

## SELENE TRANSLUNAR TRAJECTORY RECONFIGURATION PLAN PROVIDED FOR THE CASE OF MAIN ENGINE ANOMALY

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In this paper, the reconfiguration of translunar trajectory in case of main engine anomaly is investigated. The objectives of the trajectory design are to reduce the excessive velocity at the Lunar encounter as well as to reduce the total required  $\Delta v$  to complete the sequence. 3-impulse Hohmann transfer based trajectory is adopted and possible trajectories are categorized under 2-body approximation. The solutions obtained are applied to more sophisticated models (3-body approximation and 4-body) and yields feasible trajectory.

### INTRODUCTION

SELENE is a Japanese Lunar explorer, which is planned to be launched in September, 2007 (Figure 1). SELENE is a Lunar polar orbiter of 100km altitude with two sub-satellites for gravity field measurement. The mission of SELENE is to obtain the global scientific information of the Moon, which includes mapping of elemental abundance, mineralogical composition, topography and geological structure, and the gravity field data for both near and far side of the Moon with high accuracy and resolution. SELENE payload includes 14 instruments such as spectrometers and imagers. Synergistical analysis with these data will improve our understanding of the origin and evolution of the Moon.

Figure 2 shows the Lunar transfer sequence of SELENE. SELENE is once injected into a long elliptical phasing orbit around the earth, and after two rotations on the phasing orbit, it is injected into the translunar trajectory for the final approach to the Moon. At the Lunar encounter, a deceleration maneuver is performed at the perilune passage of the Lunar approaching hyperbolic orbit, and SELENE is injected into a Lunar orbit.



Figure 1 Overview of SELENE

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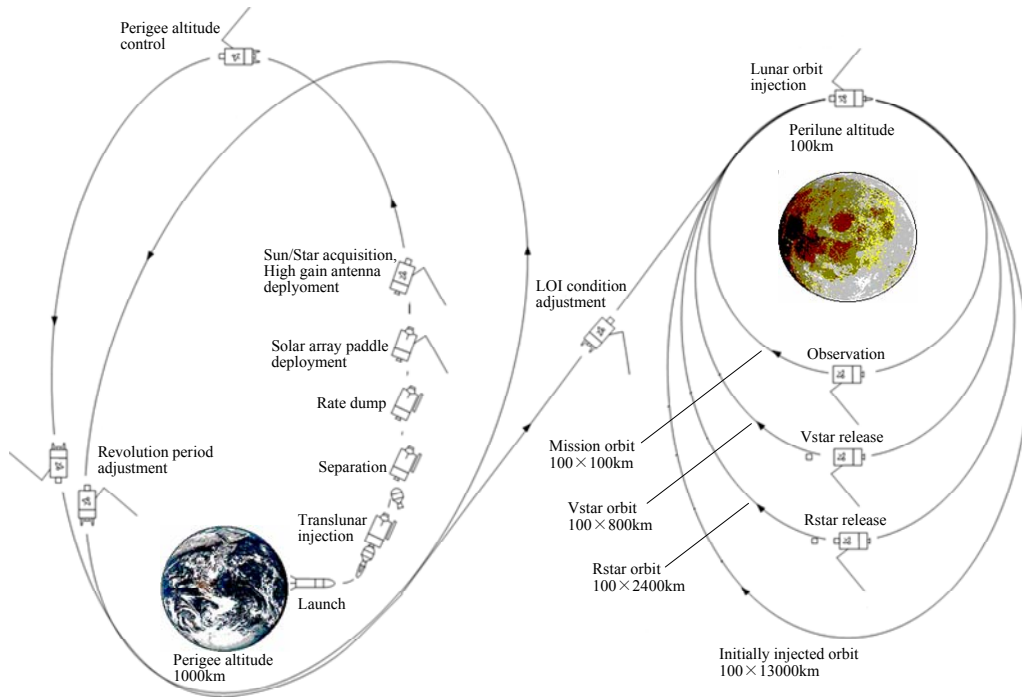


Figure 2 SELENE Lunar Transfer Sequence

The Lunar orbit injection maneuver is planned to be performed by a 500N bi-propellant main engine. It is the only large thrust engine onboard, and the remaining way to produce effective axial thrust is to use twelve 20N mono-propellant thrusters, which are originally planned to be used for the attitude control during the main engine maneuver and small orbit maneuvers during the operation.

A situation discussed in this paper is that, it turns out prior to the Lunar orbit injection that the main engine is unable to be used. The situation supposes not only faults in the main engine, but also other obstacles such as the misalignment of the main engine thrust axis and the spacecraft's mass center due to the solar array paddle deployment failure, or the double failure on the 20N thrusters in the same control direction. It must be noted that, in any case, the situation is “contingency”, which is not caused by a single fault, and the possibility of the emergence of the anomaly is extremely low.

