OPTIMIZATION OF NEW 4 S/C FORMATIONS CONSIDERING OPERATIONAL CONSTRAINTS OF THE EXTENDED CLUSTER MISSION

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ABSTRACT

This paper presents the developed constellation manoeuvre strategy for the second extension of the Cluster mission. This requires an overview about the mission in the past which introduces the regions of scientific interest together with the constellations formed. After analysis of the future orbit evolution new scientific regions of interest are identified. Based on them requirements for the future constellations are derived. Limited by a tight fuel budget a suitable constellation could be found that later allows a lot of flexibility during several years with phasing manoeuvres only. The optimization of this constellation will be detailed followed by the presentation of the future capabilities with phasing manoeuvres.

1. INTRODUCTION AND PAST MISSION

The Cluster mission is based on four almost identical spacecraft placed into eccentric polar orbits with a semi major axis of about 12 Earth radii. The orbits cross through the regions of scientific interest of geomagnetic space. They have been selected to ensure coverage of the northern polar-cusp region and the geotail and to optimize the coverage of the magnetopause, the bow shock and the solar wind. More information can be found in [1]. The mission was designed for two years and during that time the constellation should be changed twice per year during a constellation change manoeuvre campaign. Originally it was foreseen to interrupt the science during the whole constellation change which was later reduced to time windows around manoeuvres.

1.1 Two Tetrahedra Strategy Twice per Year

To control the orbit of the satellites the two constellation strategy as described in [2] was developed and applied twice per year. This strategy allows to impose, within one orbital revolution, constellations at two different points in space and this with equal orbital periods. For one constellation restrictions exist on the size and the orientation. The strategy is visualized in Fig. 1 and Fig. 2. For the cusp crossing period in spring two tetrahedra were selected at the north and at the south cusp (598/600km in Fig. 1). The tetrahedra in the figures are projected into the orbital plane. They are not drawn to scale but expanded with an expansion factor. The drawn orbit is the reference orbit. It is a Kepler orbit close to the tetrahedra centres.

During typically six weeks in the summer a full constellation change manoeuvre sequence was executed. It consists of repeated sequences of one manoeuvre at apogee and two manoeuvres at perigee with axial oriented thrusters of the spin stabilized satellites and of two manoeuvres with the radial thrusters within the ascending and descending part of the orbit. More details are given in [3].

Afterwards the constellation for the tail crossing period was kept for six months with two regular tetrahedra located a bit before and after a mean tail surface (2,000 km in Fig. 2).

Then in winter the next full constellation manoeuvre sequence followed to establish the next cusp configuration. This strategy was applied one and a half years.

