PRACTICAL IMPLEMENTATION OF INVERSE OPTIMAL SATELLITE ATTITUDE CONTROL TECHNIQUES

Nadjim M. HORRI (1), Philip L. PALMER (1), Mark ROBERTS (2)

1 Surrey Space Centre, University of Surrey, Guildford, GU2 7XH, United Kingdom, n.horri@surrey.ac.uk

2 Department of Mathematics, University of Surrey, Guildford, GU2 7XH, United Kingdom

ABSTRACT

Despite the theoretical superiority of nonlinear optimal control, operational satellite attitude control is still predominantly based on simpler control techniques that are easier to design, validate, tune and implement, such as PD type control. This is due to the complexity of numerically implementing nonlinear optimal control techniques. As an alternative, we present here an inverse optimal Lyapunov approach, which circumvents the task of numerically solving partial differential equations onboard a satellite. We exploit the fact that the convergence rate of Lyapunov functions can be a natural player in nonlinear optimisation problems. The control design is based on phase space geometry. The optimisation objective is the minimisation of the norm of the control torque subject to a constraint on the convergence rate of the Lyapunov function. A gain scheduling scheme is adopted to avoid the practical risk of torque saturation, while maintaining the trade-off improvement. Experimental results on an air bearing table show the feasibility and practical benefit of the approach.

1. INTRODUCTION

Optimal control of nonlinear systems has a long research history but the attitude control of operational satellites is still predominantly based on less effective classical linear control laws, such as PD controllers, which are simpler to implement and present low implementation risk for a space mission. The implementation complexity of nonlinear optimal control by numerical techniques has been hindering their practical implementation for satellite attitude control.

Numerical implementation of nonlinear optimal attitude control can be based on two formulations: The Euler Lagrange approach from calculus of variations which is numerically complex and only guarantees local stability or the Hamilton Jacobi Bellman (HJB) formulation from dynamic programming, which is global but requires solving a numerically intractable system of partial differential equations. Both are not considered suitable for operational satellite attitude control.

Inverse optimal control theory circumvents the practically complex task of numerically solving a Hamilton-Jacobi-Bellman equation, while achieving global stabilisation. The starting point of the approach is to construct a stabilizing feedback controller based on a Control Lyapunov Function. The underlying theory shows that the Lyapunov function solves the Hamilton-Jacobi-Bellman equation. An optimisation problem is then formulated to apply an optimising transformation to the benchmark controller. The resulting controller is optimal with respect to a ‘meaningful’ cost-function.

Lyapunov based inverse optimal control techniques are being increasingly considered for a large range of control system applications but their application has surprisingly remained limited so far in the aerospace field, with few exceptions (refer to [2], [4] and [5]).
We show here the application of minimum norm optimisation theory to the attitude control of a satellite with three orthogonal reaction wheels. The Lyapunov optimal approach exploits the fact that the convergence rate of Lyapunov functions can be used as a natural player in optimization problems.

Based on a phase space analysis, the optimal control objective is the minimization of the norm of the control torque subject to a constraint on the convergence rate of a Lyapunov function with a benchmark controller. To demonstrate the possibility of a performance improvement, the optimisation problem is reformulated with a gain scheduling approach.

The proposed gain-scheduled formulation of minimum norm optimisation demonstrates significant enhancement of the torque-rapidity trade-off, compared to a standard PD law, used as the benchmark. The torque-rapidity trade-off is enhanced in the sense of higher control rapidity for a set level of overall torque expenditure. The underlying theory is generic and extends to any stabilising benchmark controller other than a PD law.

Experimental results on an air bearing table demonstrate the feasibility and the possibility of a torque-rapidity trade-off enhancement, over a benchmark controller, despite the fact that the gravity torque of the table is higher than the control torque. The performance enhancement would theoretically be even higher for satellite rest to rest manoeuvres, having more favourable double integrator natural dynamics.

2. DYNAMIC MODEL

The dynamic model of a satellite’s rotational motion, in the disturbance free case, is described by the following equation:

\[ \dot{L} + \omega \times L = 0 \] (1)

Where the total angular momentum in the body frame is given by:

\[ L = I \omega + h \] (2)

By substituting \( L \) from equation (2), into the equation (1), we obtain the well-known Euler’s rotational equation using three orthogonal reaction wheels:

\[ I_1 \dot{\omega}_1 = (I_2 - I_3)\omega_2 \omega_3 + N_1 - \omega_2 h_3 + \omega_3 h_2 \]
\[ I_2 \dot{\omega}_2 = (I_3 - I_1)\omega_3 \omega_1 + N_2 - \omega_3 h_1 + \omega_1 h_3 \]
\[ I_3 \dot{\omega}_3 = (I_1 - I_2)\omega_1 \omega_2 + N_3 - \omega_1 h_2 + \omega_2 h_1 \] (3)

With:

\( I = [I_1, I_2, I_3]^T \): Inertia tensor of the body of the satellite about its centre of mass.
\( \bar{\omega} = [q, \dot{q}]^T = [q_1, q_2, q_3, q_4]^T \): Attitude quaternion of the satellite.
\( \omega = [\omega_1, \omega_2, \omega_3]^T \): Vector of the angular velocity in body fixed reference frame.
\( h = [h_1, h_2, h_3]^T \): Angular momentum generated by the reaction wheels in the body frame.
\( L \): Total angular momentum in the body frame.