The problems in this case are summarized as the following two points. Firstly, the main engine is the only bi-propellant engine on board. Its unavailability makes the oxidizer useless and the substitution by the mono-propellant thruster lower the specific impulse of the maneuver. As a result, the total  $\Delta v$  attainable is reduced and it becomes impossible to reach the originally planned mission orbit (that is, a circular orbit with the altitude 100km.) Secondly, there is a restriction in the continuous operation time of the 20N thrusters, which limits the  $\Delta v$  attainable by a single maneuver to be less than approximately 100m/s. On the other hand, the original lunar encounter condition requires more than 200m/s of  $\Delta v$  to be captured by the Moon.

To cope with this situation, a trajectory reconfiguration plan is investigated. The primary objective of the trajectory reconfiguration is to reduce the excessive velocity at the lunar encounter to lower the minimum required  $\Delta v$  to be captured by the moon. The second objective is to reduce the total required  $\Delta v$  to complete the sequence, so as to enable the improvement of the observation condition or the enhancement of the operation life.

In the scope of the paper, the anomaly is supposed to be detected sufficiently early so that the lunar encounter condition is able to be modified by a moderate correction maneuver in advance. It is assumed that the anomaly is detected before the final trajectory correction maneuver planned at 3 days before LOI (it is approximately 2 weeks from the launch). Since some maneuver operations (using the main engine) are planned prior to this maneuver, if any problem, it is likely to be detected prior to this time limit.

The basic concept of the reconfigured trajectory is described as follows (The number coincides with the notation in Figure 3).

1. Main engine anomaly is detected on the way to the Moon of the original trajectory.
2. Correction maneuver is performed to modify the lunar encounter condition.
3. SELENE flies by the moon and swung out to a distant apogee by the gravity assist of the Moon.
4. Deep space maneuver (DSM) is performed nearby the apogee to adjust the orbit to re-encounter the Moon.
5. SELENE re-encounters the Moon.

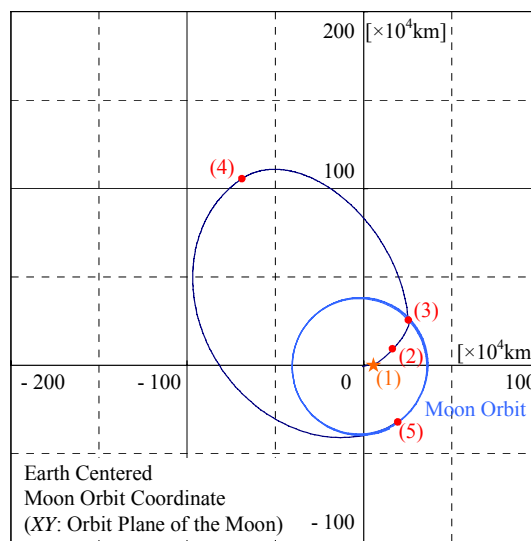


Figure 3 Basic Concept of the Reconfigured Trajectory

By carefully tuning the first Lunar encounter condition, the amount of DSM and the excessive velocity ( $v_{\infty}$ ) at the re-encounter can be reduced sufficiently. In addition, if the geometric relation between the trajectory and the Sun is desirable, the perturbation by the Sun gravity can be applied effectively, which results in the further reduction of the DSM.

This concept is basically the same as the sequence proposed by Belbruno and Miller (Ref. 7 and 8) or Yamakawa (Ref. 9 and 10). However, there is a small but an important difference in case of SELENE, that the condition of the first lunar encounter is strongly constrained by the original nominal translunar trajectory. Basically, the position and the time of the first lunar encounter cannot be shifted widely from those of the original condition. In the previous works, the encounter condition was important design parameters, because it determines the geometric relation between the trajectory and the Sun, which closely related to the effective application of the perturbation by the Sun gravity. Therefore, the constraint in the first lunar encounter condition in the case of SELENE means that the way to control the effect of the perturbation by the sun gravity is strongly restricted.

Under this situation, to widen the design flexibility and to find a better (less  $\Delta v$ ) solution, wider range of the trajectory configurations are considered in this work. All of the trajectory configurations conserve essential features of the basic concept shown in Figure 3. That is, the time of the Lunar swing-by ( $t_{LSB}$ ), the inclusion of one lunar swing-by and one DSM, and small  $v_\infty$  at the Lunar re-encounter ( $v_{\infty LRE}$ ). On the other hand, the trajectory configurations differ in the remaining features, such as the time of DSM ( $t_{DSM}$ ), the time of the Lunar re-encounter ( $t_{LRE}$ ), and the number of rotations before and after DSM. Therefore, the effect of the perturbation by the sun gravity differs by the trajectory configurations even though their start point are the same (that is, the constraint in the first lunar encounter condition in the case of SELENE is satisfied.)

Various configurations of the trajectories are initially designed under 2-body approximation based on the concept of 3-impulse Hohmann transfer. By using the trajectories obtained under 2-body approximation, the detailed trajectories are designed under the 3-body and 4-body approximation. The design parameters, such as the swing-by condition at the first lunar encounter,  $t_{DSM}$  and  $t_{LRE}$ , are tuned and  $\Delta v$  at DSM ( $\Delta v_{DSM}$ ) and  $v_{\infty LRE}$  are estimated for each trajectories. The performance of the trajectory differs by the trajectories depending on the effect of the perturbation by the Sun gravity during the sequence. The trajectory with the better performance is selected as the candidate of the reconfigured trajectory.

