FUZZY BOUNDARY JUPITER MOON TOUR TRAJECTORIES USING THE BIFURCATION METHOD

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ABSTRACT

The consideration of transfers to the Fuzzy Boundary region represents one of the more advanced concepts when trying to reduce the propellant requirements to obtain an interplanetary goal. DEIMOS Space, under ESA contract, has developed a tool to generate such transfers to inner planets, giant planets and natural moons of giant planets. The method is based on the Systematic Scan Search of Bifurcations, with a three-step approach consisting on: selection of strategy, generation of initial solutions and numerical optimisation.

The generation of the initial solution by systematic search of bifurcations is accomplished by splitting the trajectory into smaller arcs. The initial guess of the trajectories must be obtained always by backwards propagation from the final targeting conditions. Forward propagation is used from the initial conditions to match the backwards propagation previously derived.

Different values of the orbit eccentricity are used when propagating (typically ranging between 0.9 and 1.1, close to the parabolic orbit). This leads to three different types of trajectory: close orbits below the Fuzzy region, escape trajectories and trajectories reaching maximum and minimum distances within the Fuzzy region. The actual change of nature in the resulting orbit corresponds to a bifurcation.

Matching of forwards and backwards propagation will take place within the Fuzzy region, using in general a manoeuvre or a low-thrust arc. Finally, an optimisation process is started to obtain a full continuous numerically integrated trajectory, with minimum required propellant consumption.

One of the key advantages of this new method is the large number of solutions found, thanks to its systematic scan approach.

In particular, it has been applied to systematically explore trajectories between the different moons of a giant planet by using the Fuzzy Regions of those moons. A Tour of the Jupiter Galilean Moons has been created, allowing a spacecraft to visit a sequence of moons with a reduced fuel consumption when compared to classical solutions.

A direct application of the method is the USA JIMO mission or ESA's proposed Europa mission.

1. DESCRIPTION OF THE METHOD

The numerical algorithm proposed to solve the problem is based on a three-step strategy.

1.1 Selection of WSB transfer strategy

For each particular problem, a sequence of events is defined according to previous theoretical analyses of the problem. This sequence of events completely defines the selected WSB strategy, including the potential use of gravity assists or low thrust arcs.

1.2 Generation of initial solutions

Once the WSB transfer strategy is selected, simplified methods are used to generate a database of, in general, non-optimal transfer trajectories. The whole trajectory is divided in various arcs, and initial solutions are piecewise computed. This is the most critical phase of the method.

For classical interplanetary trajectories, patched conics are used to generate the initial guess of the final solution. However, the very complex dynamics of the weak stability boundaries makes impossible the use of simple analytical calculations.

This is a very critical phase, as the final refinement of the trajectory must converge to the local minimum closest to the provided initial guess of the trajectory. A Systematic Scan Search of Bifurcations provides a very robust way to generate optimum transfers. This method is based on the division of the whole trajectory into smaller arcs and the individual generation of initial solutions for all of them. Normal arcs out the WSB regions are generated by Keplerian approximation or Lambert solver modules. Trajectories within the WSB regions are generated using a numerical integrator and single or multiple shooting methods.

The key factors of the method are:

- The initial guess of the WSB trajectories must be obtained always by backwards propagation from the final targeting conditions.
- The trajectory must be propagated forwards from the initial conditions (i.e. interplanetary trajectory) to match the backwards propagation previously derived.
- Matching of forwards and backwards propagation will take place within the WSB region, in general.
- If a manoeuvre or a low-thrust arc is allowed at the WSB region, a high flexibility in finding the final solution is obtained. However, the final optimisation process tends to remove this manoeuvre whenever possible.

In order to illustrate how the bifurcations are practically obtained, let assume a transfer from an interplanetary trajectory to a final orbit around a given planet via WSB of the planet-Sun system.

The generation of the initial solution by systematic search of bifurcations is accomplished by splitting the trajectory up into two arcs; one arc from the WSB region to the final injection conditions, which is obtained by backwards propagation since a large information on the final state is given and another arc from the arrival hyperbola up to the WSB region, which is obtained by forwards propagation.

The bifurcations are linked to the final target orbit (pericentre and apocentre radii, inclination, right ascension of the ascending node and argument of pericentre) and the orbit injection date.

They are found by performing backwards propagation from the final orbit injection point (pericentre) for different values of the arrival orbit eccentricity (typically ranging between 0.9 and 1.1, close to the parabolic orbit). For increasing values of the arrival orbit eccentricity, the backwards propagation leads to three different types of trajectory (Fig. 1):

- Type 1: the trajectory remains below the WSB region.
- Type 2: the trajectory reaches maximum and minimum distances within the WSB region.

