

# STUDY ON THE CHARACTERISTICS OF TWO-BURN TRANSLUNAR TRAJECTORY

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## ABSTRACT

In this paper, translunar trajectory of the simple sequence is investigated. The trajectory discussed in this paper is two-burn ballistic trajectory from the low earth parking orbit to the low lunar orbit. The problem is practical in that the geometric relation from the launch site to the moon is fully considered. The paper includes three subjects. The first is the structure of the problem and the solution space. The problems is defined and the solutions are grouped by the properties of lunar transfer sequence. The second is the characteristics of the optimal solutions. The topics discussed are, the transition of the required velocity increment by the launch date, and the difference by the property of the lunar transfer sequence. The third is the sensitivity analysis of a number of items to the deviation of the parameters from the optimal solution.

## 1. INTRODUCTION

With the opening of this new century, the moon attracts the attention again as the target of the space exploration. SMART-1 launched by ESA, which is now on the long way to the moon, will be the lead-off visitor to the moon of this century. Japan is developing two lunar explorers, LUNAR-A and SELENE, which are planned to be launched within a few years. China and India are planning their first mission to the moon, and the United States refocuses on the human exploration to the moon.

Looking from the point of lunar transfer trajectory, most of the explorers, including scores of explorers in the last century, approach the moon directly. Two exceptions are LUNAR-A and SMART-1, though their methods are quite different, they take a round-about but efficient way to the moon. Investigated in this paper is the former, two-burn direct translunar trajectory.

As is shown in the next section, the problem is basically formulated as parameters optimization under constraints. Any well known numerical tool will easily converge the parameter set to the stationary point, that is, the local optimum solution. However, it is sometimes problematic to use the result without the knowledge as to the structure of the problem and the solution space. Overlooking the better solution is the typical case. In Section 2, the problem is defined and the structure of the problem and the solution space is investigated. As a

result, the solutions are grouped by the two properties of the lunar transfer sequence. The optimal solution is basically Hohmann type transfer connected to the lunar approaching hyperbolic trajectory, however, it has several characteristics to be pointed out. Discussed in Section 3 are, the transition of the required velocity increment ( $\Delta v$ ) by the launch date, and the difference by the property defined in the previous section. Although " $\Delta v$  optimal" is the most significant factor to be considered, in practical problem, many other items must be taken into account. And sometimes, the parameters that deviate from the optimal solution are chosen to improve the other items. Discussed in Section 4 is the effect of the parameters' deviation from the optimum solutions.

## 2. PROBLEM DEFINITION

Fig.1 describes the lunar transfer sequence investigated in this paper. The sequence starts from the launcher's injection to the low earth parking orbit (LEO). After coasting for a while, the spacecraft is injected into the translunar trajectory by impulsive maneuver of the launcher. The explorer approaches the moon without any additional maneuver, and the sequence terminates at the explorer's perilune passage (PLP).

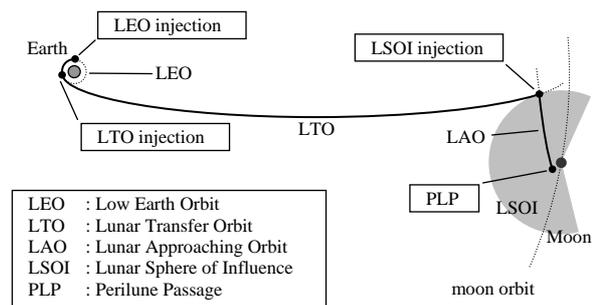


Fig. 1. Lunar transfer sequence

The state at LEO injection is fully given in the earth fixed coordinate system. This means that the launcher, the launch site and the launch trajectory from the launch site to LEO injection is fully specified. The only variable here is the time of LEO injection, which determines the direction of LEO orbital plane in inertial coordinate system. The time of translunar trajectory injection determines the state just before the injection,

and with  $\Delta v$  of the injection maneuver, the state just after the injection, that is the initial state of the translunar trajectory, is determined. As to the translunar trajectory, the patched conic method is applied. The trajectory is divided into two sections. The lunar transfer orbit (LTO) from the translunar trajectory injection to the lunar sphere of influence (LSOI) injection, and the lunar approaching orbit (LAO) from the LSOI injection to PLP. The two orbits are connected at LSOI injection both in position and velocity. The state at PLP is partially assigned, but the detail is discussed later.