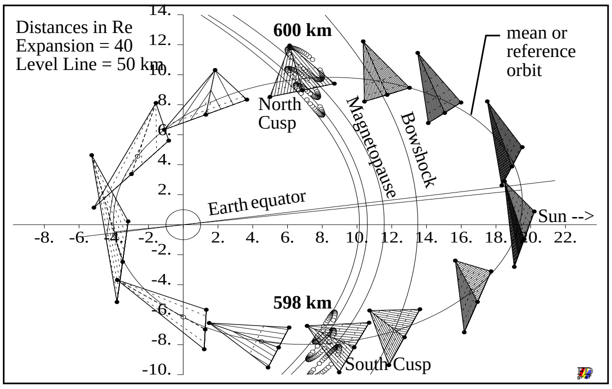

Fig. 1. Cusp crossing constellation March 2001

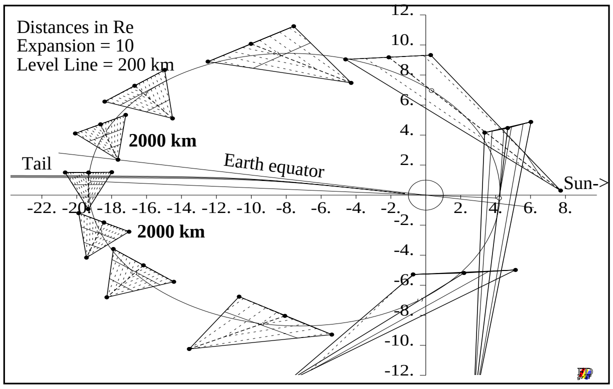

Fig. 2. Tail crossing constellation September 2001

1.2 Two Tetrahedra Strategy Once per Year

During constellation change manoeuvres instruments have to be switched off which reduces the science data return. For the following years an increased data return got priority over a perfect tetrahedron at the south cusp. Due to the evolution as presented later in Section 2.1 the orbit anyhow has been moving more into the less interesting region exterior to the south cusp. Therefore from now on the two tetrahedra were placed at positions such that the first tetrahedron was set up for the tail crossing and the second for the cusp crossing half a year later avoiding one constellation change per year. This scenario was kept during the first mission extension. This strategy is visualized in Fig. 3 and Fig. 4. Basically the cusp tetrahedron is already in place during the tail crossing.

This strategy was proposed for the first mission extension but was applied already a bit earlier. See also Fig. 5.

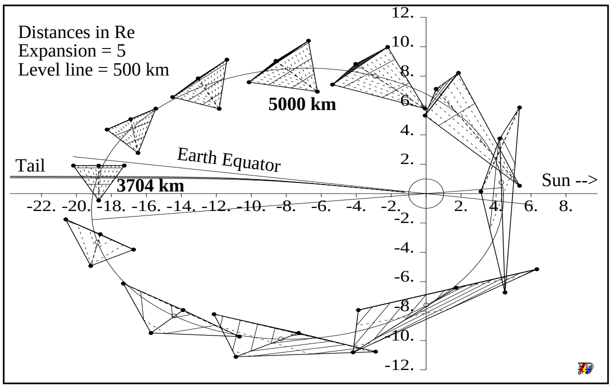

Fig. 3. Tail crossing constellation September 2002

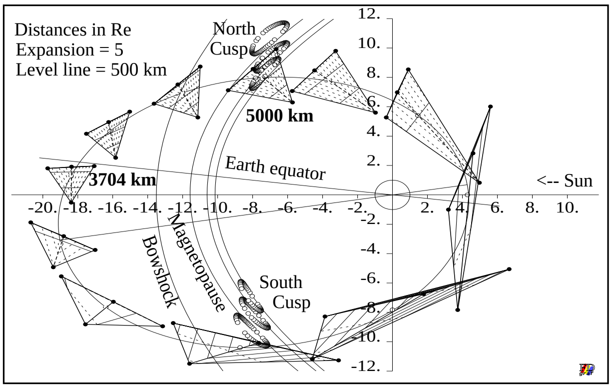

Fig. 4. Cusp crossing constellation March 2003

2. FORMATION FOR 2ND EXTENSION

Due to the uniqueness of the Cluster mission the scientist showed a big interest to get another extension of the mission and ESOC flight dynamics was engaged to work out a proposal including a configuration with a large separation of 10,000 km. This big constellation was promised to the cusp scientist already at the beginning of the mission but postponed several times to allow more science at smaller distances. Going back to smaller distances in between 10,000 km constellations would have cost too much fuel.

2.1 Orbit Evolution

Table 1. Cost of change of mean orbit

Argument of perigee	1.8 kg / degree
Ascending node	3.5 kg / degree
Inclination	4.3 kg / degree
Perigee height	2.4 kg / 1,000 km
1 kg corresponds to velocity change of 7.2 m/s	

Looking at the fuel evolution shown in Fig. 5 and the cost of a pure change of the mean orbit in Table 1 allows the immediate conclusion that only very minor changes of the general orbit evolution could be done. Further 0.4 kg/year are needed for attitude maintenance and 0.2kg/year for phasing manoeuvres.