In the following sections, the trajectory design process is introduced step by step with the examples of the designed trajectories.

## TRAJECTORY DESIGN PROCESS

### Two body problem approximated solutions

The concept of the reconfigured trajectory introduced in the previous section is based on the 3-impulse Hohmann transfer. Figure 4 depicts the concept of 2-impulse and 3-impulse Hohmann transfer. It is known that, when  $r_2/r_1 > 15.58$ , for arbitrary selection of  $r_m (> r_2)$ , total  $\Delta v$  required for 3-impulse Hohmann transfer is smaller than that required for 2-impulse Hohmann transfer. Here,  $r_1$ ,  $r_2$  and  $r_m$  denotes the radius of the initial orbit, the terminal orbit and the intermediate apogee respectively. In case of the Lunar transfer, the ratio  $r_2/r_1$  is approximately 60, and it is in the range where 3-impulse Hohmann transfer is the superior. Additionally, there are two features of this problem which may give advantage to 3-impulse Hohmann transfer. Firstly, by the usage of the Lunar swing-by,  $\Delta v$  to inject from the initial orbit to the transfer orbit ( $\Delta v_1$ ) can be reduced compared to the case of direct transfer to the intermediate apogee. Secondly, if the geometric relation between the trajectory and the Sun is desirable, the perturbation by the Sun gravity could be utilized effectively to reduce  $\Delta v$  at intermediate apogee ( $\Delta v_2$ ) or  $\Delta v$  to inject from the transfer orbit to the final orbit ( $\Delta v_3$ ).

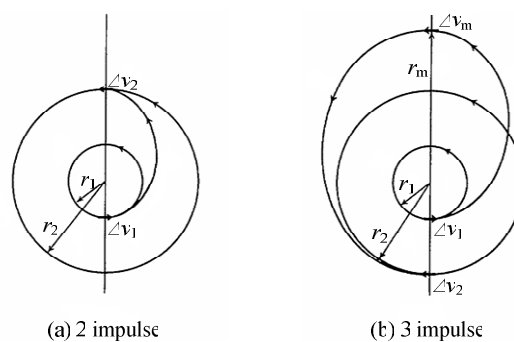


Figure 4 2 Impulse and 3 Impulse Hohmann Transfer

Based on the concept of 3-impulse Hohmann transfer, the trajectory (from the Lunar swing-by) to transfer to the orbit of the Moon is designed by the following procedure (Figure 5).

1. The time of the Lunar swing-by ( $t_{LSB}$ ) and  $v_\infty$  at the Lunar swing-by ( $v_{\infty LSB}$ ) are given from the Lunar encounter condition of the nominal trajectory. By setting the azimuth and the elevation of  $v_\infty$  after the Lunar swing-by ( $\alpha_{LSB}, \delta_{LSB}$  respectively), the trajectory after the Lunar swing-by is determined (green line in Figure 5).  $\alpha_{LSB}$  is the angle between  $v_{\infty LSB}$  after the Lunar swing-by and the velocity of the Moon within the orbit plane of the Moon, and  $\delta_{LSB}$  is the angle between  $v_{\infty LSB}$  after the Lunar swing-by and the orbit plane of the Moon.  $\delta_{LSB}$  is constantly set to 0 (i.e. the trajectory after the Lunar swing-by is limited in the orbit plane of the Moon), and  $\alpha_{LSB}$  is restricted so as the perilune radius at the Lunar swing-by to be larger than the radius of the Moon.
2. At the apogee of the trajectory after the Lunar swing-by, a deep space maneuver (DSM) is performed in tangential direction (the time of DSM ( $t_{DSM}$ ) is determined).  $\Delta v$  at DSM ( $\Delta v_{DSM}$ ) is determined so that the trajectory after DSM (orange line in Figure 5) tangent at its perigee to the orbit of the Moon.
3. At the perigee of the trajectory after DSM, SELENE arrives at the orbit of the Moon (blue point in Figure 5). The angle at the arrival and the phase angle ( $\theta$ ) at the arrival are determined.  $\theta$  is the angle from the perigee direction of the Moon measured within the orbit plane of the Moon.