The parameters are LEO injection time, LTO injection time, and LTO injection  $\Delta v$ . LTO injection  $\Delta v$  has three degrees of freedom, two for its direction and one for its magnitude. In the following discussion, the  $\Delta v$  magnitude is replaced with the transfer angle on LTO ( $\theta$ ) for the handling convenience. Both parameters determine the shape of LTO ellipse, however, the latter is more useful in that it can distinguish between the LSOI injections before and after LTO apogee passage. There are 5 parameters in all, however, possible ranges of their values are strongly restricted from the point of minimizing the energy for the transfer.

Roughly speaking, LTO is the transfer orbit between noncoplanar circular orbits, LEO and the moon orbit. Minimum energy two impulse transfer of this type is generalized Hohmann transfer. The transfer orbit is elliptical orbit cotangential with the two circular orbits at the apses, and the plane is changed at the apogee maneuver. This indicates that the direction of LTO injection  $\Delta v$  should be tangential to LEO and  $\theta$  should be 180 deg. for the minimum energy transfer. The terminal of LTO, that is the apogee of LTO, is assigned from the position of the moon at the arrival. This indicates that LEO injection time should be assigned so that the assigned LTO terminal is included on LEO orbital plane, which is identical to LTO orbital plane. In addition, LTO injection time should be assigned so that LTO injection position, that is the perigee of LTO, is the opposite side of the assigned LTO terminal. LEO, which is fixed to the earth, rotates around the earth's axis once a day. Therefore, there are two chances of LEO injection in a day when the assigned LTO terminal is on the LEO orbital plane (Fig.2). These two cases are called "short coast" and "long coast" after the length of the coasting on LEO.

The rough investigation above assigns the approximate values of the parameters for the minimum energy transfer. Next, the values of the parameters are adjusted to satisfy the boundary condition at the terminal, PLP. At this point, two of the parameters that define the direction of LTO injection  $\Delta v$  are fixed to the optimal values, that is, tangential to LEO. The reason is that the magnitude of LTO injection  $\Delta v$  is more sensitive to these two parameters. The effect of the deviation from

the optimal values is investigated in Section 4 for these two parameters. The remaining three parameters, LEO injection time, LTO injection time, and  $\theta$ , are available to adjust three boundary conditions at the terminal.

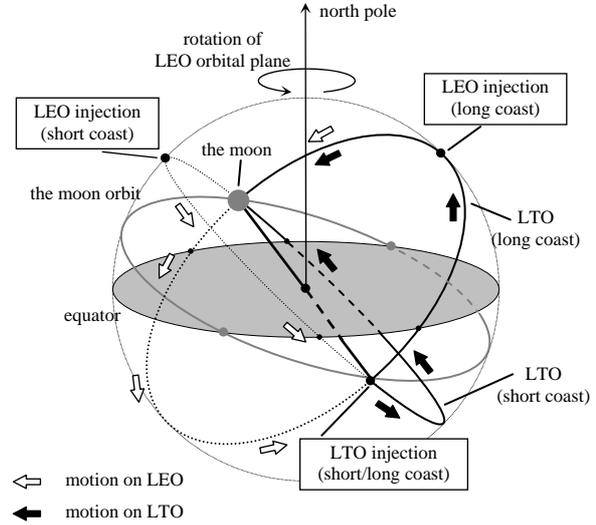


Fig. 2. Short coast and long coast

However, to keep the transfer energy within reasonable level, the values of the parameters cannot be far from that assigned for the minimum energy transfer. Therefore, the possible range of PLP state is limited. For example, two of the orbital elements of LAO, the semimajor axis and the direction of the nodes with the lunar equatorial plane, are determined by the relative velocity of the spacecraft against the moon at LSOI injection. However, the relative velocity at LSOI injection is almost fixed for the minimum energy transfer. Its magnitude is approximately in the range from 800m/s to 900m/s and its direction is almost against the moon's orbital motion. As a result, the values of these two orbital elements are almost fixed for the transfer with reasonable energy.

The combination of the three boundary conditions is adaptable. The examples of the possible combinations are, to assign the values of three state variables, or to assign the values of two state variables and minimize one performance index, and so on. Boundary conditions used in the following discussion are, to assign the altitude and the inclination at PLP and minimize the velocity at PLP. This is general type of boundary condition for the lunar orbiting mission. For a given relative velocity vector at LSOI injection, there are two LAO orbital planes that include the velocity vector. Two cases are called "ascending approach" and "descending approach" after the direction of the motion at PLP.

