SUN-EARTH L1 REGION HALO-TO-HALO ORBIT AND HALO-TO-LISSAJOUS ORBIT TRANSFERS

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ABSTRACT

Practical techniques for designing transfer trajectories between Libration Point Orbits (LPOs) are presented. Motivation for development of these techniques was provided by a hardware contingency experienced by the Solar Heliospheric Observatory (SOHO), a joint mission of the European Space Agency (ESA) and the National Aeronautics and Space Administration (NASA) orbiting the L1 point of the Sun-Earth system. A potential solution to the problem involved a transfer from SOHO's periodic halo orbit to a new LPO of substantially different dimensions. Assuming the SOHO halo orbit as the departure orbit, several practical LPO transfer techniques were developed to obtain new Lissajous or periodic halo orbits that satisfy mission requirements and constraints. While not implemented for the SOHO mission, practical LPO transfer techniques were devised that are generally applicable to current and future LPO missions.

1. INTRODUCTION

To date there have been several spacecraft orbiting either the L1 and/or L2 Lagrange, or libration, points of the Sun-Earth/Moon barycenter system. In the future we may expect many more solar and astrophysics missions to take advantage of the observational benefits associated with these locations. Renewed interest in the possible use of the collinear points of the Earth-Moon system has arisen recently as well. The use of Libration Point Orbits (LPO) has been accompanied by a rich and growing literature on the design, modeling, and analysis of a variety of orbits in the restricted three-body problem (RTBP). Less attention has been paid, however, to the problem of transfers between distinctly different LPOs; in particular, the problem of transferring between two LPOs that do not intersect. The problem of how to design such transfers recently arose as a practical problem for a currently operational L1 mission. The question was posed: how does one design such a transfer, and conduct it efficiently with the existing operational support software and with regard to mission constraints including remaining fuel?

The event spurring this question was a contingency occurring with the ESA/NASA Solar and Heliospheric Observatory (SOHO), history's second and longest serving L1 orbiter (launched December 1995). In May 2003, trouble with the High Gain Antenna (HGA) gimbal motor led to the HGA being left in a fixed position. This condition made high data rate contacts unachievable over long portions of a revolution, which jeopardized not only the science return but the continued viability of the mission itself. In response, a study was conducted to investigate the possibilities for transferring SOHO to an LPO of different dimensions in hopes of improving the HGA-to-Earth coverage geometry.

The investigation took as its starting point the established SOHO LPO—the periodic type commonly referred to as a 'halo' orbit—and considered transfers to both halo-type and the non-periodic Lissajous-type orbits of distinctly different dimensions. Though techniques are possible involving a return to the Earth for a lunar swingby-assisted transfer to a different LPO, the work described here is limited to transfers conducted entirely within the L1 region. Practical transfer solutions were constructed for both types and found to be affordable fuel-wise but not advisable for SOHO due to past anomalies that have seriously degraded its capability to perform orbit maneuvers of large size [1]. Nevertheless, practical LPO transfer techniques were devised that have general applicability.

2. THE SOHO MISSION HALO ORBIT

SOHO's halo orbit is referred to as a Class 2 orbit, indicating that its sense of revolution about L1 is counter-clockwise as seen from Earth. Both halo and Lissajous orbits are described with respect to the usual synodic reference frame of the RTBP, commonly called the Rotating Libration Point (RLP) frame in work at Goddard Space Flight Center (GSFC). The x-axis of the L1-centered RLP frame points toward the Earth-Moon barycenter, the z-axis points up toward the North Ecliptic Pole (NEP), and the y-axis completes the right-handed triad. The RLP system should be assumed herein, except for impulse variables for which the reference frame is described in Section 3.

Because of solar radio interference constraints, SOHO is required to avoid the region within 4.5 degrees of the Sun called the Solar Exclusion Zone (SEZ). The minimum halo z-axis amplitude, A_Z, needed to satisfy the above SEZ requirement was 120,000 km, the value selected during mission design. Richardson shows in his development of the formulation for periodic halo orbit motion about the collinear points [2] that the non-linearities of the problem force a relationship between the z-amplitude and y-amplitude. Richardson provides an algebraic expression that yields a y-amplitude (A_Y) of 666,672 km when A_Z is 120,000 km. Also, because the motion in the xy plane is coupled, the corresponding x-amplitude (A_X) for this case is 206,448 km.

