SAFE SWITCHING OF THE GRACE FORMATION USING AN ECCENTRICITY/INCLINATION VECTOR SEPARATION

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ABSTRACT

After more than two years in orbit, a longitude swap maneuver is required to exchange the leading and trailing spacecraft of the GRACE formation. While the two satellites are nominally separated by about 200km in along-track direction, a close encounter will take place during the swap sequence. Based on the successful use for co-location of geo-stationary satellites, the concept of eccentricity/inclination vector separation is suggested for safe proximity operations in this mission phase. Taking care of the natural evolution of the relative orbital elements of GRACE 1 and 2, optimum maneuver dates are identified. By proper timing of the maneuvers a safe limit for the minimum distance during the encounter can then be guaranteed even in case of arbitrary thruster performance errors. This allows the use of a fuel optimal maneuver sequence with single drift start and stop maneuvers.

1. INTRODUCTION

The GRACE mission, which was launched at March 17, 2002, is among the first missions ever to exercise close formation flying to achieve its advanced science goals. The joint US-German project aims at the exploration of temporal variations of the Earth's gravity field and the investigation of the Earth's atmosphere. Key payload elements comprise a GPS receiver, a high-precision accelerometer and a star sensor onboard each spacecraft as well as a K-band inter-satellite ranging system. Both spacecraft are flying at a 300-500 km altitude with a relative along-track separation of about 220±50 km (or 30±6 secs) to build up a gradiometer that is particularly sensitive to high-order harmonic components in the Earth's gravity field.

Mainly to balance the surface erosion of the on-board K-band radar, both satellites have to exchange their position (leader/trailer) at least once during the mission lifetime. GRACE has now entered the third year of a nominal 5 years mission and it is planned to do such a position exchange maneuver in the second half of 2004. In an effort to maximize the operational safety of the satellite swap maneuver with no increase in fuel expenditure, an eccentricity and inclination vector separation is applied. The concept of e/i-vector separation has originally been developed for the colocation of geostationary satellites, but can likewise be applied to avoid a collision risk during proximity operations of satellites in low Earth orbit. Using a specific phasing and size of the relative orbital inclination and the relative eccentricity, a minimum radial or cross-track position offset between two spacecraft can always be ensured irrespective of their along-track separation. This is of particular concern if uncertainties in the maneuver calibration or differential drag modeling do not allow an accurate forecast of the along-track motions. Likewise the ever-present risk of maneuver failures can easily be covered by application of an e/i-vector separation.

As a result of natural orbital perturbations, the relative e/i-vectors of the two GRACE satellites vary in time and attain an optimal, parallel configuration once every 47 days. A fuel optimal and safe separation of both spacecraft during the fly-by can thus be achieved by proper selection of the maneuver date. In adopting the e/i-vector separation, single maneuvers are sufficient to initiate and stop the drift required for the swap of Grace 1 and 2, which notably reduces the operational complexity and workload.

This paper presents the current orbit properties with respect to the eccentricity and inclination vector variations, the algorithms needed to apply the proposed strategy and an error analysis comprising the uncertainties in the knowledge of the relative drag, the maneuver performance, the location and epoch of the initial maneuver and the prediction of the relative position.

2. CURRENT ORBIT PROPERTIES

The current plans foresee a drift time of one week between the initialization maneuver and the closest approach. Assuming an initial separation of 220 km between the satellites, a Δv of about 12 cm/s is necessary, which corresponds to a semi-major axis offset of about 200 m. First simulations of a switch maneuver showed oscillations of the relative distances with +900/-1300 m in radial and ±250 m cross direction (refer to Fig. 1), where the radial center shows the 200 m offset. The reasons for these variations are a difference in the right ascension of the ascending node

($\Delta i \approx 0$) for the cross distance as well as differences in the eccentricities and argument of perigees for the radial distance. Further simulations showed changes in the phase difference between the radial and cross curves, which suggests that the geometric conditions of the fly-by are time dependent.