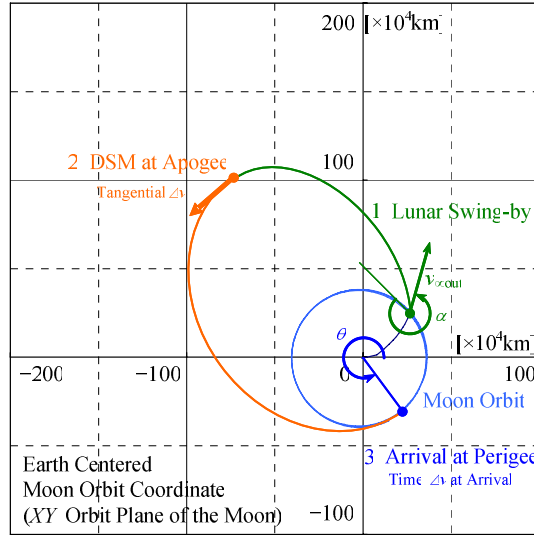


Figure 5 Trajectory Design to Transfer to the Moon Orbit (2-Body Approximation)

Obviously, designed by the procedure above is the trajectory to transfer to the orbit of the Moon, and the re-encounter with the Moon is not guaranteed for the designed trajectory. The trajectory to re-encounter with the Moon can be selected from the designed trajectories in the following way (Figure 6). Firstly, the plots in dark blue of Figure 6 denote the sets of the time and  $\theta$  at the terminal (i.e. arrival at the orbit of the Moon) of the trajectories designed in the procedure above for various  $\alpha_{LSB}$ . Though it seems intermittent plots in the right hand side region due to the discrete  $\alpha_{LSB}$ , essentially it is consistent line. Two lines (upper and lower) are observed in the figure. The upper line is the plots of the trajectory which goes out-bounds of the orbit of the Moon just after the Lunar swing-by (Figure 7 (a)). The lower line is the plots of the trajectory which goes in-bounds of the orbit of the Moon just after the Lunar swing-by (Figure 7 (b)). Next, the plots in light blue of Figure 6 denote the sets of the time and  $\theta$  of the Moon. Then, the intersection of the dark and the light blue lines (marked by red circles) means that SELENE and the Moon is at the same place at the same time, namely, the re-encounter.

Each intersection (marked with red circle) in Figure 6 denotes individual trajectory that re-encounter with the Moon. The labels (ex. *out\_0*) denotes the direction of the trajectory (out-bound or in-bound) and the number of the rotation of the Moon from the swing-by to the re-encounter (the integer after the underscore). Figure 7 shows the trajectories that re-encounter with the Moon selected in Figure 6. Five out-bound trajectories and four in-bound trajectories are shown in the figure.

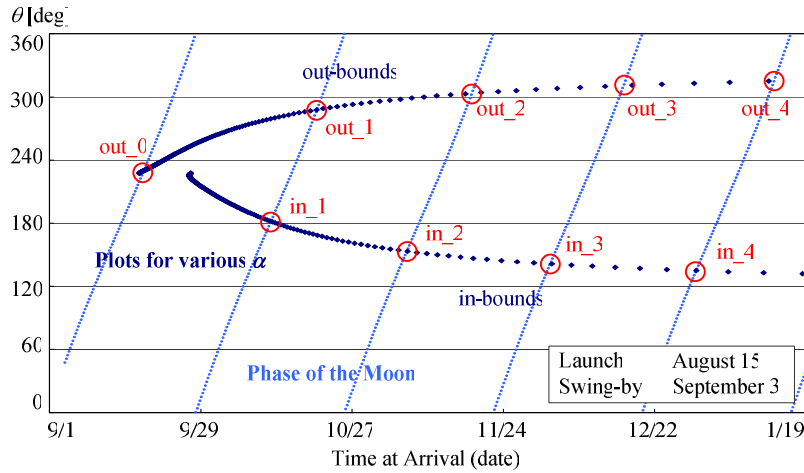


Figure 6 Selection of the Trajectory to Re-encounter the Moon

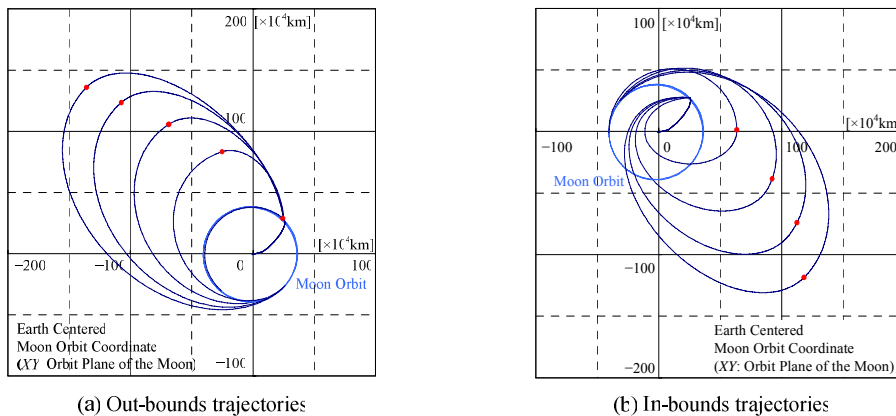


Figure 7 Designed Trajectories to Re-encounter the Moon

The trajectories shown in Figure 7 are the typical set of the trajectories based on the concept of 3-impulse Hohmann transfer, however, they are not all of them. Other series of “3-impulse Hohmann transfer based trajectories” can be constructed by increasing the number of rotations (around the Earth) before and after DSM. In the case of trajectories shown in Figure 7, the numbers of rotations before and after DSM are both 0. Figure 8 shows three examples of the other trajectory configurations. It can be seen that the number of rotations in the trajectory before DSM (green line) and the trajectory after DSM (orange line) are different in the three examples.