The average fuel consumption in Fig. 5 is about 1.5 kg per 1000 km change of tetrahedron size. The exceptional high value 2004/2005 comes from an argument of perigee change of the mean orbit which was done to stay closer to the north cusp. It also improved the average visibility from northern ground stations, delayed the re-entry and triggered the trink sensors. From the latter the remaining fuel could be estimated and the fuel book keeping of the command generation subsystem was confirmed. Nevertheless the remaining fuel level can only be estimated to a certain accuracy level of about 5 kg. This amount of fuel has to be kept for each satellite to avoid running out of fuel with any satellite.

The analysis of the orbit evolution was discussed together with the principal investigators. Especially the eccentricity and argument of perigee increases steadily until re-entry of the satellites as can be seen clearly in Fig. 6.

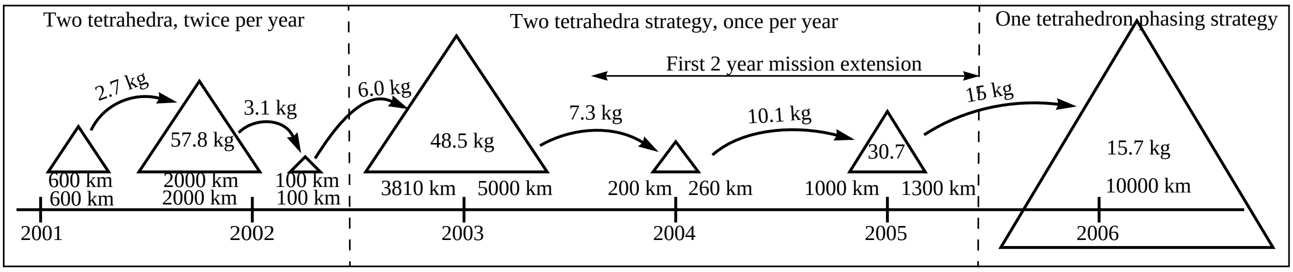

Fig. 5. Tetrahedron size and fuel consumption evolution

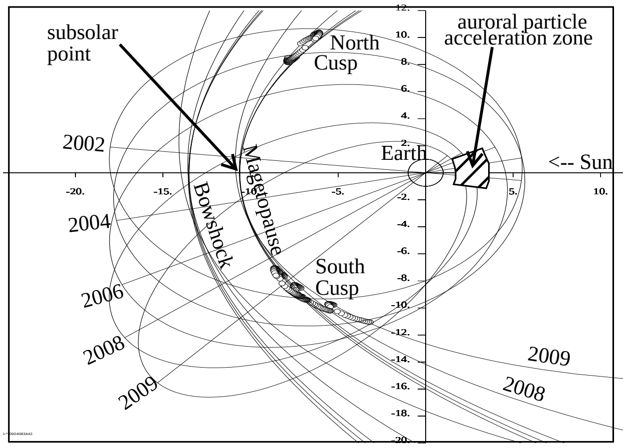

Fig. 6. Orbit evolution

A smaller impact has the inclination increase from 90 deg to 100 deg and further during the last 18 months to 120 deg. The projection of the scientific regions into the orbital plane in Fig. 6 varies a bit more in 2008 and 2009. The reduced perigee allows the study of the auroral particle acceleration zone. The change in the argument of perigee leads to a crossing of magnetopause and bowshock at lower altitudes above the ecliptic and in 2008 even very close to the subsolar point. Also the tail, being close to the ecliptic in autumn, is crossed at lower distances from the Earth. These new regions allow a lot of new research. Together with the outstanding constellation of 10,000 km useful science could be done for at least four further years. Therefore a delay of the re-entry of the satellites should be considered as well. A delay is possible by either increasing the perigee height or by reducing the rate with which the height decreases. The approximate average rate caused by the Sun is given by Eqn. 1. The rate can be reduced by a decrease of the argument of perigee or apogee height.