Discussion as to the structure of the problem and the solution space is summarized as follows. There are five parameters in all, however, at this point, two of them are fixed to the optimal values. Therefore, the number of free parameters is three. The number of boundary conditions to be satisfied is also three. This indicates that the number of parameter sets that satisfy the conditions is finite. Two properties characterize the lunar transfer sequence. The first is the property as to the coasting on LEO, that is, “short coast” and “long coast”. The second is the property as to the approaching direction of LAO, that is, “ascending approach” and “descending approach”. These properties can be selected independently, therefore, there are four possible set of parameters (in a day) that satisfy the boundary conditions.

### 3. OPTIMAL SOLUTION CHARACTERISTICS

Discussed in this section are the characteristics of the solutions obtained under the condition defined in the previous section.

Previous to the discussion, the conditions used in the analysis are described in the followings. As to the initial condition, LEO, it is assigned as the circular orbit with the altitude 290km, launched eastward from Tanegashima space center (131.0degE, 30.4degN). For the convenience of handling “short coast” and “long coast”, the injection point is assumed to be the north edge of the orbit. As to the connection of LTO and LAO, the radius of LSOI is assigned to 66280km. As to the terminal condition, PLP, the altitude is assigned to 100km and the inclination is assigned to 90deg. against the moon equatorial plane.

Firstly discussed is the transition of  $\Delta v$  by the launch date. Fig.3 shows the transition of two  $\Delta v$ 's by the launch date from July to September in 2005. The two  $\Delta v$ 's are, LTO injection  $\Delta v$  (squares) and the low lunar orbit (LLO) injection  $\Delta v$  (solid circles). LLO is the circular orbit with the altitude of 100km. The information as to the position of the moon at LSOI injection is also marked in the figure. They are the marks that express the relation with the apses of the moon orbit (perigee and apogee) and the marks that express the relation with the nodes of the moon orbit with the earth equatorial plane (ascending node and descending node). The figure shows the result of the case “short coast” and “descending approach”. However, the latter property doesn't matter so much for these  $\Delta v$ 's, since these  $\Delta v$ 's are determined mainly by LTO, whereas the latter property is related mainly to LAO.

The transition of LTO injection  $\Delta v$  is simple. Since this  $\Delta v$  is determined mainly from the size of LTO (note that the direction of LTO injection  $\Delta v$  is fixed to be tangential to LEO),  $\Delta v$  is large when the moon is far (at

the apogee) and is small when the moon is near (at the perigee).

The transition of LLO injection  $\Delta v$  reflects the transition of the relative velocity of the spacecraft against the moon at LSOI injection. The relative velocity is determined not only from the difference in the velocity magnitude, but also from the difference in the velocity direction. The difference in the velocity magnitude is large when the moon is at the perigee and is small when the moon is at the apogee. The difference between the velocities of the moon at these two points overtakes the difference between the velocities of the spacecraft at these two points. The difference in the velocity direction is mainly determined from the angle between LTO orbital plane and the moon orbital plane at their intersection. In the case of “short coast” here, the spacecraft is injected into LTO in the descending direction (note that the discussion here is the case that the LEO injection point is in the northern hemisphere), and it moves in the ascending direction at the terminal of the LTO (See Fig.2). Therefore, the angle between the velocities at the intersection is large when the moon is at the descending node and is small when the moon is at the ascending node. The transition of LLO injection  $\Delta v$  reflects the transition of these two factors. There are peaks at the perigee and the descending node, and the bottoms at the apogee. Since the magnitude of the component by the velocity direction difference is small, the bottom corresponds to the ascending node does not appear. It is observed that the interval between the two peaks extends little by little. This is the effect of the perturbation of the solar gravity to the moon orbit.

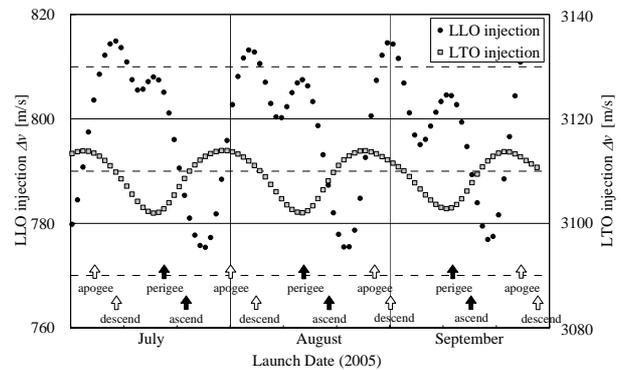


Fig. 3. LTO injection  $\Delta v$  and LLO injection  $\Delta v$

Secondly discussed is the difference by the property of “short coast” and “long coast”. Since these two cases differ from the start of the sequence, difference by this property can be found in many aspects. However, the essence is the difference of LTO orbital plane and the difference of LTO injection point.