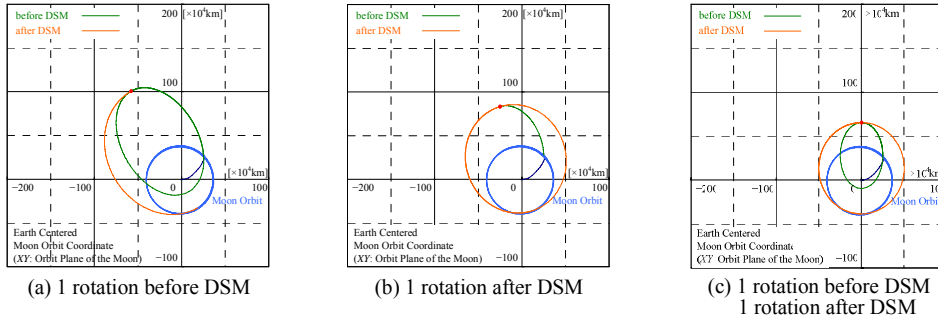


Figure 8 Expansion of Possible Trajectories under 2-Body Approximation

Considering the various configurations of the trajectories (i.e. the number of rotations before and after DSM), much more possible trajectories are obtained to re-encounter with the Moon. Figure 9 is the same plot as Figure 6, but includes more plots of various configurations of the trajectory. Many intersections of the dark and the light blue lines denote the possible trajectories that re-encounter with the Moon. Considering the conditions related to the practical operation below, 19 feasible trajectories (marked in red solid circle in Figure 9) are selected from the possible trajectories (the unselected trajectories are marked in blue cross in Figure 9). The conditions considered are,

- Distance at DSM < 2,000,000km.
- $\Delta v_{DSM} < 300\text{m/s}$ .
- $t_{LRE} < t_{LSB} + 4$  months
- $v_{\infty LRE} < 300\text{m/s}$

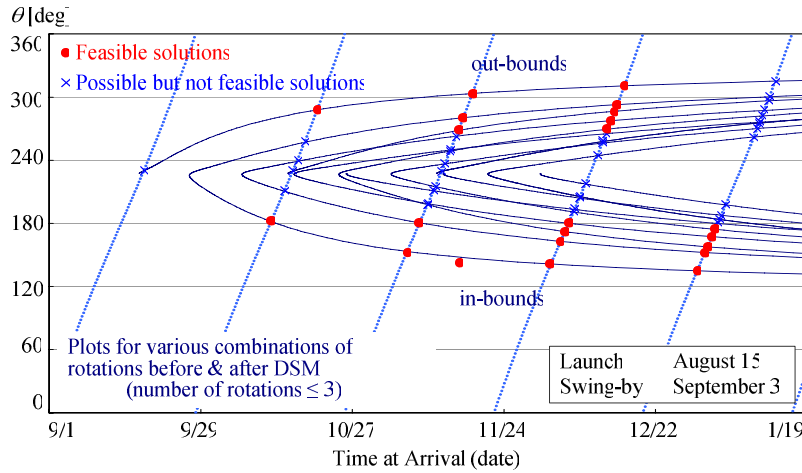


Figure 9 Possible Solutions under 2-Body Approximation

Figure 10 shows the 19 feasible trajectories selected in Figure 9. The labels (ex.  $00\_out\_0$ ) denotes the number of rotations before and after DSM (the two integer before the first underscore), the direction of the trajectory (out-bounds or in-bounds), and the number of the rotation of the Moon from the swing-by to the re-encounter (the integer after the second underscore).