The nominal SOHO halo orbit is depicted in the RLP xy- and xz-projections in Fig. 1 and Fig. 2, respectively. For contrast, Fig. 1 and 2 also show the small-amplitude Lissajous orbit belonging to NASA's Advanced Composition Explorer (ACE) mission. Additional information on the SOHO mission can be found in [1].

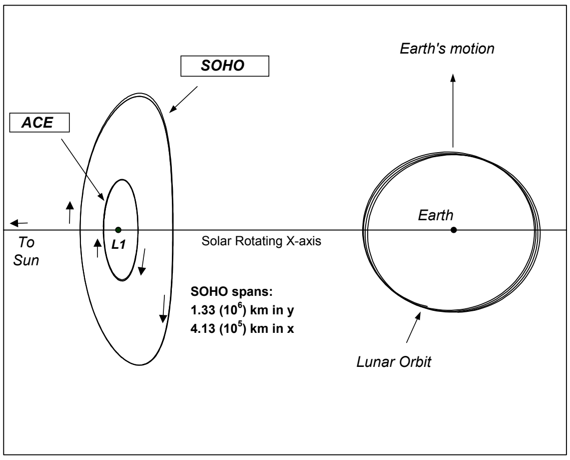

Fig. 1. SOHO Halo RLP XY Plane Projection

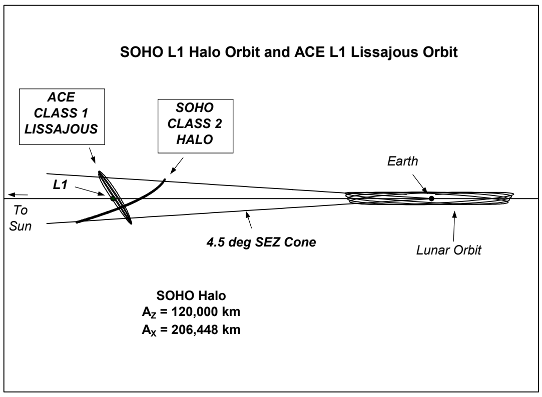

Fig. 2. SOHO Halo RLP XZ Projection

3. LPO TRANSFER TECHNIQUE

Peculiarities of the SOHO anomaly led to two lines of inquiry, both of which were well suited to an initial exploration of the transfer problem. The first asked what is required to change A_Y (in particular, to reduce A_Y) while leaving A_Z unchanged? The second asked, what is required to change A_Z (in particular, to increase A_Z) with no change required to A_Y?

The transfer problem is seen to have two main parts. First, a target LPO must be selected, which means that mission requirements, goals, and constraint criteria must be considered to arrive at an appropriate choice of target LPO type and overall dimensions. The second part of the transfer problem then consists of designing a transfer trajectory arc connecting the initial and final LPOs. The design of the final LPO also requires selection of a target point, or patch point, for the transfer arc to intersect.

For the problem of target—or destination—LPO selection, two things are apparent at once. First, the target LPO should be of the same class as the initial LPO. Attempting to jump onto an LPO of the opposite class—that is, one with motion in the opposite direction—would necessarily be prohibitively costly. Second, a minimum of two impulses—an impulse to initiate departure from the original LPO and an insertion impulse on the target LPO—would seem necessary. Thus, in addition to selecting target LPO dimensions, departure and insertion locations must be chosen. Finally, coordinates (position and velocity components) for the target insertion location must be calculated.

The trajectory propagation and targeting software used for this work is the same as that used for flight operations—namely, the program called '*Swingby*' [3]. Targeting within *Swingby* employs an iterative differential corrections (DC) process. The user constructs a targeting scheme consisting of independent targeting variables (components of the impulses) and dependent variables (targeting goals). Generally a scheme is constructed such that the number of independent variables equals the number of dependent variables to provide the differential corrector with a 'square' problem to solve. Dependent variables may be Cartesian position components, velocity components, or a mixture. Independent and dependent variables both may be separated in space and time. Swingby propagates trajectories numerically with full operations-level force modeling invoked.