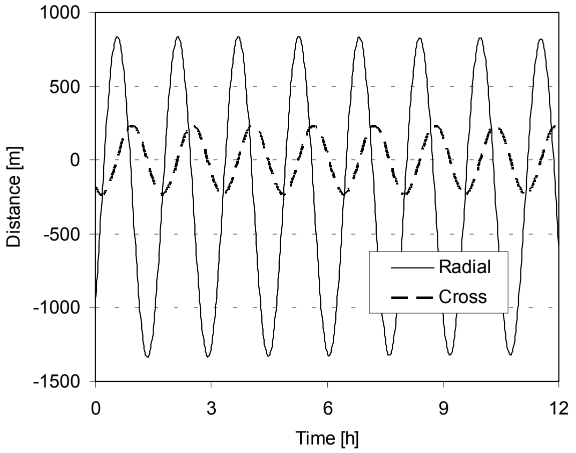

Fig. 1 Radial and cross distance around closest approach

In the following, the relative motions in radial direction (relative eccentricity) and cross-track direction (relative inclination) are analysed.

2.1 Eccentricity Vector

The asphericity of the Earth results in a variety of short periodic, long-periodic and secular perturbations of the orbital elements of a LEO satellite. According to [1, 2, 3], it is sufficient to consider only the long-periodic eccentricity vector variations.

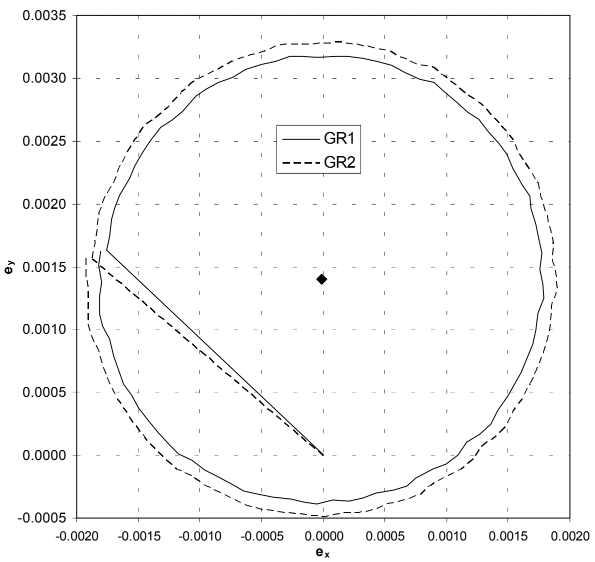

Fig. 2 Eccentricity vector evolution of both orbits

For near-circular satellite orbits, the mean Keplerian elements e (eccentricity) and ω (argument of perigee) are commonly replaced by the mean eccentricity vector

$$\bar{\vec{e}} = \begin{pmatrix} \bar{e}_X \\ \bar{e}_Y \end{pmatrix} = \bar{e} \cdot \begin{pmatrix} \cos\bar{\omega} \\ \sin\bar{\omega} \end{pmatrix} \qquad (1)$$

Fig. 2 shows the evolution of the mean eccentricity vector for both orbits over 94 days, where the first points with the epoch of 2004/05/01 are marked with a line to the origin. The plot likewise shows a difference in the eccentricity itself as well as in the argument of perigee $\bar{\omega}$.

Building the difference

$$\Delta\bar{\vec{e}} = \bar{\vec{e}}_2 - \bar{\vec{e}}_1 = \delta\bar{e} \cdot \begin{pmatrix} \cos\bar{\varphi} \\ \sin\bar{\varphi} \end{pmatrix} \qquad (2)$$

for two spacecraft, one obtains the mean relative eccentricity vector $\Delta\bar{\vec{e}}$ that characterizes the relative motion within the orbital plane. While $\delta\bar{e}$ measures the size of the relative trajectory, the angle $\bar{\varphi}$ defines the relative pericenter. In Fig. 3, the mean relative eccentricity vector is plotted for the same period. The phase $\bar{\varphi}$ shows a clear difference to the argument of perigees of Fig. 2, whereas the rate is identical. With the present eccentricity difference of about $1.2 \cdot 10^{-4}$ and a mean semi-major axis of about 6870 km, the altitude difference varies roughly between ±825 m.