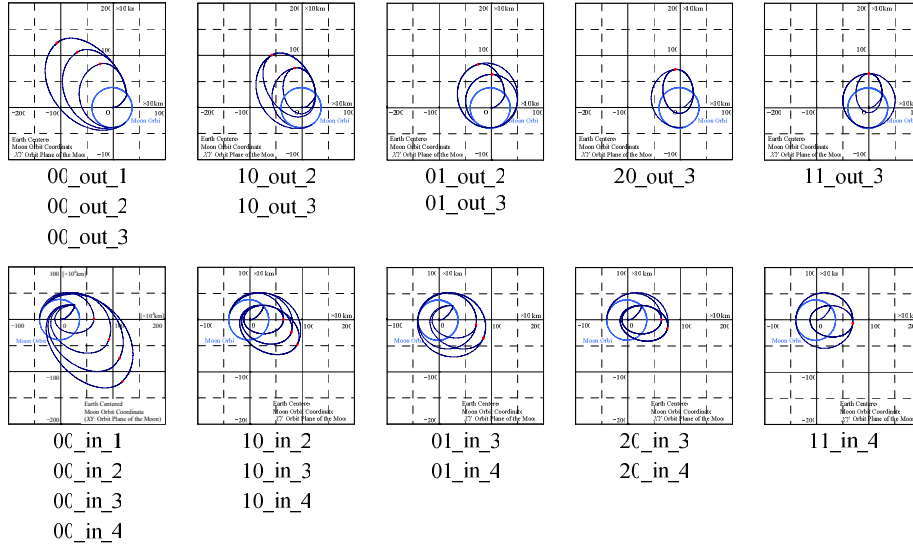


Figure 10 Possible Trajectories under 2-Body Approximation

### Trajectory Design under 3-Body Approximation

Following the results of the investigation under 2-body approximation, the trajectory design is performed taking into account the perturbation by the Sun gravity. Namely, it is the trajectory design under 3-body approximation. The consideration of the Sun gravity makes possible to assess if the perturbation by the Sun gravity can be utilized effectively to reduce  $\Delta v_{DSM}$  and  $v_{\infty LRE}$ . Naturally, the effect of the perturbation strongly depends on the geometric relation between the trajectory and the Sun. Therefore, it is expected that the effect differs by the trajectories shown in Figure 10.

The trajectories are designed based on the results under 2-body approximation. As to the four parameters,  $\alpha_{LSB}$ ,  $\delta_{LSB}$ ,  $t_{DSM}$  and  $t_{LRE}$ , the values obtained under 2-body approximation are used as they are in the trajectory design under 3-body approximation. And  $\Delta v_{DSM}$  is tuned to target the final position of the trajectory (i.e. the position of the Moon at  $t_{LRE}$ ), so that the closed trajectory is obtained under 3-body approximation.

This procedure is applied to the 21 possible trajectories obtained under 2-body approximation, however, not all the cases give the expected designed trajectory under 3-body approximation.

For 6 cases,

$$00\_in\_1, 00\_in\_4, 00\_out\_1, 00\_out\_2, 01\_out\_2, 01\_out\_3$$

the designed trajectory converged to the configuration as expected. The numbers of rotations before/after DSM in the obtained trajectory hold the numbers in the respective original trajectory obtained under 2-body approximation. Figure 11 shows the two examples in this category ( $00\_out\_1$  and  $01\_out\_2$ ), in which the trajectory obtained under 2-body approximation (blue line) and that obtained under 3-body approximation (red line) are drawn in the same figure. It is observed that the shape of the trajectory transformed by the perturbation of the Sun gravity, however, the configuration of the trajectory (i.e. the numbers of rotations before/after DSM) is remain unchanged in the trajectory obtained under 3-body approximation.



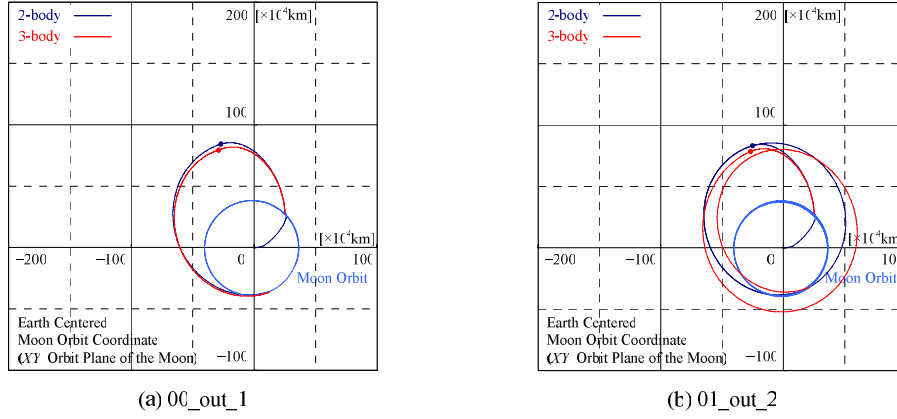


Figure 11 Cases that 2-Body Solutions Give Good Initial Estimation

For the other 15 cases, the designed trajectory does not converged to the configuration as expected. In some cases, the targeting process is converged (that is,  $\Delta v_{DSM}$  to satisfy the terminal position is obtained), however, the configuration of the designed trajectory is not the same as that of the original trajectory obtained under 2-body approximation. In other cases, the targeting process is not converged and the closed trajectory is not obtained under 3-body approximation. Two reasons are supposed to explain the situation of these 15 cases. First is that, the value of the four parameters inherited from the results of the 2-body approximation is inappropriate for the trajectory design under 3-body approximation. Second is that, the geometric relation between the trajectory and the Sun is inappropriate, so that the perturbation by the Sun gravity effects in undesirable way. At this moment, the detail is under investigation.

In the design above, the values of four parameters are fixed to the values inherited from the results under 2-body approximation. Naturally, those values are not optimized for the trajectories under 3-body approximation and there remains the space for improvement. In other words, it is possible to reduce the total  $\Delta v$  required to complete the sequence by tuning the value of these parameters.

