THEONA THEORY OF RELATIVE SATELLITE MOTION FLYING IN THE FORMATION

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ABSTRACT

Satellite formation flights are considered in the past few years as an effective alternative to large expensive satellites. The dynamics of relative satellite motion flying in the formation is complex and requires detailed study. This paper presents the approach based on the THEONA semi-analytical satellite theory and developed for orbits with arbitrary values of the eccentricity.

The intermediary orbit corresponds to precise solution of the Generalized Problem of two fixed centers (GP2FC) which includes the effects of 2nd, 3rd and partially 4th zonal harmonics of the Earth gravity field. The osculating orbital elements (named Eulerian orbital elements) correspond to the intermediary orbit of GP2FC. The theory of relative perturbations of Eulerian orbital elements can take into account all essential perturbation effects. The analytical integrals use the special transcendental functions of orbital elements.

Besides, THEONA allows predict the motion of both passive and active satellites. Therefore the proposed model considers all essential secular, long-periodic and short-periodic perturbations for these primary disturbances of relative satellite motion in a cluster.

1. INTRODUCTION

In recent years, satellite formation flying is commonly considered as a key technology for advanced space missions. The advantages of the missions with formation flying clusters result in additional flexibility and efficiency of the space-based programs leading to the reducing of the size, complexity and cost of the spacecrafts. Satellites flying in the formation have close to each other orbits, with similar orbital elements. These satellites work together to accomplish the same mission hence the formation flying requires bounded relative motion for long time intervals. The dynamics of relative motion of satellites in a cluster is complex and requires detailed study.

There are many different approaches to study motion of the satellite flying in formation. Usually they are based on the Hill's [2] or Clohessy-Wiltshire [3] (HCW) equations with their perturbations. Many studies often modify the HCW equations to consider the nonlinear effects, an eccentric reference orbit, the primary gravitational perturbation J_2, etc. Kechichian [4] derived the general, nonlinear equations of motion in the rotating Hill's reference frame (Local Vertical – Local Horizontal, LVLH) subjected to J_2 and the drag with respect to an eccentric reference orbit. Gim and Alfriend [5],[6] gave the station matrix in closed form for the mean elements and in series-expanded form for the osculating elements with both the reference orbit eccentricity and the J_2 effects.

The COWPOKE (Cluster Orbits With Perturbations Of Keplerian Elements) equations [7],[8] use the mean orbit elements with modifying the in-plane mean motion M_{xy} and the out-plane mean motion M_z by adding secular terms due to orbit perturbations. The paper [9] describes the method used to account for only J_2 (first-order non-spherical gravity) perturbation, and compare the difference in relative position between using the COWPOKE/J_2 model and a model based on Hill's equations with J_2. It was shown escalating troublesome error in the along-track and cross-track directions.

An orbital elements approach [10],[11] uses the mean orbit elements. From the mean elements the osculating Keplerian elements are obtained for each satellite and then transformed to the rotating reference LVLH frame to view the relative motion. This approach was used to identify the J_2 invariant orbits [12] and control using orbital elements has been demonstrated [13].

In the paper [1] of our previous Symposium (Moscow, June 2003) I had presented the orbital elements approach for the study of the relative motion of formation flying satellites. It is based on the THEONA semi-analytical satellite theory with osculating Eulerian orbit elements. This method takes into account all essential secular, long-periodic and short-periodic perturbations for the primary disturbances effects of relative satellite motion in a cluster. Moreover, this method allows to estimate the separate contribution of considered perturbations of the relative motion of formation flying. The proposed paper is a continuation of the work presented [1] at 17th ISSFD.

2. THEONA'S APPLICATIONS

The numeric-analytical satellite theory [14], [15], [16] was developed in the 80s-90s by E.L.Akim and A.R.Golikov at Keldysh Institute of Applied Mathematics (KIAM), Russian Academy of Sciences. The theory is used as a rapid and efficient orbit propagator in various space problems at Ballistic Center of KIAM with a success.