  Although this region is somehow not very well defined “a priori”, typical values lie between 0.75 and 1.5 times the distance of the lagrangian L_1 (or L_2) point.
- Type 3: the trajectory escapes from the planet influence

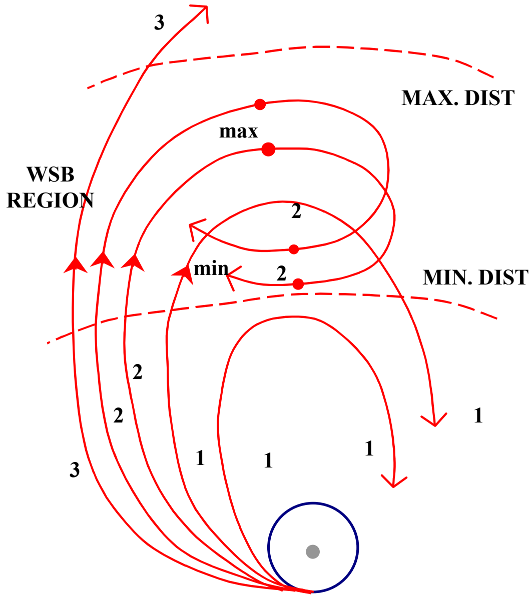

Fig. 1. Search of bifurcations

The actual change of nature in the resulting orbit (‘jump’ from one type to another) corresponds to a bifurcation. Fig. 2 illustrates this behaviour.

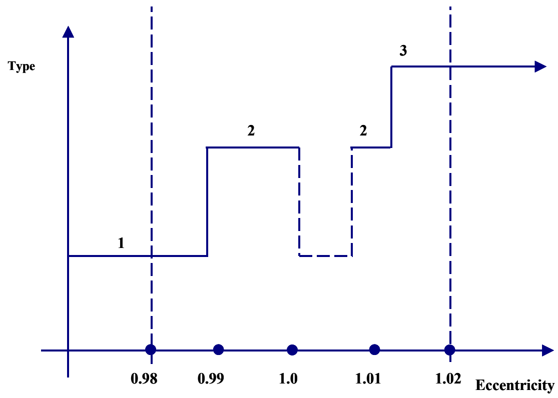

Fig. 2. Behaviour of bifurcations w.r.t. eccentricity

The forward bifurcations are computed from the hyperbolic arrival velocity vector and the minimum allowed pericentre altitude. At first pericentre passage, a manoeuvre is allowed to achieve the required eccentricity range. This approach provides an additional degree of freedom since the phase angle θ of the impact geometry in the target B-plane is not fixed. For each value of θ, bifurcations are calculated in an identical way to that presented for the backwards propagation.

The last step prior to the optimisation process is to match the forward and backwards trajectories. If the separation in time, the distance in position and the difference in velocity are smaller than a given set of thresholds, the trajectories are considered as matched and the optimisation process is started to obtain a full continuous numerically integrated trajectory, with minimum required propellant consumption.

In order to validate the described method to obtain initial solutions, an alternative approach has been followed, which consists on the use of a Genetic Algorithm Optimiser. Genetic algorithms are very well suited for problems with several local minima and a very complex structure. The main problem of this method is the large number of evaluations of the cost function and the slow convergence close to the final solution. However, those problems are avoided as the genetic algorithms are used only to obtain initial solutions, avoiding the need for the final step of convergence.

1.3 Numerical optimisation of trajectories

The fine optimisation of the complete trajectory by numerical integration is formulated as a constrained parameter optimisation problem. The most promising WSB transfer trajectories in terms of required propellant and mission duration taken from the database of initial solutions are selected for fine optimisation where numerically integrated trajectories satisfying all mission constraints are obtained. The constrained parameter optimisation is formulated as follows:

- Selection of a set of physical parameters to be optimised which completely describe the WSB trajectory.

- From above parameters, the trajectory is numerically integrated forwards or backwards depending on the trajectory leg.

- At intermediate points, matching conditions are imposed as equality constraints; forwards and backwards integrated position vectors must be identical, velocity vectors must be also identical if no manoeuvre is applied at this point of the trajectory.
- Minimum pericentre altitude inequality constraints must be considered for all gravity assists and pericentre passages.
- The cost function to be minimised is the total required ΔV if impulsive manoeuvres are considered. In case a low-thrust propulsion system is utilised, the final mass is to be maximised.