The most important effect caused by the difference of LTO orbital plane is the difference in the transition of LLO injection  $\Delta v$  by the launch date. On the contrary to

the case of “short coast” discussed before, in the case of “long coast”, the spacecraft moves in the descending direction at the terminal of LTO (See Fig.2). Therefore, the transition of the difference in the velocity direction between the spacecraft and the moon shows the trend inverse to that observed in the case of “short coast”. That is to say, the difference in the velocity direction is large when the moon is at the ascending node and is small when the moon is at the descending node. On the other hand, the transition of the difference in the velocity magnitude between the spacecraft and the moon shows the same trend as that observed in the case of “short coast”. The transitions of these two factors result in the transition of LLO injection  $\Delta v$  shown in Fig. 4. In the figure, the transitions of LLO injection  $\Delta v$  of the two cases are shown (“short coast” in solid circles and the “long coast” in squares) with the marks that signify the position of the moon at LSOI injection. In the case of “long coast”, there are large peaks results from the combination of the adjacent peaks of the perigee and the ascending node. In addition, about half of a month, the LLO injection  $\Delta v$  in the case of “long coast” is less than that in the case of “short coast”.

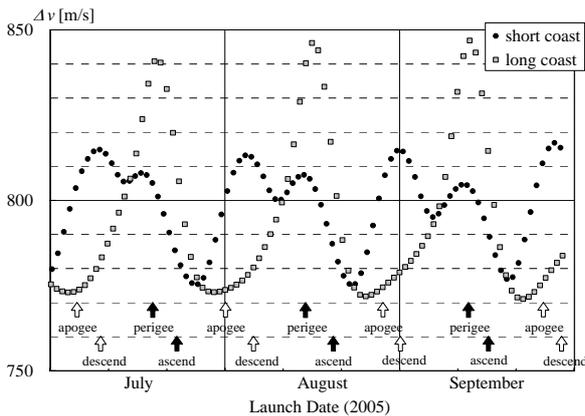


Fig. 4. LLO injection  $\Delta v$  of short/long coast

Other examples of the effect caused by the difference of LTO orbital plane are, the difference in the visibility of the spacecraft on LTO from the ground station in northern/southern hemisphere, the difference in the argument of perilune at PLP, and so on.

Next, the most important effect caused by the difference of LTO injection point is the duration of the coasting on LEO. In most case, the launcher is still in operation on LEO. Therefore, the performance of the launcher (for example, the amount of the gas for the attitude control thrusters, or the accuracy of the inertial navigation system) usually limits the duration of the coasting. In practice, “long coast” is often not applicable from this limit. The difference of LTO injection point itself is also important. The factors to be considered are, the visibility of the LTO injection maneuver from the

ground station, the eclipse soon after the spacecraft’s separation from the launcher, and so on.

Finally discussed in this section is the difference by the property of “ascending approach” and “descending approach”. The difference by this property is limited compared to the difference by the former property. The reason is that this property is related mainly to LAO, the final phase of the sequence, and basically it does not effect to the former phase. (Of course the parameters in the former phase are adjusted to satisfy the boundary condition at the terminal, the effect of the difference in LAO by this property is small.) The difference by this property pointed out here is the difference of the argument of perilune at PLP. If the spacecraft is injected directly into the circular LLO, this value is not important. However, if the spacecraft is injected into the elliptical orbit, this value is closely related with the effect of the perturbation of the earth gravity.

Fig. 5 shows LAO of the two cases projected on  $x$ - $z$  plane of the moon centred moon orbital plane coordinate system. The thick line is LAO in the case of “ascending approach” and the thin line is LAO in the case of “descending approach”. The launch date is August 4 in 2005, and the property as to the coasting on LEO is “short coast”. Also drawn in the figure are the straight lines and the ellipses to make clear the relation between the major axis of LAO and the moon orbital plane. The straight dash-and-dotted line is the major axis of LAO, and the ellipse in dashed line is the elliptical orbit with eccentricity 0.4 whose perilune is identical with that of LAO. (The ellipse looks warped since the orbit is not viewed from the right in front of its orbital plane.) It indicates that the effect of the perturbation of the earth to the elliptical orbit is different between the two cases.