Four trial cases were developed for both the A_Y reduction and the A_Z amplification transfer problems. In all cases the impulse variables used are with respect to a spacecraft-centered delta-V coordinate frame. In this frame, the x-axis points to the Sun, the z-axis points toward the NEP, and the y-axis completes the right-handed system. All references to impulses (i.e., delta-Vs, or burns) herein are with respect to this system, which was originally defined for the SOHO mission. (It was naturally suited to SOHO's solar-pointing orientation and thruster configuration, but is well suited to LPO work generally because its axes are virtually parallel with the RLP and related synodic systems.)

3.1 A_Y Reduction Transfer Design

The question of reducing the A_Y of a halo orbit is examined first. In the case of SOHO, its halo is very near the minimum A_Y halo. Periodic halo orbits must have an $A_Y \geq 654{,}276$ km [4]. Smaller *y*-amplitude LPOs are necessarily of the Lissajous type. Therefore, for SOHO, any substantial reduction of A_Y necessarily means the transfer would be to a Lissajous orbit.

In his development of the third-order solution to the equations of motion for LPOs, Richardson [2, 5] shows that for Lissajous orbits both A_Z and A_Y are free parameters (unlike for periodic halo orbits). (For both halo and Lissajous orbits, A_X and A_Y are related via a proportionality constant.) In fact, to fully specify a Lissajous orbit, four parameters are required. In addition to A_Z and A_Y, two phase angles ϕ_{xy} and ψ_z—pertaining to the in-plane and out-of-plane oscillations—must be specified. In short, specifying the amplitudes establishes the overall dimensions, while the phase angles establish a given point on the orbit. The selection of ϕ_{xy} and ψ_z also establishes the 'class' of orbit, i.e., its sense of revolution about L1 as seen from Earth. (The angle ϕ_{xy} is measured in the RLP *xy*-plane clockwise from the –*x*-axis, and the angle ψ_z is the phase with respect to the z-axis.) In summary, given the four parameters of the desired Lissajous, the third-order theory yields the state coordinates of the desired target insertion point.

How to choose the target insertion point and the transfer arc duration become the next two questions. An immediate answer to the first was lent by the past practices of targeting both Earth-to-L1 transfers and LPO stationkeeping maneuvers to target coordinates on the $y = 0$, or *xz*, plane. *Swingby* actually facilitates such targeting via a user-selectable propagator stopping condition feature that identifies RLP plane crossings. Experience with LPO stationkeeping and Lissajous control helped suggest an answer to the second. It seemed a minimum of one-half a revolution about L1 would be necessary to keep transfer costs reasonable. The total period of revolution in the halo orbit is about 178 days, though interestingly the flight time from the Earth-side *xz*-plane to the Sun-side *xz*-plane is 90.5 days while the return to Earth-side takes only 87.5 days.

At the time this work began, SOHO was coming up to a crossing of the Earth-side *xz*-plane occurring on July 7, 2003. It was this *xz*-plane impact point state that was used as the nominal halo departure point for all transfer cases developed. Thus, the approach devised is that a departure impulse is targeted on one or more coordinate goals (e.g., the position on the target LPO) at a specified future *xz*-plane crossing. It was decided to target three of the trial cases on the Sun-side *xz*-plane, i.e., ½ revolution from the Earth-side departure point. The fourth case targeted a return to the Earth-side crossing, making for a full revolution transfer arc. Once the *xz*-plane impact point is successfully achieved, the remaining step is to apply the LPO insertion impulse. The insertion impulse supplies the difference between transfer arc arrival velocity vector components and the three components of the target LPO velocity.