Following with the practical case of a transfer from an interplanetary trajectory to a final orbit around a given planet via WSB region of the planet-Sun system, Fig. 3 presents a scheme of the optimisation variables.

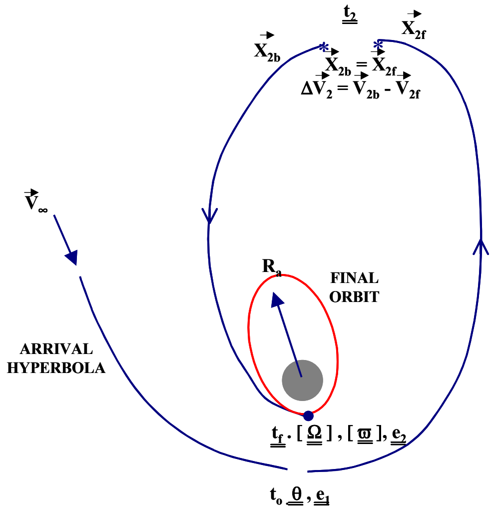

Fig. 3. Description of an optimisation case

2. APPLICATIONS

2.1 WSB Inner Planet Capture

The proposed method has been first applied to the capture by inner planets.

One of the most important features of the problem is the lack of natural moons, which translates into two major topics:

- Impossibility of performing gravity assists manoeuvres, apart from the initial pericentre passage.

- Very difficult to obtain full flexibility from the WSB trajectory since there is not any possibility to fly over a four-body dynamics region, which gives potential for big propellant saves as the case of the Earth-Moon-Sun system

The strategy for a spacecraft to be captured by an inner planet is described in Fig. 4 below.

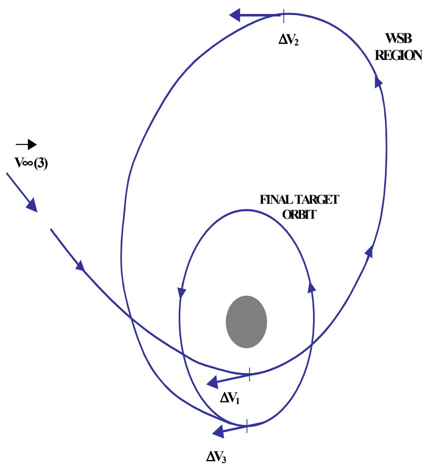

Fig. 4. WSB inner planet capture strategy

The hyperbolic arrival velocity vector provides the initial conditions for the problem. Since the pericentre altitude must fulfil certain constraints, the B-plane target conditions are selected such that this constraint is not violated. At planet pericentre, a tangential braking manoeuvre ΔV1 injects the spacecraft into a large eccentricity trajectory leading to the planet-Sun WSB region. At the planet-Sun WSB a second manoeuvre ΔV2 is allowed to fine-tune the arrival conditions. At the subsequent pericentre, an insertion manoeuvre ΔV3 injects the spacecraft into the final target orbit.

The method has been applied to:

- Bepi Colombo
- Venus Express
- Mars Express

Resulting in the following general conclusions for the inner planet capture problem:

- It is feasible to build trajectories to Venus, Mars and Mercury making use of the WSB regions.
- The improvement in ΔV is not significant, although gravity losses are reduced.
- The greatest advantage results from the increased flexibility in the selection of the parameters characterising the final orbit (essentially Ω and ω) with no ΔV penalty in most cases (but with increased transfer time).

2.2 WSB Giant Planet Capture

The WSB capture method described for inner planets can also be applied to the problem of the capture in a giant planet. However, the presence of several natural moons provides the possibility of combining the general strategy with a sequence of gravity-assisted manoeuvres. It is important to note that the geometry of the arrival hyperbolic velocity vector imposes a strong constraint over the problem with flybys, since the optimum results are obtained when the arrival velocity is in the plane of the moon w.r.t. the planet.

The proposed strategy for a Jupiter insertion trajectory is a three-manoeuvre scheme presented in Fig. 5.