Further improvement of this theory led to the creation of a new version of the semi-analytical satellite theory named THEONA (THÉOrie Numérique-Analytique) with various applications in spaceflight dynamics. Among such applications we could mention the following essential trends:

- Satellite motion calculations: Orbit *propagation* for the satellites (singles and constellations). *Design* of trajectories and traces. *Study* of orbit *evolution*. Calculation of the tasks of *secondary ballistics* (zones of optical visibility and radiovisibility, passages of the shadow, the issues to needful points, etc.). An accuracy and rapidity of the Numeric-Analytical satellite theory permits to support many tasks with voluminous computations.
- Orbit measurements: *Interpretation* of the measurements obtained by tracking stations and space observers. Orbit *determination* and *prediction* taking into account these measurements. *Determination* of maneuver parameters. The Numeric-Analytical satellite theory propagates not only the satellite orbit parameters (state vector $\vec{X} = \{\vec{r}, \vec{V}\}$, osculating elements $\vec{q}$), but also their partial derivations (e.g. with respect to initial orbit parameters, ballistic coefficient, solar radiation pressure coefficient, maneuver components, force model parameters, etc.).
- Mission analysis: *Estimation* of mission scheme *on long-time intervals* (from the point of view of the efficiency of examined maneuvers, on-board scientific experiments, etc.). *Planning* of the mission projects. *Optimization* of the maneuver calendar. *Prognosis* of different variants of the mission with respect to the changes of physical conditions (e.g. solar radiation activity). *Estimation of errors of mission scheme realisation* (maneuvers, navigation, simulation). The performances of the Numeric-Analytical theory permit frequently (and efficiently) propagate the orbits with different parameters.

New toolkit THEONA is developed by the author at "Space Informatics Analytical Systems" (KIA Systems) company which has a wide cooperation with Russian Aviation and Space Agency, Russian Academy of Sciences, Babakin Center, CNES, Alcatel Space and other organizations in Russia and in Europe. This toolkit provides high speed and accurate propagation of spacecraft motion for the operational software on different phases of preparation and implementation of the project: mission design, orbital perturbation analysis, maneuver planning, orbit determination, motion model matching, operative ground control, on-board flight dynamics tools, etc.

Crucial advantage of the THEONA toolkit is a support of efficient calculations of absolute and relative motion of satellite formation flight for all phases of mission analysis. It provides the acceptable accuracy of calculations especially for the description of satellite relative motion because of "physical" character of numeric-analytical theory. The key points of the THEONA satellite theory and its applications for the problems with formation flying were briefly described in the paper [1] of 17th ISSFD (Moscow, 2003). Now we will concentrate on the computing scheme of THEONA for formation flying tasks.

3. COMPUTING SCHEME

The satellite motion in THEONA, as it discusses in the paper [1], is described by the orbital elements of the intermediary orbit from the Generalized Problem of Two Fixed Centers (GP2FC) created by E.P.Aksenov, E.A.Grebenikov, V.G.Demin [17]. Two fixed centers with complex masses $m_1 = \frac{m}{2}(1 + i\sigma)$, $m_2 = \frac{m}{2}(1 - i\sigma)$ are placed at complex distances: $\vec{r}_1 = \{0; 0; c(\sigma + i)\}$, $\vec{r}_2 = \{0; 0; c(\sigma - i)\}$ in the Earth-Centered, Earth-Fixed (ECEF) coordinate frame of reference. The asymmetric ($\sigma \neq 0$) mode of GP2FC has the gravitational potential:

$$U^{\otimes} = \frac{Gm}{r} \cdot \left\{ 1 + \sum_{n=2}^{\infty} \gamma_n \left(\frac{r_e}{r} \right)^n P_n(\sin\varphi) \right\},$$

where G is the universal gravitational constant, m is the Earth's mass, r_e is the equatorial radius of the Earth, r is the orbital distance, φ is the latitude, and zonal harmonics:

$$\gamma_n = \frac{i}{2} (c/r_e)^n (1 + \sigma^2) \cdot \left[(\sigma + i)^{n-1} - (\sigma - i)^{n-1} \right].$$