Detailed development of the A_Y cases is now considered. The first three cases—AY1, AY2, and AY3—involve reducing A_Y to 500,000 km (resulting in a Lissajous) using three targeting scheme variations. Since it was not the intent to change A_Z, the amplitudes used to construct the Lissajous were $A_Z = 120{,}000$ km and $A_Y = 500{,}000$ km. Cases AY1 and AY2 involved ½ revolution transfers to the Sun-side *xz*-plane crossing, hence to complete specification of the target Lissajous insertion state the selected phase angles were $\phi_{xy} = 0$ and $\psi_z = -90$ (herein, units for the phase angles are understood to be degrees). For SOHO, this target phase had the additional attraction that it yielded a Lissajous that would have a halo-like trace in the *yz*-plane, avoiding the SEZ for at least a revolution or two. For Case AY3, the transfer involves a full revolution return to the Earth-side *xz*-crossing, so the phase angles needed were $\phi_{xy} = 180$ and $\psi_z = +90$. Also different for Case AY3 is the introduction of a *z*-axis impulse located at the –*y*-extremum, which is mid-way between the departure and insertion burns. The fourth case, AY4, involves a ½ revolution transfer to the Sun-side to achieve an even smaller Lissajous, i.e., $A_Y = 300{,}000$ km, with insertion state specified by $A_Z = 120{,}000$ km, $\phi_{xy} = 0$, and $\psi_z = -90$.

The four targeting schemes—a unique scheme for each of the four A_Y cases—are given in Table 1. In the second column of Table 1, D = Departure, I = Insertion, and Z = *z*-axis burn. The Cartesian velocity components are denoted *Vx, Vy,* and *Vz.* To interpret the table, we take Case AY3 as an example. Recall that the departure burn is located at the Earth-side *xz*-plane and is targeted

Table 1. A_Y Reduction Targeting Schemes

Case	ΔV	Impulse (ΔV) Variables (components)	Goal Variables at xz-plane	Goal xz-plane at Impulse plus
AY1	D	*x, y, z*	*Vx, Vy, Vz*	½ rev
	I	*y*	*Vx* = 0	1 rev
AY2	D	*x, y*	*x, y*	½ rev
	I	*x, y, z*	*Vx, Vy, Vz*	n/a
AY3	D	*x, y*	*x, y*	½ rev
	Z	*z*	*z*	¼ rev
	I	*x, y, z*	*Vx, Vy, Vz*	n/a
AY4	D	*x, y*	*x, y*	½ rev
	I	*x, y*	*Vx, Vy,*	n/a

n/a = not applicable

on the x and y position coordinates of the target Lissajous located at the Sun-side xz-plane (½ revolution away from departure). The z-burn, located at the $-y$-extremum (¼ revolution removed from the xz-plane, as the motion of this Class 2 orbit is from the Earth-side crossing toward the $-y$-extremum), is targeted on the z-coordinate at the same Sun-side xz-plane. However the departure burn and the z-burn are differentially corrected simultaneously, making for a 3-by-3 DC problem. Once the xz-plane insertion point is achieved, the insertion burn—also a 3-by-3 DC problem for Case AY3—follows. For brevity's sake, variable values are not given.

For AY3, the z-axis impulse is used to exert precise control over the z-coordinate at the target xz-plane to achieve exactly the Lissajous position as calculated by the third order approximation. For the other three cases, a looser strategy is allowed with regard to the z-position.

3.2 A_Z Amplification Transfer Design

The technique for changing A_Z follows a development similar to that discussed in Section 3.1, though in order to modify A_Z the z-axis burn becomes essential as part of a 3-impulse transfer problem. The z-axis burn is placed at the y-extremum for maximum effect on A_Z and to produce a transfer arc that approaches the destination LPO tangentially. For SOHO, changing A_Z meant increasing it (decreasing it would incur SEZ violations). But in increasing A_Z, one may go to either another (larger) halo or to a Lissajous. Choosing a halo requires that Richardson's third-order analytic solutions for periodic orbits [2] be used to construct an insertion state for the target halo. Richardson shows how both A_Z and A_Y are constrained by an algebraic relationship, and that the phase angle ψ_z is functionally related to ϕ_{xy} as well. Specifically, $\psi_z = \phi_{xy} + (n\pi/2)$, where $n = 1, 3$. (The 'n' serves as a solution bifurcation "switch", where $n = 1$ generates the Class 1 solution and $n = 3$ the Class 2 solution.) Thus in the periodic formulation, halo orbits need be fully specified by just two parameters, A_Z and ϕ_{xy}.