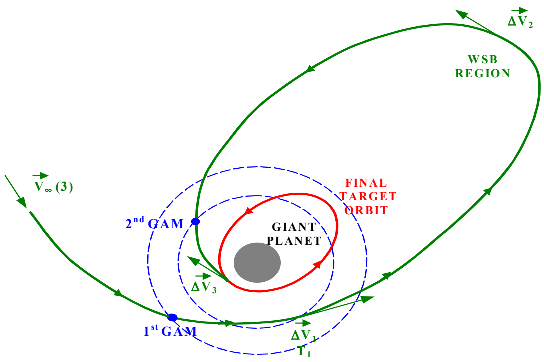

Fig. 5. Three-manoeuvre scheme with a double flyby

The constrained parameter optimisation problem is based on the following parameters:

- Impact vector at first Moon encounter.
- Pericentre radius at planet without considering the flyby, linked to the moon position in its plane around the planet.
- At planet pericentre and after the first flyby, a manoeuvre is performed to achieve a particular eccentricity.
- A second manoeuvre is performed at a certain distance to match the forwards and backwards orbits. The distance is constrained to limit the total time. In the analysed cases, the distance is such that it is not really within the WSB region.
- At second flyby, impact vector at the moon plus the arrival velocity vector.
- At planet pericentre and after the second flyby, a third manoeuvre is performed to insert the probe into the final orbit, specified by its orbital period.

Moon 1 / Moon2	Io	Europa	Ganymede	Callisto
Io	934.2 / 369.8	1144.8 / 342.4	1075.9 / 372.5	1179.4 / 360.6
Europa	1136.8 / 188.8	1349.0 / 369.5	1213.6 / 366.4	1345.9 / 228.0
Ganymede	1090.2 / 372.4	1205.9 / 345.3	1114.0 / 336.3	1206.4 / 298.1
Callisto	1153.0 / 372.4	1357.2 / 355.6	1218.9 / 353.0	1271.2 / 367.0

Table 1. ΔV (m/s) / insertion time (days) for a WSB Jupiter capture into a 30-day orbit using two gravity assist manoeuvres. Transfer distances range from 15 to 25 million kilometres.

From the analysis of Table 1, the best combination of moons to perform the insertion is the one formed by Io and Io, with a minimum ΔV of 934 m/s and a total transfer time of around 370 days. These figures might be expected from the fact that it combines a low height w.r.t. the planet, thus obtaining the maximum braking, with a good flyby effect, just slightly worse than that in Ganymede. However, radiation in the vicinity of Jupiter might advise to replace the Io-Io solution by the Ganymede-Ganymede solution with higher flybys. The case of Ganymede amounts to 1114 m/s and 336 days.

On the other hand, it seems that there is no possibility to perform a direct insertion without any manoeuvre (only with flybys), unless a low-thrust trajectory is used before the Jupiter encounter to reduce the incoming V_∞ to practical levels.

A similar analysis has been performed for Saturn, Uranus and Neptune.

3. MOON TOUR USING WSB

Following a similar approach as the one used to create previous planet capture trajectories, an additional feature has been incorporated to the software tool to systematically explore trajectories between the different moons of a giant planet by using the WSB regions of those moons. An application for the Jupiter system is presented.

It has been checked that the WSB of the different moons overlap between them, thus making possible the passage from one region to the other.

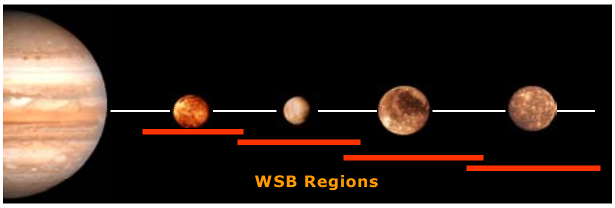

Fig. 6. WSB regions of the Jovian System

Table 2 shows the WSB boundaries for the moons of Jupiter.

Natural Moon	SMMA (km)	WSB min (km)	WSB max (km)
Io	**421600**	**314899**	**578619**
Europa	**670900**	**527741**	**866993**
Ganymede	**1070000**	**756974**	**1561307**
Callisto	**1810000**	**1380032**	**2633379**

Table 2. WSB regions of Jupiter moons.

3.1 Moon Tour Strategy

The strategy is similar to the inner or outer planet capture, searching for forward or backwards bifurcations from a given moon.

From an orbit around the moon, the algorithm proposed makes a loop in eccentricities detecting when the trajectory escapes the moon and calculating the apocentre or pericentre of the orbiter around the planet.

Once forward and backwards bifurcations are calculated, the optimisation process tries to match both legs, as Fig. 7 shows. Normally, a ΔV is required to actually connect them.