If $c = r_e \sqrt{J_2 - \left(\frac{J_3}{2J_2} \right)^2}$, $\sigma = -\frac{J_3}{2J_2} \bigg/ \sqrt{J_2 - \left(\frac{J_3}{2J_2} \right)^2}$, the asymmetric mode of GP2FC includes the effects of the 2nd, 3rd and partially 4th zonal harmonics ($\gamma_2 = -J_2 = c_{20}$, $\gamma_3 = -J_3 = c_{30}$, $\gamma_4 \approx 0.72 \cdot c_{40}$) of the Earth gravity field described as:

$$U = \frac{Gm}{r} \cdot \left\{ 1 + \sum_{n \geq 2}^{\infty} \sum_{m \geq 0}^{n} \left(\frac{r_e}{r} \right)^n \cdot \left[c_{nm} \cos m(\lambda - S) + d_{nm} \sin m(\lambda - S) \right] \cdot P_n^m(\sin\varphi) \right\}$$

where λ is the longitude, S is the sidereal time, c_{nm}, d_{nm} are the coefficients of the gravity field.

The authors of GP2FC have named the intermediary motion (orbit) based on the solution of GP2FC as Eulerian motion (orbit) because Leonard Euler was the first to formulate the Problem of two fixed centers and found its solution in the plane case. That's why the corresponding orbital elements are named Eulerian orbital elements [18]. The cartesian coordinates x, y, z (in the ECEF frame) of Eulerian orbit are described by the formulae:

$$x = \rho \cdot \left(\cos u \cdot \cos\Omega - \cos i \cdot (\sin u + \beta) \cdot \sin\Omega\right)$$

$$y = \rho \cdot \left(\cos u \cdot \sin\Omega - \cos i \cdot (\sin u + \beta) \cdot \cos\Omega\right)$$

$$z = c\sigma + \xi\eta = c\sigma + \xi \cdot \left(\sin i \cdot \sin u + \gamma\right) / \left(1 + d \cdot \sin u\right),$$

where $\rho = \dfrac{\sqrt{\left(1-\varepsilon^2\sigma^2\right)\left(\xi^2+c^2\right)}}{1 + d \cdot \sin u}$, $\dfrac{\xi}{p} = \dfrac{1}{1 + e\cos\psi} + \delta$, $p = a \cdot \left(1 - e^2\right)$ is the Eulerian semilatus rectum, i is the Eulerian inclination, Ω is Eulerian longitude of the ascending node, $\varepsilon = c/p$, β, γ, δ, d are the coefficients of Eulerian orbit, u, ψ are the Eulerian argument of latitude and rhe Eulerian true anomaly, correspondingly.

To consider the motion of satellite formation flight the THEONA toolkit propagates the orbit of the guiding centre (*Chief*) with its orbit elements $q_i^{(C)}$. This Chief might be *real* or *virtual* (to control the appearing non-homogeneity of the satellite moving away during the flight). The motion of other satellites (*Assistants*) within the formation is described by the deviations of own orbital elements $q_i^{(A)}$ from the ones of the Chief: $\delta q_i^{(A)} = q_i^{(A)} - q_i^{(C)}$. The perturbations of the Chief's orbital elements $q_i^{(C)}$ and the changes of the deviations $\delta q_i^{(A)}$ are recalculated analytically by using the same analytical "perturbative" integrals and special functions (with their partial derivatives) which are already obtained in the expressions of THEONA for orbit propagation of the Chief. So we don't spend any additional computing time with respect to the Chief's orbit prediction. And hence, the large number of satellite in formation (constellation) will show in the most advantageous way processing speed of THEONA in the formation flying tasks.