A_Z amplification Cases AZ1 and AZ2 both employ ½ revolution transfers to Sun-side xz-plane insertions ($\phi_{xy} = 0$ and $\psi_z = -90$), and do not aim to change A_Y (hence, $A_Y = 667{,}000$ km). The Case AZ1 transfer is to a large Lissajous specified by $A_Z = 205{,}785$ km. The Case AZ2 transfer target is an even larger Lissajous specified by $A_Z = 291{,}570$ km.

Cases AZ3 and AZ4 both target a transfer to a large halo orbit of $A_Z = 296{,}000$ km. Given the amplitude constraints of halo orbits, from the third-order theory the new A_Y and A_X become 726,437 km and 224,954 km, respectively. Case AZ3 involves a ½ revolution transfer to the Sun-side xz-plane, which requires $\phi_{xy} = 0$. Given the halo phase angle constraint relationship for Class 2 orbits (i.e., $n = 3$), then necessarily $\psi_z = -90$. Finally, Case AZ4 targets a full revolution return to the Earth-side xz-plane, which is specified by $\phi_{xy} = 180$ (and $\psi_z = +90$).

The targeting schemes for the four A_Z amplification cases are given in Table 2. Note the scheme is the same for AZ1 through AZ3.

Table 2. A_Z Amplification Targeting Schemes

Case	ΔV	Impulse (ΔV) Variables (components)	Goal Variables at xz-plane	Goal xz-plane at Impulse plus
AZ1	D	x	x	½ rev
thru	Z	z	z	¼ rev
AZ3	I	x, y, z	Vx, Vy, Vz	n/a
AZ4	D	x	x	1 rev
	Z	z	z	¾ rev
	I	x, y, z	Vx, Vy, Vz	n/a

n/a = not applicable

For all cases, the departure burn and z-burn (again, ¼ revolution apart) are targeted simultaneously in a DC 2-by-2 problem.

4. LPO TRANSFER CASE RESULTS

4.1 A_Y Reduction Transfer Cases

The delta-V results for the A_Y cases (including z-burn direction for Case AY3) are given in Table 3.

Table 3. Delta-V Results for the A_Y Reduction Cases

Case	Departure Burn (m/sec)	Z-axis Burn (m/sec)	Insertion Burn (m/sec)	Burn Totals (m/sec)
AY1	63.72	none	4.33	68.05
AY2	1.19	none	54.73	55.92
AY3	0.36	+1.31	61.53	63.20
AY4	6.09	none	123.00	129.09

Fig. 3 shows the Case AY1 and AY4 orbits in the yz-projection. The Case AY1 transfer arc is seen together with two close-together revolutions of the target Lissajous. Note that the Case AY4 transfer trace follows that of the nominal halo so closely that it cannot be discerned in this projection. Fig. 4 shows the AY2 case including a 5-year Lissajous trace. The transfer arc is barely discernible (it appears at the $-y$-extremum at the left) in this yz-projection. The Case AY3 transfer arc and Lissajous orbit (not depicted) are very similar to, and close neighbors of, those of both Case AY1 and AY2. This suggests that a loose z-control strategy—i.e., neglecting the z-burn—is adequate at least for the case of departing a halo for a Lissajous with the same A_Z.

For a total of 55.9 m/sec, Case AY2 represents the least costly transfer to a Lissajous of A_Y = 500,000 km.

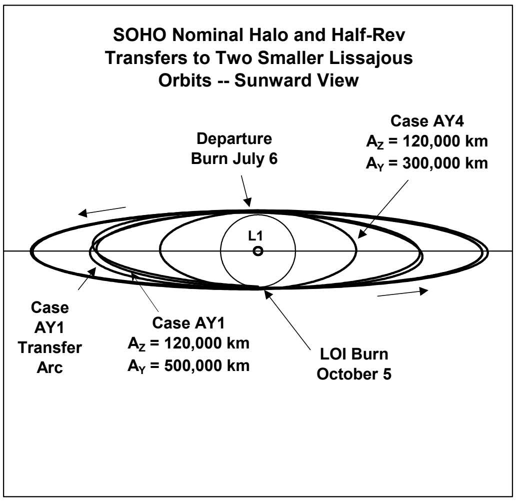

Fig. 3. Case AY1 and AY4 Transfers (RLP frame)