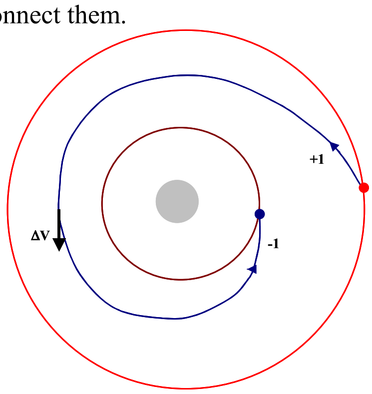

Fig. 7. Matching of fwd and bwd trajectory legs

The nominal procedure for solving the problem comprise the following tasks:

- Computation of forward bifurcations starting from the initial moon (+1). I.e. Arcs departing from an orbit around a moon and reaching the close-to-WSB region, calculated by forward propagation
- Computation of backwards bifurcations reaching the final moon (–1). I.e. Arcs departing from the close-to-WSB region and reaching an orbit around a moon, calculated by backwards propagation
- Matching and optimisation of forward and backwards trajectories.

In the final step, a continuous transfer trajectory is derived, matching the forward and backward arcs in a optimum way. Candidate arcs are further optimised to get a continuous trajectory solving the capture problem. Once the constrained parameter optimisation problem (NLP) is formulated, the selected numerical method to solve it is a Recursive Quadratic Programming algorithm developed at the Numerical Optimisation Centre of Hatfield (UK) by Bartholomew-Biggs, Dixon, Hersom et al.

3.2 Moon Tour Results

Transfers from different moon within the Jovian System have been analysed, namely those between consecutive moons. The results are similar to those obtained by Koon et al [6], but this study follows a systematic approach for all the possible combinations and finds a bigger number of solutions. Table 3 shows a comparison of the values obtained in the frame of the study and different strategies; Hohmann transfer, best case (moon aligned) and Koon's results.

Transfer	**Hohmann transfer**	**Best comb.**	**Koon's results**	**Current study**
	ΔV (km/s)			
Amalthea-Io	9.739	6.860		
Io-Europa	3.546	1.211		1.521
Europa-Ganymede	2.823	0.794	1.214	1.117
Ganymede-Callisto	2.473	0.560		1.099

Table 3. ΔV cost of a Jovian moon tour

Table 4 shows some of obtained results for the Ganymede-Europa transfer, filtered to reject ΔV bigger than 1180 m/s.

NMIS	ECC1	DV1	DTM	DV2	DTF	BOM	SOM	ECC2	DV3	TTOT	DVT
1	0.696504	0.	7.1	1167.3	2.9	170.7	0.0	0.664199	5.4	10.0	1173.
5	0.696383	0.	7.0	1160.7	3.8	129.9	0.0	0.655153	-0.2	10.8	1161.
7	0.696768	0.	7.0	1168.1	3.9	140.1	0.0	0.655682	0.0	10.9	1168.
8	0.696823	0.	7.0	1164.4	4.1	150.1	0.0	0.655377	-0.1	11.0	1165.
9	0.697223	0.	6.9	1161.3	4.2	140.3	0.0	0.750216	-0.3	11.1	1162.
90	0.692388	0.	6.7	1155.6	4.0	106.1	0.0	0.864135	-0.3	10.7	1156.
148	0.911409	6.	7.1	1155.4	3.0	170.2	0.0	0.652041	0.5	10.1	1162.
149	0.907857	8.	7.1	1112.3	3.1	178.3	0.0	0.653511	1.1	10.2	1122.
150	0.907390	9.	7.1	1108.3	3.2	190.6	0.0	0.652305	0.6	10.3	1117.
157	0.925539	-2.	6.7	1167.1	4.2	136.3	0.0	0.794112	-5.2	10.9	1174.
178	0.838681	0.	6.6	1169.2	3.3	208.3	0.0	0.675893	10.1	9.8	1179.
183	0.868982	-17.	6.9	1110.1	2.9	76.0	0.0	0.784913	-7.8	9.8	1135.
239	0.881432	12.	6.9	1132.4	3.1	175.7	0.0	0.652535	0.7	10.0	1145.
291	0.694471	0.	6.1	1173.7	2.9	172.5	0.0	0.650804	0.0	9.1	1174.
295	0.694978	-1.	6.2	1171.4	2.9	169.9	0.0	0.652883	0.9	9.1	1173.
298	0.694255	0.	6.1	1163.0	3.2	199.0	0.0	0.677597	10.8	9.3	1174.
300	0.695296	-1.	6.1	1153.3	3.3	205.1	0.0	0.682277	12.7	9.4	1167.
303	0.693867	0.	6.1	1175.6	3.9	139.9	0.0	0.655562	0.0	10.0	1176.
304	0.693412	0.	6.0	1167.2	3.9	133.0	0.0	0.693611	-2.8	10.0	1170.
308	0.693985	0.	6.0	1157.6	4.2	150.6	0.0	0.700560	0.0	10.2	1158.
310	0.693932	0.	6.1	1177.5	4.2	139.6	0.0	0.804656	-0.1	10.3	1178.
325	0.699203	0.	6.1	1161.3	2.8	162.3	0.0	0.651070	0.1	9.0	1162.
328	0.698322	0.	6.1	1165.4	2.8	160.3	0.0	0.646234	-1.8	8.9	1167.
330	0.698146	0.	6.1	1163.2	2.8	162.1	0.0	0.650893	0.1	9.0	1164.
339	0.697954	0.	6.1	1163.0	4.0	109.5	0.0	0.864848	0.0	10.1	1163.
341	0.698786	0.	6.2	1155.9	3.9	138.0	0.0	0.698671	-0.7	10.1	1157.
390	0.750232	0.	6.0	1162.7	4.2	163.4	0.0	0.655613	0.0	10.2	1163.
393	0.749944	0.	6.0	1160.1	4.3	150.5	0.0	0.751261	0.1	10.3	1160.
457	0.818379	3.	6.5	1158.7	3.1	183.8	0.0	0.652054	0.5	9.5	1162.
506	0.842159	6.	6.6	1162.9	3.0	177.8	0.0	0.656587	2.4	9.6	1171.
507	0.838705	8.	6.6	1149.1	3.1	190.4	0.0	0.649955	-0.3	9.7	1157.