Other dynamical effects (not included in Eulerian intermediary orbit) are taken into account by "step-by-step" analytical integration of the differential equations for Eulerian orbit elements: $q_i^{(\text{step}+1)} = q_i^{(\text{step})} + \Delta q_i^{(\text{step})}$, where $q_i^{(\text{step})}$ are the orbital elements, and $\Delta q_i^{(\text{step})}$ are their perturbations whithin the step.

The analytical "perturbative" integrals are expressed in THEONA (the terms of the 3rd order with respect to J_2) for essential perturbations due to:

- the rest of zonal, tesseral and sectoral harmonics of the geopotential model (with arbitrary degree and order) $\Re_{NC} = U - U^{\otimes}$,
- air drag (with various atmospheric density models),
- gravity influence of other celestial bodies (e.g. the Moon, the Sun),
- solar radiation pressure (with the shadow effects).

Of course, to compute the relative motion of formation flying is not necessary to take into account all perturbations because these disturbances are proportional to the distances between the satellites flying in formation (see e.g. [11]) which are 3-4 orders less than the orbital radius. Hence the accuracy of the relative motion is more precise than the one of the absolute motion. Therefore also, it is enough to take into account only the terms of the 2nd order (with respect to J_2) in the expressions of THEONA's analytical "perturbative" integrals. Thus the THEONA's computing speed becomes much more effective without any loss of accuracy.

But we should not forget that to insure high accuracy of calculations of the relative motion we are to dispose of well predicted absolute orbit of the Chief. Therefore we have to take into account the mentioned above essential perturbations of Chief's orbit (in case we do not have another way to receive this precise orbit).

The differential equations (analogous to Gaussian equations, Lagrange's equations, or Hamilton's canonical equations) for Eulerian intermediary orbit are developed by E.P.Aksenov, E.A.Grebenikov, and V.G.Demin (see e.g. [18]) for various sets of orbital elements. The similar systems of equations are used in the THEONA theory for non-singular Eulerian orbital elements (including circular and equatorial orbits).

To obtain simple and efficient formulae of analytical integration, the THEONA satellite theory uses the special functions of orbital elements:

- functions of inclination (well-known functions of Kaula, new supplementary functions for including third order terms of THEONA),
- functions of two arguments (eccentricity and mean motion) proposed by author [15] (the Hansen's coefficients are the private case of these functions),
- Newcomb polynomials, Legendre functions, Jacobi functions, etc.

The THEONA's scheme allows to predict the motion of both passive and active satellites, with calculation of their maneuvers. Moreover, the numeric-analytical scheme is constructed so that it can take into account other corrections in the satellite motion (see [1]).

4. CONSIDERATION OF RELATIVE MOTION'S DISTURBANCES

In the paper [1] of 17th ISSFD we show an example of the estimation of various contributions of the force model for the relative motion of formation flying. This analysis is based on the THEONA satellite theory and can be efficiently used to study the choice of the force models for different space problems with multi-iterative processes: planning algorithms from several to many satellites within a cluster, methods of control for formation keeping, for autonomous reconfiguration, for collision avoidance, etc.

As we may see from similar analyses, most essential disturbances of the relative motion of formation flying are the zonal harmonics (especially, 2nd and 3rd) of the gravity field, some tesseral harmonics (for resonance orbits), the air drag forces (for low altitudes), and the solar radiation pressure (with shadow effects).