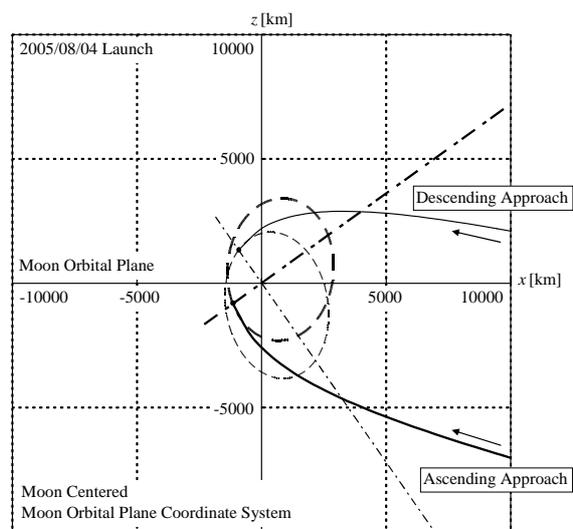


Fig. 5. LAO of ascending/descending approach

#### 4. SENSITIVITY ANALYSIS

Discussed in this section is the sensitivity of a number of items to the deviation of the parameters from the optimal solutions. However, the discussion is not for the estimation of the influence of some unintentional deviation, but for the estimation of the effect of some intentional deviation.

Firstly discussed is the deviation of the two parameters that defines the direction of LTO injection  $\Delta v$  from the optimal values, that is, tangential to LEO. Two parameters are introduced to express the direction of LTO injection  $\Delta v$ . These are, the change of the inclination between before and after the maneuver,  $\Delta i$ , and the change of the flight path angle (from the local horizon) between before and after the maneuver,  $\Delta \gamma$ . For the optimal solution, where  $\Delta v$  (and LTO as well) is tangent to LEO, the values of these two parameters are zero. Deviation of 1.5deg. for  $\Delta i$  or  $\Delta \gamma$  is assumed and their effects are investigated. The value of deviation, "1.5deg.", is selected so that the increment of LTO injection  $\Delta v$  from the optimal solution to become about 10m/s, that is approximately the maximum difference of LTO injection  $\Delta v$  by the launch date (See Fig.3). In the analysis, one of  $\Delta i$  and  $\Delta \gamma$  is assigned to 1.5deg., and the other is assigned to zero. The remaining three free parameters are used to adjust the three boundary conditions at the terminal. The properties of the sequence used in this analysis are "short coast" and "ascending approach".

The following five items are selected to evaluate the effect of the deviation.

- (a) LEO injection time
- (b) LTO injection position
- (c) PLP time
- (d) Argument of perilune at PLP
- (e) LLO injection  $\Delta v$

These items are all closely related to the practical operation. LEO injection time is related to the time of launch, the position of LTO injection is related to the visibility of LTO injection maneuver, PLP time is related to the visibility of LLO injection maneuver, the argument of perilune is related to the perturbation of the earth, and LLO injection  $\Delta v$  is the item of the most interest.

Firstly shown is the result as to the deviation of  $\Delta i$ . The effect of the deviation varies by the launch date. Table 1 shows the result of the launch date July 28 in 2005, the launch date in that the effect is the largest. The position of the moon at the LSOI injection is about 90 deg. from the ascending node of the moon orbit, that is, the moon is at the north edge of its orbit. (a) to (e) are the selected items listed above. Obviously, the effect of the deviation of  $\Delta i$  is small even with this largest case. The effect by the deviation of  $\Delta i$  obtained in expense of 10m/s loss in

LTO injection  $\Delta v$  are about, 4 minutes in LEO injection time and 20 minutes in PLP time. LTO injection position, the argument of perilune at PLP, and LLO injection  $\Delta v$  hardly change.