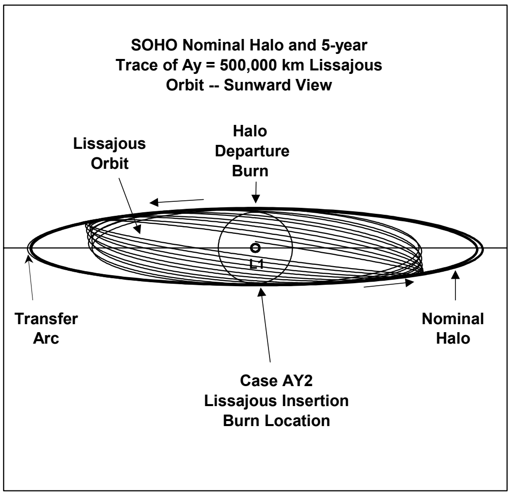

Fig. 4. Case AY2 Transfer and Lissajous (RLP frame)

4.2 A_Z Amplification Transfer Cases

The delta-V magnitude results for the A_Z cases are given in Table 4. The ± sense of the x-component of the departure impulse and of the z-axis impulse is indicated.

Table 4. Delta-V Results for the A_Z Amplification Cases

Case	Departure Burn (m/sec)	Z-axis Burn (m/sec)	Insertion Burn (m/sec)	Burn Totals (m/sec)
AZ1	–0.20	–35.0	8.75	43.95
AZ2	–0.63	–70.0	16.05	86.68
AZ3	–0.77	–70.22	29.77	100.76
AZ4	–1.11	–76.52	28.54	106.17

Fig. 5 shows the transfer arcs and resulting Lissajous orbits in the *yz*-projection for Cases AZ1 and AZ2. Again, in this projection the transfer arcs follow the traces of both the original halo and the destination Lissajous orbits so closely that only an intermediate portion of them are discernible. Fig. 6 through Fig. 8 show the halo-to-halo transfer of Case AZ4 in the three planar projections. Case AZ4 achieves a true halo orbit, as demonstrated by the 4-year halo trace (Fig. 6). (The "splitting" seen in the *yz*-projection of the destination halo (Fig. 6) is an Earth eccentricity effect.) For some reason yet unexplained, the destination orbit achieved for Case AZ3 (not depicted) was not quite a periodic orbit. Though initially halo-like, when propagated it was seen that the orbit was actually a slowly evolving Lissajous. For that reason, Case AZ4 with its full revolution transfer arc would have to be preferred (assuming a halo is required) despite the fact it was several percent more costly than Case AZ3. The total transfer cost for Case AZ4 was about 106 m/sec.

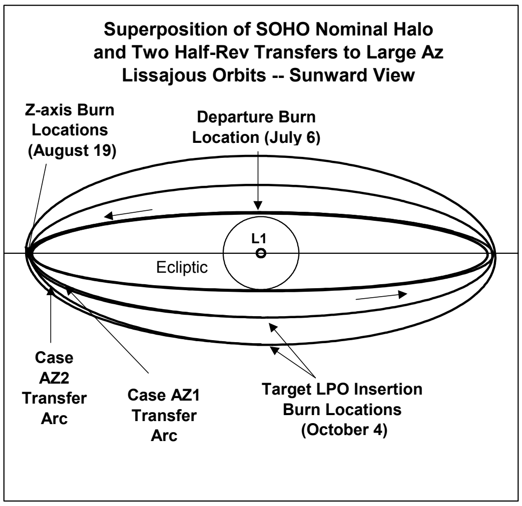

Fig. 5. Case AZ1 and AZ2 Transfers (RLP Frame)

