	8034

ISS MOTION PREDICTION BY DATA OF GPS MEASUREMENTS

Data of measurements at intervals 6 – 8 of Table 1 allows to estimate possible accuracy of such prediction over the period of about 1 day. Each of intervals mentioned integrates two measurement sessions spaced by a day approximately. In such situation it is interesting to estimate error of measurement data approximation for the second more late session by solution of system (I) or (II), defined by the measurements at the first session. Let this approximation will be prediction 1.

Note, that in all variants except one, the error of the station position prediction of 15 h = $54 \cdot 10^3$ s in advance does not exceed 400 m. For one another variant this error does not exceed 600 m.

Obtained estimates of the prediction accuracy related to refined situation. Measurement sessions used for prediction were too long and it was possible to refine the station ballistic coefficient for them. Let's consider what is the prediction without this refinement. Let's isolate on intervals 6, 7 and 8 the initial segments of the data covering about 3 hours. These segments contain 20, 35 and 18 points $(t_k, \mathbf{r}_k)$ respectively. Let's construct the smoothing solutions of systems (I) and (II) for these segments and estimate accuracy of approximation by these solution for data at full intervals. Ballistic coefficient will be considered as constant during the smoothing solution constructing at the short interval and further prediction. Value of this coefficient at interval 6 (7 or 8 respectively) will be taken from the results of measurement data processing at interval 1 (3 or 4 respectively). The procedure described will be named as prediction 2.The results analysis shows that in all considered variants accuracy of the station position prediction of 15 hours in advance does not exceed 400 m. True, points with abscissa $t = 64.8 \cdot 10^3$ s are the areas with measurements, but view of diagrams with remainders (interpolating them) allows to argue that mentioned estimate is really implemented.

Bottleneck of estimates obtained within prediction 2 is selection of ballistic coefficient. Here its values are selected in the best manner. On the other hand it is clear that making mistake in selection of this coefficient the non-acceptable large error of prediction can be obtained. Apparently to construct the reliable prediction of the station motion by GPS-measurements the enough long (up to 10 hours) interval shall be provided for these measurements accumulation to refine the ballistic coefficient.

Prediction 2 was constructed for interval 10 too. In this case the ballistic coefficient was taken from results of data processing at the whole interval. For prediction construction the system (I) was used and 61 points $(t_k, \mathbf{r}_k)$, having occurred within the three-hour initial segment. The prediction was acceptable. That is not surprising because it was performed over 75 % of measurements.

CONCLUSION

Accuracy of the station low-disturbed motion definition by data of GPS-measurements of its Greenwich coordinates corresponds to passport data for equipment ACH-2401. Mean-square values of error for these data approximation at intervals of no less than 1 day do not exceed 30 m.

Data of GPS-measurements for station velocity contain the stable systematic errors. Errors of absolute velocity components calculated by measured phase vector within the natural trihedral are in the following form. Component on normal to the trajectory (this normal is practically collinear to radius-vector) contains a permanent error of about 8 m/s. Component on binormal to the trajectory (normal to the orbit plane) contains a periodic error with orbital period and amplitude of about 0.8 m/s. To clear nature of these errors the analysis is required of pseudo-range and pseudo-velocity measured by equipment ACH-2401.

With correct selection of the ballistic coefficient the data of GPS-measurements of station coordinates during the three-hour interval allow to predict its motion of 15 hours in advance with an error in radius-vector does not exceeding 400 m.

REFERENSES

1. **M.Yu. Belyaev, E.S. Medvedev, D.N. Rulev,V.V. Sazonov** "Taking into account of atmosphere density variations caused by solar activity to the precision of determination of motion parameters of ISS.", KIAM preprint ,RAS, № 23 , 2004.