The 2nd and 3rd zonal harmonics take into consideration in Eulerian intermediary motion, the other gravitational perturbations are calculated by the analytical "perturbative" integrals for "similar-Lagrange's equations" where the THEONA satellite theory makes good use of the functions of inclination:

$$Q^n_{mk}(\zeta) = \sqrt{\frac{(n+m)!}{(n-m)!}\cdot\frac{(n-k)!}{(n+k)!}}\cdot P^k_n(0)\cdot i^{n-m} P^n_{mk}(\zeta) =$$

$$= \begin{cases} \sqrt{\dfrac{(n+m)!}{(n-m)!}\cdot\dfrac{(n-k-1)!!\,(n+k-1)!!}{(n-k)!!\,(n+k)!!}}\cdot i^{k-m} P^n_{mk}(\zeta) & ,(n-k)-even \\ 0 & ,(n-k)-odd \end{cases}$$

where $P^k_n(0)$ is the associated Legendre polynomial, and $P^n_{mk}(\zeta)$ is the Jacobi polynomial:

$$P^n_{mk}(\zeta) = \frac{1}{2\pi}\int_0^{2\pi}\left(c+s\cdot\mathbf{e}^{i\varphi}\right)^{n+k}\left(c+s\cdot\mathbf{e}^{-i\varphi}\right)^{n-k}\mathbf{e}^{i(m-k)\varphi}d\varphi$$

with abbreviations $\zeta = \cos\theta,\ c = \cos\dfrac{\theta}{2},\ s = \sin\dfrac{\theta}{2}$.

This function of inclination $Q^n_{mk}(\cos i)$ is different from well-known inclination function of Kaula only by multiplier. To take into account the terms of 3rd order with respect to J_2 the THEONA satellite theory introduces new supplementary functions of inclination:

$$\bar{Q}^n_{mk}(\zeta) = \sqrt{\frac{(n+m)!}{(n-m)!}\cdot\frac{(n-k)!}{(n+k)!}}\cdot P^{k+1}_n(0)\cdot i^{n-m} P^n_{mk}(\zeta) =$$

$$= \begin{cases} 0 & ,(n-k)-even \\ \sqrt{\dfrac{(n+m)!}{(n-m)!}\cdot\dfrac{(n-k)!!\,(n+k)!!}{(n-k-1)!!\,(n+k-1)!!}}\cdot i^{k-m} P^n_{mk}(\zeta) & ,(n-k)-odd \end{cases}$$

Besides, the THEONA satellite theory has developed special functions of two arguments (eccentricity and mean motion) proposed by the author [15]:

$$\mathrm{X}^n_s(x,e) = \frac{\left(1-e^2\right)^{-n/2}}{2\pi}\int_0^{2\pi}\left(1+e\cos\psi\right)^n\cdot\mathbf{e}^{ix(E-\psi-e\sin E)}\cdot\mathbf{e}^{\pm is\psi}$$

where $E = 2Arctg\left(\sqrt{\dfrac{1-e}{1+e}}\cdot tg\dfrac{\psi}{2}\right)$ is the eccentric anomaly, $x = m\,\omega_e/n_0$, ω_e is the angular velocity of the Earth's rotation, and n_0 is Eulerian mean motion.

These functions $\mathrm{X}^n_s(x,e)$ possess many interesting properties which will be describe in a separate paper. Here, we note the relationship of these functions with the modified Bessel functions of the first kind $J_n(z)$ and the Jacobi functions $\mathrm{P}^{\,n}_{mk}(z)$:

$$\mathrm{X}^n_s(x,e) = \sum_{\nu=-\infty}^{\infty} J_\nu(xe)\cdot\mathrm{P}^{\,n}_{s-x,\nu-x}\left(\frac{1}{\sqrt{1-e^2}}\right),$$

where the integral representation of Jacobi functions:

$$\mathrm{P}^{\,n}_{mk}(\vartheta) = \frac{1}{2\pi}\int_0^{2\pi}\left(c+s\cdot\mathbf{e}^{i\varphi}\right)^{n+k}\left(c+s\cdot\mathbf{e}^{-i\varphi}\right)^{n-k}\mathbf{e}^{i(m-k)\varphi}d\varphi$$

with abbreviations $\vartheta = \cosh\tau,\ c = \cosh\dfrac{\tau}{2},\ s = \sinh\dfrac{\tau}{2}$.