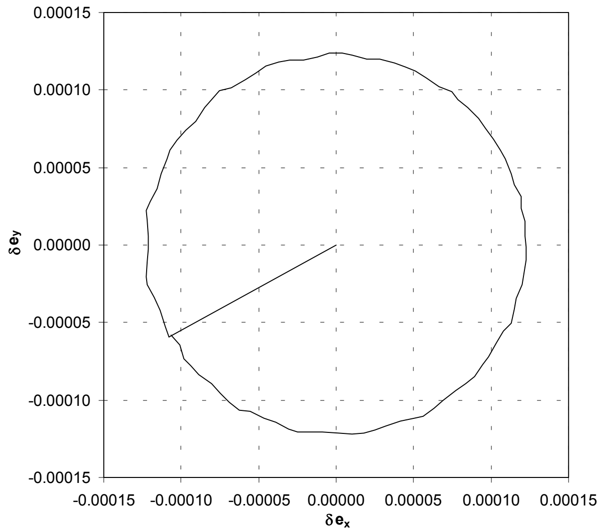

Fig. 3 Relative eccentricity vector evolution

2.2 Inclination Vector

As inclination difference the angle between the two orbital planes $\delta\bar{i}$ has to be understood (see also Fig. 4).

According to [1], the mean relative inclination vector can be expressed as

$$\Delta\vec{i} = \sin(\delta i)\cdot\begin{pmatrix}\cos\theta \\ \sin\theta\end{pmatrix} \tag{3}$$

which simplifies to

$$\Delta\vec{i} \approx \begin{pmatrix}\Delta i \\ \sin i\Delta\Omega\end{pmatrix} \tag{4}$$

for small differences in the orbital elements. It's modulus equals the sine of the angle δi enclosed by the two orbital planes while θ is the argument of latitude at which s/c_2 crosses the orbital plane of s/c_1 in ascending direction.

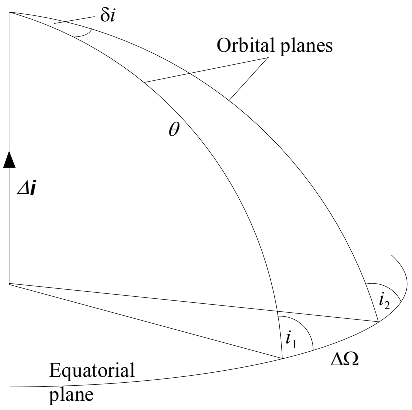

Fig. 4 Relative inclination vector

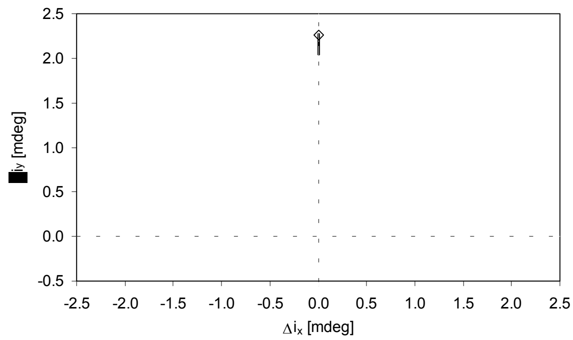

Fig. 5 Relative inclination vector evolution

The vector is plotted in Fig. 5 for the same period as in section 2.1, where the first point is marked by a diamond. Two effects can be recognized by the plot:

1. The phase is constantly 90°, which means the intersection point is always at the most northern point. This is expected as the inclination difference is nearly zero (refer to Eqn. 4).
2. The value of the inclination difference is slowly decreasing, approximately 10% in about 94 days. The reason is a difference in the regression of the nodes caused by the very small inclination difference.

Thus, the largest out-of-plane (cross-track) distance is **always** at the equator crossing. With 6870 km for the semi-major axis and 2 mdeg for the inclination difference, the value for the largest distance is about 240 m.

3. INCLINATION/ECCENTRICITY VECTOR SEPARATION

As shown in the previous chapter, the location of the relative pericenter $\overline{\varphi}$ rotates with a period of about 94 days whereas the argument of latitude θ of the ascending intersection point is more or less constant. The optimum situation would be, if the maximum distance in radial direction (relative pericenter/apo-center) will be reached a fourth orbit after/before the maximum in the cross direction. This can be guaranteed by the separation of the eccentricity and inclination vectors, where $\overline{\varphi}$ has the value of θ or $\theta+\pi$. Thus, the optimum separation occurs each 47 days because of the current orbit properties.