$$\delta r = \mathrm{Const}\, a^3 a e \sqrt{1-e^2}(\sin i)^2 \sin 2\omega \qquad (1)$$

What can be achieved is of course very limited by the fuel. A moderate change of the mean orbit for this must be combined with the constellation change to 10,000 km since it is much cheaper then. The final optimization presented in Section 3.2 increases the lifetime by a 2,000 km perigee height increase and a 0.7 degree argument of perigee reduction with an additional cost of only 3.9 kg instead of otherwise 6.1 kg (2 · 2.4 + 0.7 · 1.8 from Table 1).

2.2 Requirements for the extended mission

When flying through planar shaped new regions a spacecraft separation of 10,000 km normal to the plane is unfavourable whereas a multi-scale configuration is highly desirable. In such a multi-scale configuration two satellites shall be close together (less than 1,000 km) and form with the remaining two a triangle (10,000 km) within the researched surface.

Together with the knowledge from Section 2.1 the most important requirements for the remaining mission can be summarized as follows:

- Tetrahedron of 10,000 km size for cusp crossing 2005.
- Lifetime extension until end of 2009 if possible.
- Possibility to form multi-scale constellations, but still alternation with more spatial formations shall be possible.
- Keep at least 5 kg of fuel + 0.6 kg / year.

Additional some further operational requirements make the practical execution of constellation changes more difficult:

- Restricted size of radial thruster burn length.
- Ground station visibility for radial burns.

The alternation between multi-scale and spatial constellations is only feasible if the two satellites being close together are nearly in the same orbit. Then the change can be done with cheap phasing manoeuvres. Within the two constellation strategy it is not possible to have two satellites in the same orbit. It has to be given up.

3. MANOEUVRE OPTIMIZATION

3.1 Introduction to Cluster Optimization Software

With an iterative method the constellation manoeuvre calculation software solves a set of equality equations with linear constraints and performs a simultaneous manoeuvre calculation to select the fuel optimal transfer in the case of more unknowns than equations. Targets are specified w.r.t. positions, velocities, periods and arrival times on the reference orbit which is a mean Kepler orbit. Implemented are position targets (each imposing 12 equations), velocity targets (12 equations) and a period target (4 equations). The 24 parameters representing the spacecraft states are unknowns to be solved for. Some of the target parameters can be treated as free so that they also form unknowns.

Associated with a position target are 4 satellite positions in the definition coordinate system. By applying a scaling, a deformation matrix and a change of the orientation by three rotation angles the coordinates are transformed into the work system. Often the work system of the position targets is selected such that the x-coordinates are along the velocity and the z-coordinates are normal to the orbit.

3.2 Target and Constraint Formulation 2005

The requirements of the next constellation change can be fulfilled by optimizing a full manoeuvre sequence, as briefly described in Section 1.1, together with the reference orbit (T0), a period target (T1), three position targets (T2, T3, T5) and one velocity target (T4). The arrival time of T5 is end of year 2009 and the point on the reference orbit is at perigee. For all other targets the reference point is at 172 degrees w.r.t. the Sun direction in September (Fig. 7) and the epoch is September 2005. As long as not mentioned neither deformation nor orientation change are applied between definition and work coordinate system. Here the work system is chosen as described in Section 3.1. Apart from a common time shift the following parameters are treated as free:

- T0: Perigee radius, apogee radius, inclination, ascending node and argument of perigee of the reference orbit.
- T1: No free parameter.
- T2: The definition coordinates of this target are fixed and form a regular 10,000 km tetrahedron. A change of the orientation (three angles) from definition to work system is applied and treated as free.
- T3: Coordinates y,z and satellite specific time shift are free. The x-coordinates are fixed to 0.
- T4: The velocity magnitude and satellite specific time shift are free. Fixed are the velocity directions along track the reference orbit.
- T5: Coordinates y,z and satellite specific time shift are free.