The control torques of the wheels is defined as: \( N_i = -\dot{h}_i, i=1,3. \)

Based on the quaternion parameterization of attitude kinematics, the kinematic model of a satellite is given by:
Despite the fact that mathematical stability proofs are often harder to establish based on this parameterization, the absence of singularities and the fact that quaternions are readily available in the onboard attitude control software of most satellites makes quaternions more attractive than the attitude parameterization used in [1].

For the sake of convenience, equations (3) and (4) are written with the affine control system representation:

\[
\dot{x} = f(x) + g(x)u
\]  

\[(5)\]

Where:

\[x = \begin{bmatrix} \vec{q}, \omega \end{bmatrix}^T\]

\[u = \begin{bmatrix} N_1, N_2, N_3 \end{bmatrix}^T\]

\[f(x) = \begin{bmatrix} \vec{q}, \dot{\omega} \end{bmatrix} \quad g = \begin{bmatrix} 0_{3x3} \\ 1_{3x3} \end{bmatrix}\]

3. MINIMUM NORM ATTITUDE CONTROL

The design of a stabilizing controller consists of constructing a control Lyapunov function \(V\), and the corresponding control input \(u\), satisfying:

\[
\frac{\partial V}{\partial x}(f(x) + g(x)u(x)) < 0
\]  

\[(6)\]

However, the control law \(u(x)\) is not unique in general. Inverse optimal construction is a way of determining a specific stabilizing control input, with optimality properties in the sense of minimising a ‘meaningful’ cost function.

A particularly interesting nonlinear control construction based on the existence of a control Lyapunov function \(V\) is the minimum-norm control law proposed by Freeman and Kokotovic in [2].

It is constructed from the solution to a static pointwise optimization problem, which can be formulated as a nonlinear programme:

Minimise \[\|u\|\]

subject to \[V(x) = \frac{\partial V^T}{\partial x} (f(x) + g(x)u) \leq \sigma(x) \leq 0\]

\[(7)\]

Where the function \(-\sigma(x)\) can be viewed as describing a nonlinear stability margin ([1], [2]).

This minimum-norm control problem is a pointwise optimization problem, which consists of minimizing the control effort subject to a constraint on the convergence rate of \(V\). We shall demonstrate in this paper that solving modified formulations of this problem is a natural way of achieving an inverse optimal trade-off between torque and rapidity.

This nonlinear program can be solved analytically. We can write it more compactly as a least norm problem:

Minimise \[\|u\|\]  

\[st. \quad \langle a, u \rangle \leq b \]  

\[(8)\]

With:

\[b = -L_1V(x) - \sigma(x)\]

\[a = L_1V(x)\]

\[(9)\]
$L_f V \equiv \frac{\partial V}{\partial x}^T f(x)$ and $L_g V \equiv \frac{\partial V}{\partial x}^T g(x)$ respectively represent the Lie derivatives of $V$ in the directions of the vector fields $f$ and $g$.

The solution of this least norm problem is given by:

$$u = \begin{cases} b a^T & \text{if } b < 0 \\ a a^T & \text{if } b \geq 0 \end{cases}$$

(10)

The solution is a projection onto the space spanned by the vector $a^T$ if the constraint is not solved by turning the controller off. Otherwise, when the constraint $b \geq 0$ is satisfied, then the minimum norm solution satisfying the constraint of equation (7) is simply $u = 0$.

The minimum-norm control law is therefore given by:

$$u_{\text{opt}} = \begin{cases} (L_f V(x) + \sigma(x))(L_g V(x))^T & \text{if } L_f V(x) > -\sigma \\ 0 & \text{if } L_f V(x) \leq -\sigma \end{cases}$$

(11)

Furthermore, Freeman and Kokotovic have proven in [2] using differential game theory that every min-norm controller is robust and inverse optimal with respect to a meaningful cost function of the form $J(x,u) = \int_0^\infty \{l(x) + r(x,u)\} dt$, where $l(x)$ is a continuous positive definite function of $x$ and $r(x,u) = \gamma(x)\|u\|$ is positive definite continuously increasing convex function of $\|u\|$.

The cost functional is unspecified when stating this inverse optimal problem (see ref [2]). However, the fact that an optimization trade-off is being achieved is evident from the nonlinear program stated in equation (7), which is solved analytically. Indeed, a norm of the torque is minimized subject to a rapidity constraint, which represents the other side of the trade-off.