Starting from the last data point of the vector plots with an epoch of 2004/08/02, the next possibilities for an optimal fly by in the current year would be at the following dates

2004/09/01
2004/10/18
2004/12/03 .

The initial maneuver for the fly-by is nominally planned one week before the closest approach. The strategy for the maneuver is to increase the length of the eccentricity vector in order to gain more margin in radial direction. This leads to a maneuver location at an argument of latitude calculated by the following equation

$$\overline{\varphi}_{opt} = \overline{\varphi}_0 + \dot{\overline{\varphi}}\cdot(t_{opt} - t_0 - 7) \tag{5}$$

where $\overline{\varphi}_0$ is the argument of latitude (phase) at t_0, $\dot{\overline{\varphi}}$ is time derivative of $\overline{\varphi}$ with a constant value of 2π/94 rad/day and t_{opt} is one of the dates for the optimum fly by.

With the definition of the relative eccentricity vector in Eqn. 2, the requested increase of the relative eccentricity at $\overline{\varphi}_{opt}$ can be achieved only with a positive (in flight direction) maneuver of satellite #2 or a negative maneuver of satellite 1. If the maneuver direction has to be reversed by other constraints, $\overline{\varphi}_{opt}$ has to be corrected by π.

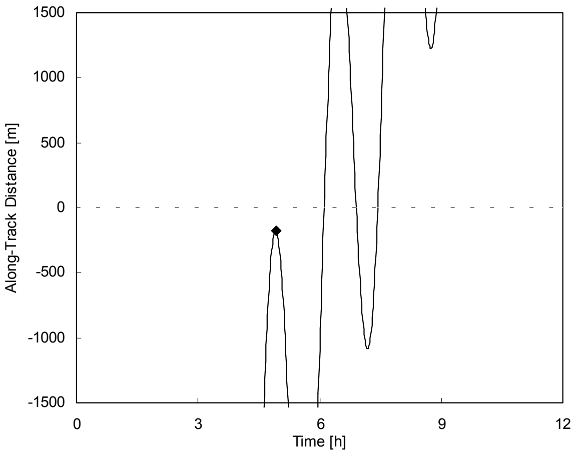

Fig. 6 Along track distance around closest approach

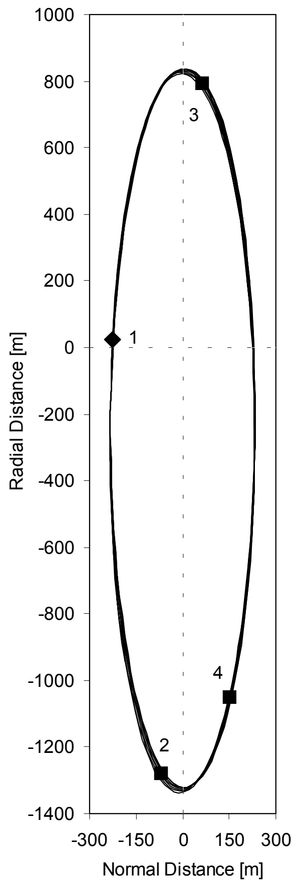

Fig. 7 Relative distances in radial and normal direction around closest approach

For a nominal fly-by, Fig. 6 and Fig. 7 show the relative distances about ±6 h around the closest approach, where the minimum distance is marked with a solid diamond. In addition, the points, where the along track distance is zero, are marked by a solid rectangle in Fig. 7 only and the added numbers indicate the sequence in time.

4. ERROR ANALYSIS

The main uncertainties for the satellite switch maneuver are addressed and discussed in the following.

4.1 Uncertainties in the Knowledge of the Relative Drag

The uncertainty in the knowledge of the relative drag is more ore less based on the uncertainty of the aerodynamic properties of the flying configuration. As the predicted differences in the ballistic coefficient show nominal values of about 1%, an uncertainty of 0.5% was used.

4.2 Maneuver Performance

Assuming the along track separation of the satellite is near to zero, both satellites have the same semi-major axis and the epoch is one of the optimum (see chapter 3), the relative motion of one satellite w.r.t. the reference satellite is along an ellipse, where the maximum radial and normal distances have the values 825 and 240 m (refer to chapter 2).