Table 4. Ganymede-Europa transfer (equatorial orbits)

Fig. 8 and Fig. 9 shows an example of the trajectory, labelled as mission number 1 in the previous table, in the MEE2000 reference frame.

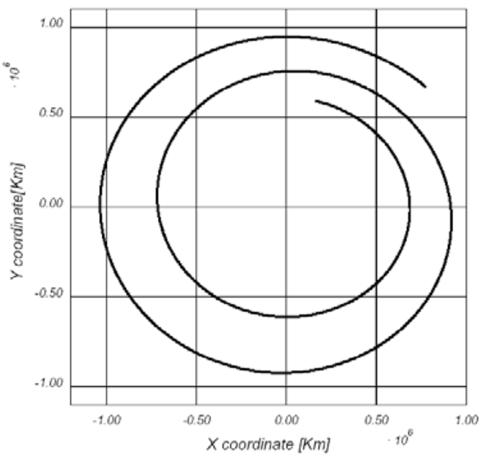

Fig. 8. Ganymede-Europa transfer (XY)

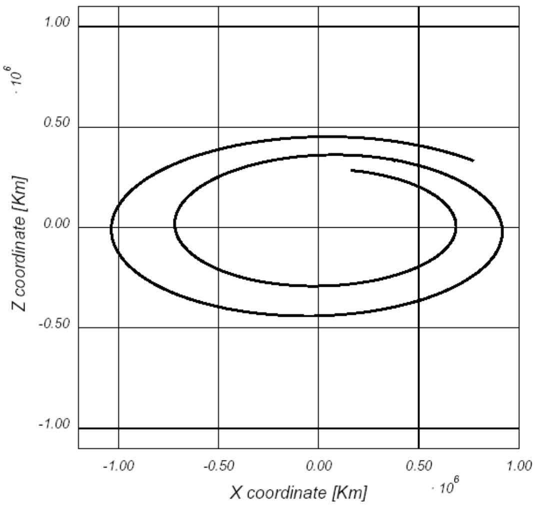

Fig. 9. Ganymede-Europa transfer (XZ)

4. CONCLUSIONS

It has been proven that trajectories making use of the Weak Stability Boundary region can be built systematically, offering the mission analyst an increased range of possibilities when designing missions to the solar system planets and moons.

The use of WSB for planets without natural moons does not decrease the total ΔV required for the capture, but provides greater flexibility when selecting the geometry of the target orbit. The method has been applied successfully for Bepi Colombo, Mars Express and Venus Express.

It has also been shown that when natural moons are available, gravity assists combined with WSB can be used to create giant planet / moon capture trajectories.

In addition to this, a method to build inter-moon trajectories using their WSB has been presented. One of the key advantages of this new method is the large number of solutions found; thanks to its systematic scan approach

The combination of these methods provides an alternative way for exploring giant planets and their moons.

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