The following linear constraints are set:

- C1: Sum of perigee and apogee height in T0 is constant.
- C2: Equal satellite specific time shifts in T3, T4.
- C3: Equal y,z-coordinates of satellites 3 and 4 in T3.
- C4: The differences between perigee height of the mean orbit and the y-coordinates of satellites 2 and 3 in T5 are equal.
- C5: The average satellite specific time shift in T5 is zero.

Equal velocity directions (T4), positions (C3,T3) and periods (T1) together with C2 forces satellites 3 and 4 to be in the same orbit. Therefore they have the same perigee height. Satellite 2 is forced to get the same perigee height (T5, C4, C5) as it turned out that it would become lower otherwise. T2 forces the constellation to form a regular tetrahedron. Furthermore almost all parameters of the reference orbit are optimized (T0). Only the mean semi major axis has to be fixed (C1) for the reason of convergence. Then the software automatically chooses the cheapest combination from most of the possibilities for extending the lifetime. The possibilities were listed in Section 2.1.

4. CAPABILITY OF PHASING MANOEUVRES

After establishing the 10,000 km tetrahedron with the optimization described in Section 3.2 only phasing manoeuvres, changing the satellite along track position, will be conducted during the following years. In the first subsection of the manoeuvre optimization approach, a short introduction to the deformation matrix will be given. In the subsequent subsections the achievable constellations will be shown. This will demonstrate the wide ranging possibilities for scientific measurements that can be achieved with simple phasing manoeuvres.

4.1 Phasing Manoeuvre Optimization Approach

As already mentioned in Section 3.1 a scaling, deformation matrix and rotation can be applied to calculate the work coordinates of a position target from the definition coordinates. The deformation matrix is a symmetric 3x3 matrix and is the separation matrix of the constellation. The eigenvectors and eigenvalues of the separation matrix can be interpreted as the axes of an ellipsoid. As described in more detail in [2] the distance from the centre of the ellipsoid to a point on its surface is a measure of the satellite separation in that direction. This separation is proportional to the standard deviation of the spacecraft coordinates along that direction centred around the mean coordinate value. In case of a perfect tetrahedron the ellipsoid is a sphere and the eigenvalues are 1. The closer the eigenvalues to 1 the better the spatial constellation.

The optimization software can treat the elements of the deformation matrix as unknowns which allows the following approach for the phasing manoeuvres to maintain a tetrahedron constellation: A sequence of at least two along track manoeuvres at perigee for each satellite is optimized together with one period and two position targets. The arrival time is identical for the three targets and can be chosen to be somewhere in the middle of the period where the orbital plane crosses a region of scientific interest. The point on the reference orbit can be kept as in Section 3.2 at 172 deg w.r.t. the Sun or be shifted by up to 20 degrees as will be shown later.
The x,y,z-coordinates in the definition coordinate system of the first target are set to the same values as in T2 in Section 3.2. Unknowns are three dimensional translation of the origin of the definition coordinate system, scaling factor, orientation (3 angles) and symmetric deformation matrix (6 elements).

Unknowns of the second target are 4 x,y,z-coordinates, scaling factor and three dimensional translation. No deformation is applied to this target.
Te first constraint for the second target is that the sum of all x coordinates is 0. The same applies to y and z. This ensures that a common coordinate shift is put in the three translation parameters. Three other constraints formulate coordinate differences between three pairs of x-coordinates to re-establish the original separation between the satellites apart from a scaling. The x-separation sequence is centred within the current constellation by setting the constraint x-translation equal to 0. The scaling is derived from the first position target by constraining the scaling factors of both targets to be equal. Furthermore the

average of the two diagonal elements of the deformation matrix of the first target describing the across track deformation is constrained to be 1. By this the separation along x is scaled to be between the across track separations.