The planning foresee a maneuver executed with the trailing satellite (sat. #2). Thus the semi-major axis has to be reduced by the maneuver to start the drift to the leading satellite. For a nominal drift of one week until the closest approach, the maneuver has to change the semi-major axis by about -220 m, which shifts the center of the ellipse by this value in radial direction and the maximum negative radial distance by the double value.

A change in the maneuver size influences the increment in the radial distance and the flight time between maneuver and closest approach. For the analysis an error of ±1% was used, which changes the radial distance by about ±2 m and the arrival by ±1.7 h.

4.3 Location of the Initial Maneuver

With a shift in the maneuver location by half a revolution, the center of the ellipse is shifted by the same value as in the nominal case, but now the relative eccentricity would be reduced and not be increased as planned, and as a consequence the maximum radial distance on the positive site would be reduced by the double value of the semi-major axis change!

4.4 Prediction of the Relative Position

It is well known, that a predicted orbit position even based on numerical integration is more or less inaccurate. The main contribution to the total error referred to the reality comes from the uncertainty of the atmospheric model. Both satellites fly in a nearly close formation, so that they cross almost the same environment and the orbit propagation is doing more or less the same absolute errors for both satellites. It can be assumed, that the relative errors are much more less.

To verify this assumption, a comparison of the predicted position with the so-called RSO (rapid science orbit) kindly supplied by JPL for both satellites was done. The position error of the RSO itself is about 5 cm.

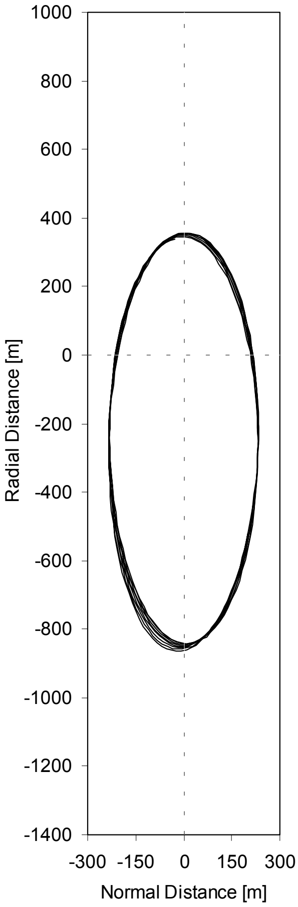

Fig. 8 Relative distances in radial and normal direction around the closest approach for an error of half an orbit in the location of the maneuver

The orbit propagation was performed including the following perturbation:

- Sun/Moon
- Air drag
- Solar radiation
- Grace Gravity Model GGM01S with an order and degree of 120x120
- Solid tides.

The initial orbital elements used for the simulations were generated by an orbit determination using the on-board GPS navigation measurements of one day.

Table 1 clearly demonstrates, that the assumptions are realistic. For the safe fly-by, only the radial and normal directions are important. Compared with the limits of the ellipse, the relative prediction errors are less than 1%.

Table 1. Maximum position errors in radial (r), along-track (t) and normal (n) directions with meter as unit

Period:	1 d			7 d		
Direction:	r	t	n	r	t	n
GR1	9.9	164	2.2	98	12984	3.6
GR2	10.1	176	1.9	99	12930	3.6
Δ	**0.3**	**13**	**0.4**	**3**	**60**	**0.1**

4.5 Error Summary

If the size of the nominal fly-by ellipse is considered as a safe configuration, the reduction of the semi axes can be used for an assessment. The change of mainly the radial axis in chapter 4.1, 4.2 and 4.4 is less than 1%, so that it is not meaningful to show the ellipses. Only the error concerning the location of the maneuver shows a distinct change (refer to Fig. 8). Although the positive max. radial distance is reduced to the size of the max. normal distance, it is recommended to use the optimum location. Referring to the experience of the last executed maneuvers, an exact positioning of a maneuver can be promised.

5. CONCLUSIONS

Based on the current orbit properties of the GRACE formation, the inclination/eccentricity separation can be used for a fuel and operational effort optimized switch maneuver. The shape and size of the relative motion ellipse is stable taking into account the discussed uncertainties for the satellite fly-by.

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