Four parameters,  $\alpha_{LSB}$ ,  $\beta_{LSB}$ ,  $t_{DSM}$  and  $t_{LRE}$  are tuned to minimize the specific  $\Delta v$  ( $\Delta v_{sp}$ ) defined as

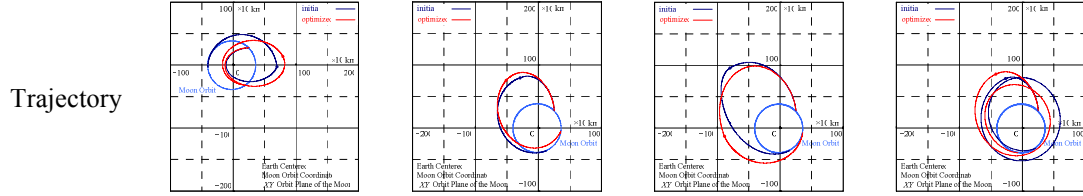
$$\Delta v_{sp} = \Delta v_{DSM} + \Delta v_{LOI} \quad (1)$$

where  $\Delta v_{LOI}$  is  $\Delta v$  required for the Lunar orbit injection, calculated from  $v_{\infty LRE}$  assuming the injection into the Lunar orbit of perilune radius 1838km and apolune radius 30000km. In each step of the optimization (i.e. whenever the parameters are updated),  $\Delta v_{DSM}$  is tuned to target the final position of the trajectory (i.e. the position of the Moon at  $t_{LRE}$ ), so that the closed trajectory is obtained.

This optimization procedure is applied to the 4 cases ( $00_{in_1}$ ,  $00_{out_1}$ ,  $00_{out_2}$ , and  $01_{out_3}$ ) which give the expected designed trajectory in the previous step. The result of the optimization is summarized in Table 1. It is observed that the shape of the optimized trajectory (red line) transformed from that before the optimization (blue line), however, the configuration of the trajectory (i.e. the numbers of rotations before/after DSM) is remain unchanged in the trajectory after optimization. The value of performance index  $\Delta v_{sp}$  is improved largely as a result of the optimization. The quantity of the reduction depends on the cases. The order of the  $\Delta v_{sp}$  among the four cases seems unchanged between “before” and “after” the optimization, however, it is questionable that this rule can be applied in general. More moderate rule (ex. “There is a tendency that ...”) may be possible, however, further investigation is required for this point. Additionally, it must be noted that the  $\Delta v_{LOI}$  required for the first Lunar encounter is estimated approximately as 210m/s.  $\Delta v_{sp}$  of  $00_{out_1}$ ,  $00_{out_2}$ , and  $01_{out_3}$  are smaller than this value, and it means that the total required  $\Delta v$  to complete the sequence is reduced at this step of 3-body approximation.

**Table 1 Results of the Parameter Optimization**  
 (\*)  $\Delta v_{sp\_0}$  denotes  $\Delta v_{sp}$  before optimization)

	<u>00_in_1</u>	<u>00_out_1</u>	<u>00_out_2</u>	<u>01_out_3</u>
$(\Delta v_{sp\_0})^{(*)}$	(420.3m/s)	(178.9m/s)	(356.9m/s)	(281.9m/s)
$\Delta v_{sp}$	283.8m/s	106.8m/s	183.3m/s	120.7m/s
$\Delta v_{DSM}$	86.4m/s	1.0m/s	108.9m/s	46.5m/s
$\Delta v_{LOI}$	197.4m/s	105.8m/s	74.4m/s	74.2m/s



### Trajectory Design under 4-Body Approximation

Following the results of the investigation under 3-body approximation, the trajectory design is performed taking into account the perturbation by the Lunar gravity. Namely, it is the trajectory design under 4-body approximation. The consideration of the Lunar gravity refines the orbit dynamics modeling at the Lunar swing-by and the Lunar re-encounter and it enables more practical  $\Delta v$  estimation. The  $\Delta v$  at the maneuver prior to the Lunar swing-by to adjust the Lunar swing-by condition ( $\Delta v_{adj}$ ) is calculated, and the third body (i.e. the Earth) effect at the Lunar re-encounter is reflected on the value of  $\Delta v_{LOI}$ .

The trajectories are designed based on the results under 3-body approximation. As to the five parameters,  $\mathbf{r}_{DSM}$  (include 3 components),  $t_{DSM}$  and  $t_{LRE}$ , the values obtained under 3-body approximation are used as they are in the trajectory design under 4-body approximation.  $\Delta v_{adj}$  is tuned to target  $\mathbf{r}_{DSM}$  and  $\Delta v_{DSM}$  is tuned to target the final condition of the trajectory, so that the closed trajectory is obtained under 4-body approximation. The final condition is defined on the Moon-fixed coordinate system at  $t_{LRE}$ . The inclination, the radius, and the true anomaly are assigned to 90deg., 1838km, and 0deg. respectively.