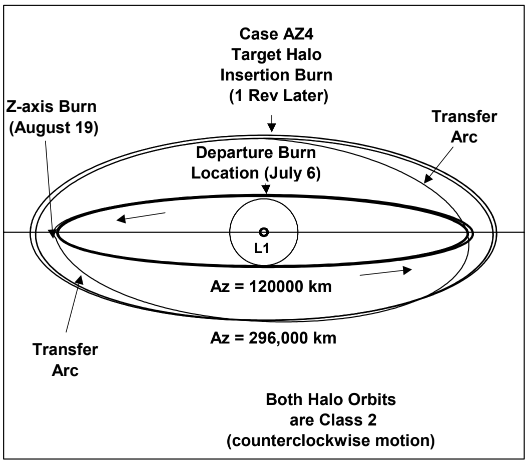

Fig. 6. Case AZ4 Transfer RLP *yz*-projection

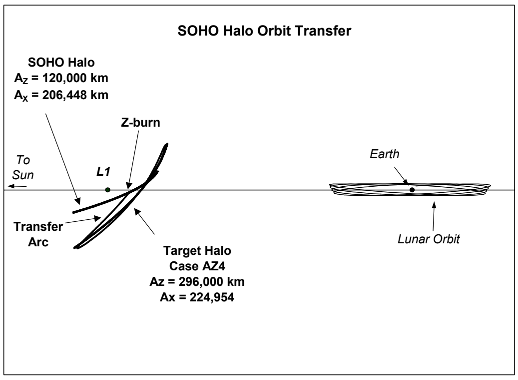

Fig. 7. Case AZ4 RLP *xz*-projection

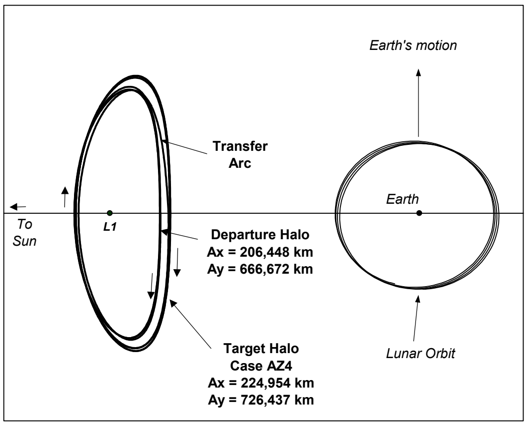

Fig. 8. Case AZ4 RLP *xy*-projection

5. CONCLUSION

All variations of the LPO transfer targeting techniques proved successful, though those corresponding to Cases AY3 (for A_Y reduction) and AZ4 (for A_Z amplification to a periodic halo) are likely best for general applicability. For A_Y modification, however, if somewhat looser *z*-axis control over the achieved Lissajous is tolerable, the Case AY2 variation (which dispenses with the *z*-impulse) should be considered for the sake of potentially smaller delta-Vs. Though the Lissajous Case AY1 DC targeting problem (with its large departure impulse) proved readily tractable, it is not clear that such behavior could be expected generally. But as it required the most delta-V of the three A_Y reduction techniques, it likely would not be the technique of choice in other cases.

As regards the delta-V magnitudes of the various cases, it is notable that the spread is modest. For the cases with transfers to the same (or neighboring) LPOs, the most expensive cases were no more than 22 percent more costly than the least expensive. It is not claimed that these techniques are necessarily fuel-optimal, as that larger issue was not explored. The techniques were developed during a short-window effort to address an urgent operations problem. The goal was to develop practical transfer techniques using existing software and methodologies. Since the transfer arcs developed have flight times between *xz*-plane crossings comparable to that of the original halo (within 3.3 percent) *and* achieve virtually tangential approaches to the target LPOs, they at least must be considered reasonably efficient.

An important point relevant to practical operations is that SOHO—as well as most spacecraft—would need to perform these impulses in components. For example, for SOHO the Case AY2 cost rises from 55.9 to 71.2 m/sec when performed in components. Likewise for Case AZ4, the cost increases from 106.2 to 115.1 m/sec.

In the end, SOHO's orbit was not changed thanks to a less risky alternative solution to the contingency having been found. Yet the LPO transfer techniques described here have potential applicability beyond just SOHO to virtually any LPO mission. Though the cases examined were limited in number and scope, it seems likely that these transfer techniques could be just as successful for modifying the *y*- and *z*-amplitudes in either direction and achieving any set of desired amplitudes. Applicability to L2 and lunar LPOs is also expected. However, these suppositions beg further research. Finally, in cases where A_Z amplification is desired, the suitability of a Lissajous versus a periodic halo should be investigated, as the delta-V costs could be less.

6. REFERENCES

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