Similar approaches are taken to optimize more planar configurations where two satellites form a reasonable triangle together with the midpoint of the other two satellites which are close together. In this case the point on the reference orbit is either close to the tail (Fig. 8) or somewhere between bowshock and magnetopause (Fig. 10). In the latter case the work system differs from the one described in Section 3.1. The first axis is oriented radial which means along the position vector since bowshock and magnetopause are almost normal to the radial direction. The second axis is in the orbital plane and the third axis completes a right handed coordinate system. Along the radial axis the satellite separations are targeted such that e.g. satellites 1,2 and the midpoint of satellites 3,4 do not have any separation at all and 3,4 are separated by a small amount. Then one eigenvalue of the deformation matrix gets close to zero. If the other two remain close to one a good triangle multi-scale configuration is achieved.

4.2 Tetrahedron configuration

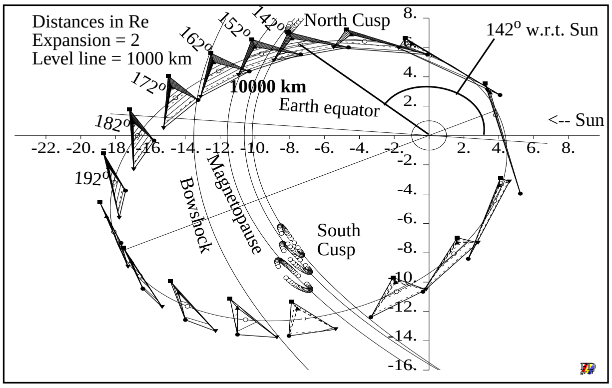

Fig. 7. Cusp crossing constellation March 2006

In the year 2005 and probably 2006 the preferred constellation in Spring (cusp crossing period) is still a tetrahedron. In Fig. 7 an almost regular tetrahedron is located at 162 degrees w.r.t. the Sun between bowshock and magnetopause. By applying a different phasing the regular tetrahedron can be shifted e.g. between 152 and 192 degrees if very small deformations are accepted. This has been proven by a set of optimization runs as described in Section 4.1.

The results are summarized in Table 2. The first two rows contain the three diagonal and three off-diagonal (d_{12}, d_{23}, d_{31}) elements of the deformation matrix. The third row contains the three orientation angles and the bold row gives the scaling factor in km and the smallest and biggest eigenvalue of the deformation matrix.