This procedure is applied to the case *00\_out\_1*, which gives the smallest  $\Delta v_{sp}$  as a result of the optimization process under 3-body approximation. Figure 13 shows the result of the trajectory design under 4-body approximation. Drawn in the figure are the trajectories obtained under 3-body approximation (blue line) and that obtained under 4-body approximation (red line). It is observed that the transformation of the trajectory from 3-body to 4-body is smaller than that from 2-body to 3-body. It can be concluded that the value of the five parameters inherited from the results of the 3-body approximation is sufficiently effective as initial estimates under 4-body approximation. Figure 14 shows the sequence of events of the reconfigured trajectory denoted on the trajectory designed under 4-body approximation. After the main engine anomaly is detected on the way to the Moon, 39.0m/s of adjust maneuver is performed on August 31 (approximately 3 days before the Lunar swing-by). SELENE flies by the Moon on September 3 and 64.0m/s of DSM is performed on September 28 at the 900,000km distant from the Earth. SELENE re-encounters the Moon on October 24 and injected into the Lunar orbit of perilune radius 1838km and apolune radius 30000km by 116.1m/s of LOI maneuver. Though the value of  $\Delta v_{LOI}$  is slightly larger than the target value (less than 100m/s approximately), it seems adjustable by enlarging the apolune altitude if necessary. From this result, it can be concluded that the feasible reconfigured trajectory is obtained.

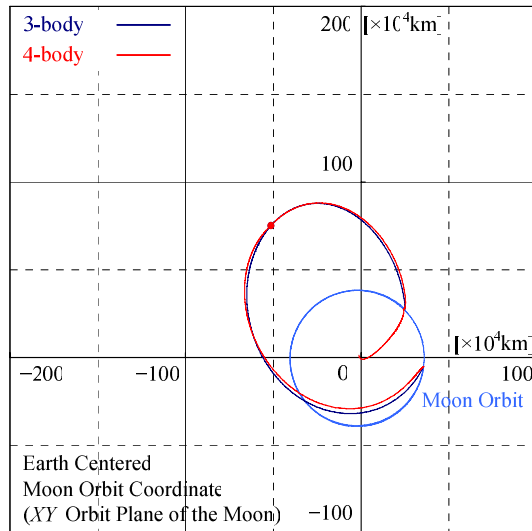


Figure 13 Results of the Design under 4-body approximation

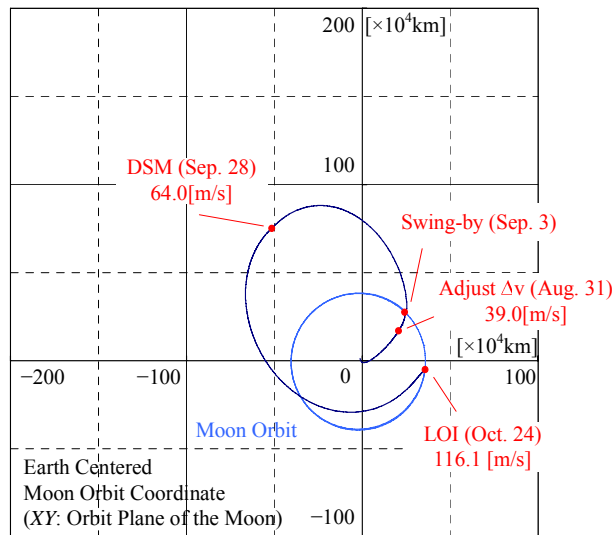


Figure 14 Example Sequence of Reconfigured Trajectory

The value of  $\Delta v_{DSM}$  and  $\Delta v_{LOI}$  obtained under 4-body approximation is larger than those listed on Table 1, the results of the optimization process under 3-body approximation. The reason is that, the values of five parameters are fixed to the values inherited from the results under 3-body approximation. Naturally, those values are not optimized for the trajectories under 4-body approximation and there remains the space for improvement. In other words, it is possible to reduce the total  $\Delta v$  required to complete the sequence by tuning the value of these parameters. It remains for further investigation.

## CONCLUSION

In this paper, the reconfiguration of translunar trajectory in case of main engine anomaly is investigated. The objectives of the trajectory design are to reduce the excessive velocity at the lunar encounter as well as to reduce the total required  $\Delta v$  to complete the sequence. 3 impulse Hohmann transfer based trajectory is adopted and possible trajectories are categorized under 2-body approximation. The solutions obtained are applied to more sophisticated models (3-body approximation and 4-body) and yields feasible trajectory.

## LIST OF SYMBOLS AND ABBREVIATIONS

DSM	Deep space maneuver
LOI	Lunar orbit injection
$\Delta v$	Velocity increment
$v_\infty$	Excessive velocity
$\Delta v_{sp}$	Specific $\Delta v$ (performance index for trajectory optimization)
$\Delta v_{adj}$	$\Delta v$ to adjust the Lunar swing-by condition
$\Delta v_{DSM}$	$\Delta v$ at DSM
$\Delta v_{LOI}$	$\Delta v$ at LOI
$v_{\infty LSB}$	$v_\infty$ at Lunar swing-by
$v_{\infty LRE}$	$v_\infty$ at Lunar re-encounter
$\alpha_{LSB}$	Azimuth of $v_\infty$ after Lunar swing-by
$\delta_{LSB}$	Elevation of $v_\infty$ after Lunar swing-by
$t_{DSM}$	Time of DSM
$t_{LRE}$	Time of Lunar re-encounter
$t_{LSB}$	Time of Lunar swing-by
$\theta$	Phase angle on the orbit of the Moon

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