To create new special functions the THEONA satellite theory utilizes methods and functions from the Representation Theory of the Inhomogeneous Lorentz Group (irreducible unitary representations) [19],[20]. From the point of view of the representations of groups, the Jacobi polynomials $P^n_{mk}(x)$ participate in matrix irreducible representations of the group *SU(2)* (for euclidean motion), the Jacobi functions $\mathrm{P}^{\,n}_{mk}(x)$ – of the group *QU(2)* (for pseudoeuclidean motion), the Bessel functions $J_n(x)$ – of the group *M(2)* of motions of euclidean plane.

Let us note once again that the Hansen's coefficients are the private (integer-valued) case of these functions: $X^{(k)}_{n,m} = \left(1-e^2\right)^{\frac{n+1}{2}}\cdot\mathrm{X}^{-(n+2)}_{k-m}(k,e)$.

All special functions of the THEONA satellite theory and their partial derivatives are calculated by using efficient recurrent relations and do not take a long computing time.

It is very important for relative motion of formation flying to take into consideration the air drag because of the two reasons:

- first, the air density ρ essentially depends on the altitude *h* of each satellite orbit,
- second, the ballistic coefficients and the attitudes of the satellites in formation can be different.

In the second case the calculation is realized simply, using the linear dependence of drag from the ballistic coefficient B : $\vec{F}_{air} = -\rho\cdot B\cdot V_{rel}\cdot\vec{V}_{rel}$, where $\vec{V}_{rel}$ is the relative velocity. So we may consider the difference of ballistic coefficients $\delta B = B^{(A)} - B^{(C)}$.

The THEONA satellite theory utilizes the own model of the Earth's atmospheric density. It is similar to Russian standard model GOST 25645.115-84 (edition 1990)

[21] of density of the Earth atmosphere (see [22]): $\rho = \rho_N K_1 K_2 K_3 K_4 K_0$, where the nocturnal density is $\rho_N = a_0 \cdot \exp\left(a_1 - a_2\sqrt{h - a_3}\right)$, and other multipliers depend on various dynamical effects: diurnal effect K_1, semi-annual effect K_2, the factor K_0 depends on mean solar activity $\bar{F}_{81}$, and K_3 – on current (diurnal) solar activity $\bar{F}$, and K_4 – on the geomagnetic index K_p.

All multipliers are the polynomials of the altitude *h*, e.g. the diurnal effect (depends on the bulge under the Sun) has $K_1 = 1 + (c_1 + c_2 h + c_3 h^2 + c_4 h^3) \cdot \cos^n(\varphi/2)$, where

$$\cos\varphi = \frac{1}{r} \cdot \left[z \sin\delta_{\odot} + \cos\delta_{\odot} \left(x\cos\beta + y\sin\beta \right) \right],$$

$\beta = \alpha_{\odot} - S_{\otimes} + \varphi_1$, $\alpha_{\odot}, \delta_{\odot}$ – right ascension and declination of the Sun, $S_{\otimes}$ – the sidereal time at Greenwich midnight.

In THEONA's density model the formula of the nocturnal density ρ_N is distinguished from aforesaid one of Russian GOST-model: $\rho_N = \rho_0 \cdot \left(1 + \frac{h - h_0}{\nu H}\right)^{-\nu}$. This distribution approximates real profile of atmospheric density better than the exponential law $\rho_N = \rho_0 \exp\left(\frac{h_0 - h}{H}\right)$ usually used by analytic theories.