Table 2.Deformation after phasing

	Year 2006			**Year 2007**			**Year 2008**		
152^{o} wrt Sun	1.06	0.93	1.00	1.07	0.92	1.00	1.06	0.93	1.00
	-0.05	0.01	0.00	-0.13	0.01	0.00	-0.22	0.02	0.00
	-0.7^{o}	0.0^{o}	6.2^{o}	-0.9^{o}	0.0^{o}	15.0^{o}	-1.4^{o}	-0.3^{o}	27.9^{o}
	7983	**0.91**	**1.07**	**7171**	**0.84**	**1.14**	**5865**	**0.76**	**1.22**
162^{o} wrt Sun	1.05	0.94	1.00	1.07	0.92	1.00	1.07	0.92	1.00
	-0.05	0.01	0.00	-0.12	0.01	0.00	-0.24	0.02	0.00
	-0.6^{o}	0.0^{o}	6.3^{o}	-0.8^{o}	0.0^{o}	15.1^{o}	-1.6^{o}	-0.2^{o}	28.2^{o}
	9016	**0.92**	**1.07**	**8283**	**0.85**	**1.14**	**6967**	**0.74**	**1.24**
172^{o} wrt Sun	1.04	0.96	1.00	1.06	0.93	1.00	1.07	0.92	1.00
	-0.04	0.00	0.00	-0.12	0.01	0.00	-0.25	0.02	0.00
	-0.3^{o}	0.0^{o}	6.3^{o}	-0.4^{o}	0.0^{o}	15.3^{o}	0.0^{o}	-0.4^{o}	29.0^{o}
	9800	**0.94**	**1.06**	**9380**	**0.85**	**1.13**	**8287**	**0.73**	**1.25**
182^{o} wrt Sun	1.04	0.95	1.00	1.06	0.94	1.00	1.07	0.93	1.00
	-0.04	0.00	0.00	-0.11	0.00	0.00	-0.25	0.02	0.01
	0.1^{o}	0.0^{o}	6.8^{o}	-0.5^{o}	0.0^{o}	15.4^{o}	-1.9^{o}	0.0^{o}	28.8^{o}
	9889	**0.93**	**1.05**	**8287**	**0.87**	**1.12**	**9678**	**0.73**	**1.26**
192^{o} wrt Sun	1.10	0.89	1.00	1.07	0.92	1.00	1.05	0.94	1.00
	0.04	0.01	0.00	-0.12	0.00	0.00	-0.25	0.02	0.01
	1.1^{o}	0.0^{o}	6.8^{o}	0.1^{o}	0.0^{o}	15.7^{o}	-1.8^{o}	0.1^{o}	29.7^{o}
	8528	**0.88**	**1.11**	**9678**	**0.85**	**1.14**	**10726**	**0.73**	**1.25**
172 / 192	0.93	1.06	0.51	0.98	1.01	0.45	1.03	0.96	0.41
	-0.01	0.32	0.03	-0.07	0.21	0.05	-0.02	0.27	0.07
	32.0^{o}	0.5^{o}	6.4^{o}	25.2^{o}	0.8^{o}	15.3^{o}	17.5^{o}	0.7^{o}	29.3^{o}
	10350	**0.50**	**1.32**	**10950**	**0.15**	**1.09**	**11049**	**0.37**	**1.21**

One can clearly see that the eigenvalues and thus the satellite separation can be maintained very well until 2007 irrespective from the exact location of the tetrahedron. The deviation of the eigenvalues from 1 is less than 15 percent. And also in the year 2008 reasonable spatial configurations are possible. The last set of four rows in Table 2 gives for comparison the deformation at 192 degrees if the nearest regular tetrahedron is located at 172 degrees. This shows the big improvement by the phasing. The third rotation angle increases to 29 degrees in 2008. The original tetrahedron (T2 in Section 3.2) in the definition coordinate system was selected such that initially all rotation angles are zero. As long as the second angle remains close to zero the first angle corresponds to a rotation around the orbit normal and the third angle to a rotation around the a long track direction of the reference orbit at 172 degrees w.r.t the Sun.

4.3 Triangle Multi-Scale Configuration

A triangle multi-scale configuration means that three satellites form a triangle with an expansion of up to 10,000 km whereas the fourth satellite is close to one of the other with less than 1,000 km separation.

Also these constellations can be established only by a change of the phasing of the satellites. Remember that this is possible only due to the fact that two satellites are nearly in the same orbit. With the manoeuvre optimization approach sketched in Section 4.1 the results presented in Fig. 8 to Fig. 11 in the following two subsections are obtained.