Table 1. Effect of deviation of  $\Delta i$

$\Delta i$	+1.5deg.	0.0deg.
(a)	2005/07/28 04:41:26	2005/07/28 04:45:16
(b)	104.6degW / 27.5degS	104.8degW / 27.4degS
(c)	2005/08/02 05:02:00	2005/08/02 05:24:17
(d)	331.7deg.	331.3deg.
(e)	778.6m/s	777.3m/s

Next is the result as to the deviation of  $\Delta \gamma$ . The effect of the deviation varies by the launch date. Table 2 shows the result of the launch date July 31 in 2005, the launch date in that the effect is the largest. The position of the moon at the LSOI injection is about the apogee of the moon orbit. The effect of the deviation of  $\Delta \gamma$  is larger than the case of  $\Delta i$ . The effect by the deviation of  $\Delta \gamma$  obtained in expense of 10m/s loss in LTO injection  $\Delta v$  are about, 15 minutes in LEO injection time, 3deg. in LTO injection position longitude, and 20 minutes in PLP time. The argument of perilune at PLP and LLO injection  $\Delta v$  hardly change.

Table 2 Effect of deviation of  $\Delta \gamma$

$\Delta \gamma$	+1.5deg.	0.0deg.
(a)	2005/07/31 08:45:14	2005/07/31 09:00:54
(b)	129.4degW / 18.1degS	126.5degW / 19.5degS
(c)	2005/08/05 09:32:54	2005/08/05 09:09:51
(d)	337.5deg.	337.5deg.
(e)	795.9m/s	795.9m/s

Although the effect of the deviation of these two parameters is small, the deviation imposes no penalty, that is, the increase of LLO injection  $\Delta v$ , on the spacecraft (note that LTO injection maneuver is generally the part of the launcher). Therefore, for example, a few minutes' launch delay can be recovered only by the launcher with small additional  $\Delta v$ , and without any effect on the spacecraft.

Secondly discussed is the effect of the deviation of  $\theta$ . In the analysis up to now, this parameter was used with the other two parameters, LEO injection time and LTO injection time, to adjust three boundary conditions at the terminal. However, in the analysis here,  $\theta$  is assigned to a fixed value that deviates from the value of the optimal solution. The two parameters that define the direction of LTO injection  $\Delta v$  are again fixed to the optimal values, that is, tangential to LEO. Therefore, the number of the remaining free parameters is two and the two boundary conditions here are to assign the altitude and the inclination at PLP. The properties of the sequence used

in this analysis are “short coast” and “ascending approach”.

Deviation of 5deg. for  $\theta$  is assumed and their effects are investigated. The value of deviation, “5deg.” is selected so that the increment of LLO injection  $\Delta v$  from the optimal solution to become about 40m/s, that is approximately the maximum difference of LLO injection  $\Delta v$  by the launch date (See Fig.3). The increment of LTO injection  $\Delta v$  from the optimal solution for this deviation is about 5m/s.

The effect of the deviation varies by the launch date. Table 3 shows the result of the launch date July 31 in 2005, the launch date in that the effect is the largest. The position of the moon at the LSOI injection is about the apogee of the moon orbit. (a) to (e) are the selected items listed before.

The effect of the deviation of  $\theta$  is far larger than the former two cases of  $\Delta i$  and  $\Delta \gamma$ . The effect by the deviation of  $\theta$  obtained in expense of 40m/s loss in LLO injection  $\Delta v$  (and 5m/s loss in LTO injection  $\Delta v$ ) are about, 3 hours in LEO injection time, 25deg. in LTO injection position longitude, 3 days in PLP time, and 2 deg. in the argument of perigee at PLP.

Table 3 Effect of deviation of  $\theta$

$\theta$	+5.0deg.	0.0deg.
(a)	2005/07/31 11:57:42	2005/07/31 09:00:54
(b)	152.6degW / 4.5degS	126.5degW / 19.5degS
(c)	2005/08/08 00:10:54	2005/08/05 09:09:51
(d)	335.4deg.	337.5deg.
(e)	829.1m/s	795.9m/s

Although the effect of the deviation of  $\theta$  is large, the deviation imposes penalty on the spacecraft, that is, the increase of LLO injection  $\Delta v$ . However, the value of the deviation used here, 5 deg., is an extreme case. If smaller effect is sufficient, the increase of LLO injection  $\Delta v$  will be smaller. The result here indicates that the conditions on these listed items are tradable with LLO injection  $\Delta v$  in wide range.

## 5. CONCLUSION

The characteristics of the two-burn translunar trajectory are investigated. The structure of the problem and its solution space are made clear, and the optimal solutions are grouped by the two properties of the sequence. The characteristics of the optimal solutions are analyzed from two aspects, the transition by the launch date and the difference by the property of the sequence. The sensitivity of a number of items to the deviation of the parameters from the optimal value is analyzed.

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