Initially, the density model of THEONA was adjusted to Russian GOST-models (1977 [23], 1984 [24], 1990 [21]). But during my work in Division Mécanique Spatiale of Centre National d'Etudes Spatiales (CNES, 1995), my respected colleague J.-C. Agnese proposed me to adapte THEONA's model for other world-famous dynamical models of the Earth's atmospheric density: DTM-77 [25], DTM94 [26], MSIS-E-90 [27], NRLMSISE-00 [28], TD88 [29], etc. Now, the THEONA Toolkit has the algorithms of approximation of these models for precise task and satellite orbit. The general type of THEONA's density model can be expressed as:

$$\rho = \rho_0 \cdot \sum_{k=0}^{K} \frac{E_k}{\left(1 + \frac{h - h_0}{\nu H}\right)^{\nu - k}} + \rho_0 \cos^n \frac{\varphi}{2} \cdot \sum_{l=0}^{L} \frac{F_l}{\left(1 + \frac{h - h_0}{\nu H}\right)^{\nu - l}}$$

where $K, L, n, E_k, F_l, \rho_0, \nu, H, h_0$ are the parametres of the density model, and φ is the same aforecited angle from the “undersolar” bulge. Such representation of density ρ allows us to express the perturbations due to air drag in series expansion of orbital elements by using the same Jacobi functions $\mathrm{P}_{mk}^{\,n}$ as we mentioned above:

$$\left(1 - \frac{ae}{\nu H}\cos\psi\right)^{-\lambda} = w^{-\lambda} \sum_{m=0}^{\infty} \gamma_m \mathrm{P}_{0m}^{\,\lambda-1}(w) \cos m\psi \;,$$

$$\cos^n \frac{\varphi}{2} = \left(\frac{1 + \cos\varphi_\sigma \cos(u - \lambda_\sigma)}{2} \right)^{n/2} =$$

$$= \left(\frac{\sin\varphi_\sigma}{2} \right)^{n/2} \sum_{m=0}^{\infty} \gamma_m \mathrm{P}_{m0}^{\,n/2} (\operatorname{cosec}\varphi_\sigma) \cos m (u - \lambda_\sigma)$$

So we utilize here the similar mathematics of special functions as in other parts of THEONA. The Jacobi functions $\mathrm{P}_{mk}^{\,n}(z)$ substitute the modified Bessel functions of the first kind $I_n(z)$ which are used for the exponential law of density model. Methodic accuracy of THEONA's calculation of air drag perturbations for LEO is characterized by relative errors of 0.3 – 0.5 % . In tasks with ballistic coefficient matching (determination) this accuracy is higher.

The analysis of force models has shown also that the solar radiation pressure can be essential for the relative motion of formation flying, especially when the satellite orbits pass along the shadow limits. There are the algorithms in the THEONA Toolkit to calculate the points of passages of shadow limits with high accuracy.

If the satellites have various coefficients C_R of solar radiation pressure or the attitudes of the satellites in formation differ, we calculate the relative deviations in much the same manner as for the case of air drag: by using the linear dependence of the force from coefficient $\kappa = C_R \cdot A/m$.

Thus we do not have significant increase of computing time for the relative motion of formation flying without the accuracy losses. The examples of the accuracy of relative motion computations (for various types of orbits and formation configurations) based on the THEONA satellite theory will be presented in the Symposium.

5. CONCLUSIONS

THEONA is efficient as a tool for the study of Satellite Formation Flight. It provides high accuracy of formation flying relative motion (less than 1 cm of position errors) with high speed of calculations (2-3 orders faster than numerical propagators for one space object). This efficiency increases for the formation flying motion because THEONA computes this motion as 1 object (= Chief) with the deviations of other satellites (= Assistants). Moreover, the feature of the THEONA satellite theory to estimate the separate contribution of considered disturbances is expanded to the relative motion analysis of the formation flight.

THEONA works with both active and passive space objects taking into account all essential forces in the satellite motion. Therefore it is effective for various space flight dynamics problems: mission design, mission analysis, formation keeping, orbit determination. It can be also applied for real-time orbit control (including on-board software).

In our opinion, the proposed approach is efficient for studying relative motion evolution of satellite clusters. It would be useful for various applications to mission design as well as, to methods of control for satellite formation flying missions.

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