4.3.1 Triangles at Tail

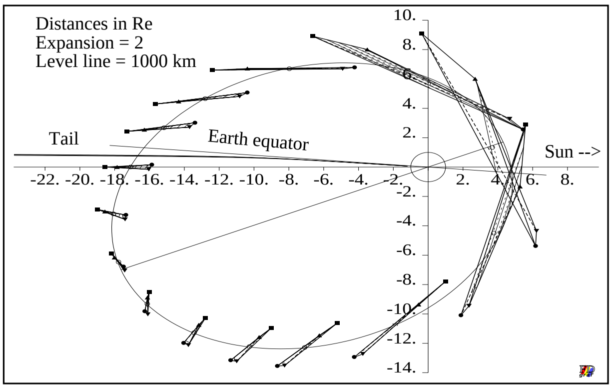

Fig. 8. Tail crossing constellation September 2005

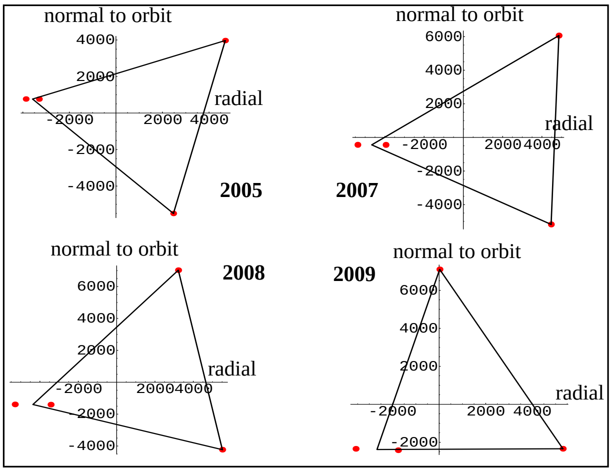

Fig. 9. Tail constellation projected into tail surface

During the tail crossing period in autumn the satellites are phased such that the plane of the triangle is parallel to the tail surface. This can be seen in the projection into the orbital plane of the mean orbit of the year 2005 in Fig. 8. The projection into the tail surface shows that the satellites always have big separations within the tail surface since they can form triangles with sufficient equal side lengths during the years 2005-2009 as shown in Fig. 9.

If you look at the rotation of the triangles over the years this confirms the change of the orientation of the deformation matrix given in Table 2 (Third angle in third row).

4.3.2 Triangles at Magnetopause

During the cusp crossing periods spring 2007 onwards at least the north cusp is now several Earth radii away from the satellite orbits but the magnetopause is crossed at low angles w.r.t. the Earth equator. This is now the region there the scientist want to focus on and therefore a multi-scale triangle configuration with the triangle parallel to the magnetopause is highly desirable. Also this can be achieved just by a different phasing as is shown in Fig. 10 and Fig. 11.

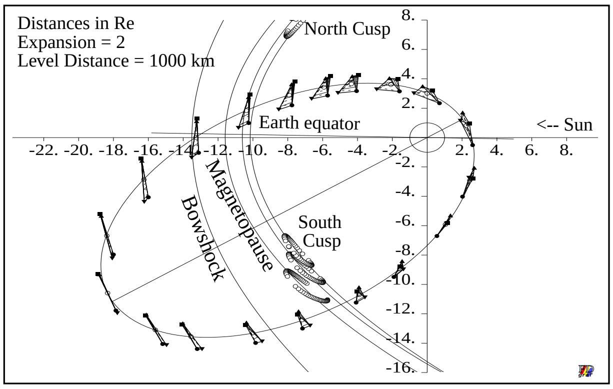

Fig. 10. Cusp crossing constellation March 2008

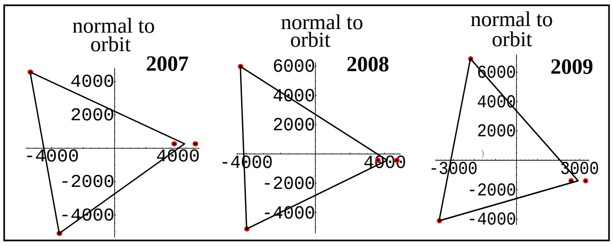

Fig. 11. Cusp constellation projected into magnetopause

5. CONCLUSIONS

It has been shown how a very suitable constellation for the second extension of the Cluster mission has been derived from the orbit evolution, the fuel budget and the wishes of the principal investigators. Due to the flexibility of the target definitions and transformations within the manoeuvre optimization no software change has been required for the new strategies. During the extension the formation can be alternated between multi-scale triangle configurations at magnetopause/bowshock or tail and reasonable tetrahedrons along the orbit arc between northern cusp and tail just by phasing manoeuvres. Based on these results the project scientist is very confident to get the extension approved by the ESA director of science during the next months.

6. REFERENCES

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