We consider the problem of the attitude control of a satellite with a minimum norm approach. A natural choice of the negativity margin function $-\sigma$ is given by the time derivative of a Lyapunov function (representing a certain convergence rate), under the effect of a benchmark controller $k(x)$:

$$\sigma(x) = -\dot{V}_{k(x)} = -L_f V - L_g V k(x)$$

(12)

As a benchmark controller, we consider without loss of generality a standard PD law that drives $q$ and $\omega$ to zero:

$$k(x) = u_{\text{PD}} = -k_1 q - k_2 \omega$$

(13)

By substituting $\sigma$ from equation (12) into the minimum norm attitude control law of equation (10), we have:

$$u_{\text{opt}} = \begin{cases} (L_g V)^T u_{\text{PD}} & \text{if } -(L_g V)^T u_{\text{PD}} > 0 \\ 0 & \text{if } -(L_g V)^T u_{\text{PD}} \leq 0 \end{cases}$$

(14)

Where the term between brackets is a scalar quantity and the projection is in the direction of $L_g V T$.

We adopt the quaternion representation of attitude kinematics in order to avoid any singularities due to the attitude parameterization. For our simulation study, we consider the following candidate
Lyapunov function with bilinear coupling:

$$V = (k_p + k_d q) \left( 1 - q_3^2 \right) + q^T q + \frac{1}{2} \omega \omega^T + \gamma q^T q$$

(15)

This candidate is a special case of the one proposed in [3], with a bilinear coupling term. Note that the switching function $L_V T$ is a linear function of $\omega$ and $q$ with this first choice of $V$. The parameter $\gamma$ of the bilinear coupling term will allow for the tuning of the minimum norm controller.

With the Lyapunov function of equation (15), the control torque given in equation (14) is switched off between two straight lines in phase space domain: $L_V V = 0$ and $u_{PD} = 0$. The first switching curve is a straight line because $L_V \omega = \omega + \gamma q$.

4. PERFORMANCE TRADE-OFF IMPROVEMENT

In this section, we compare the torque-rapidity trade-off with minimum norm optimisation against the PD law, used as a benchmark. To make a fair comparison, the rapidity of the benchmark controller and the minimum norm controller are compared for the same overall torque expenditure and by specifying the same manoeuvre.

The PD benchmark is given by:

$$k(x) = k_p q + k_d \omega$$

(16)

The control law of equation (16) is first compared to the minimum norm controller:

$$u_{opt} = \begin{cases} 
- \frac{L_V u_2}{L_V V(\sigma(x))} & \text{if } L_V V(\sigma(x)) > 0 \\
0 & \text{if } L_V V(\sigma(x)) \leq 0 
\end{cases}$$

(17)

where $u_2 = k_p q + k_d \omega$, and the gains $k_{p2} > k_{p1}, k_{d2} > k_{d1}$. Note that the gains $k_{p2}, k_{d2}$ are allowed to be higher in this comparison because the control torque is switched off for a significant amount of time.

These gains are in fact tuned to obtain the same overall torque expenditure obtained with the controller of equation (16) for the same manoeuvre. Rapidity is enhanced as shown in figure (2). However, the control torque, although admissible overall, is initially amplified and the maximum torque value, which is the initial value of the torque for a rest to rest manoeuvre, is considerably amplified and will be shown to potentially cause torque saturation in the numerical simulations.

To demonstrate a performance improvement and simultaneously overcome the issue of potential torque saturation, we propose a modified gain-scheduled minimum-norm controller. We have previously proposed a gain scheduling scheme for the specific case of rest to rest manoeuvres. A more general gain scheduling scheme (of which the abovementioned one is a special case) is proposed here to deal with the case of more complex systems, such as the air bearing table, consisting of a 3D pendulum.

The optimisation problem is reformulated as follows:
\[
\min \| u \| \quad \text{st.} \quad \begin{cases} 
\dot{V} < \dot{V}(x,u_1) & \text{if } \|u_2\|_\infty < \varepsilon \\
\dot{V} < \dot{V}(x,u_2) & \text{elsewhere}
\end{cases}
\] (18)

Where the infinity norm is defined as: \( \|x\|_\infty = \max(|x_i|, i = 1,3) \), for any vector \( x \in \mathbb{R}^3 \).

The novel control law that solves the optimisation problem of equation (18) is simply given by minimum-norm projection theory as:

\[
u_{opt} = \begin{cases}
0 & \text{if } -(L_g V)u_2 > 0 \text{ and } \|u_2\|_\infty \geq \varepsilon \\
\frac{(L_g V)(L_g V)^T}{-(L_g V)(L_g V)^T} & \text{if } -(L_g V)u_2 \leq 0 \\
\frac{(L_g V)(L_g V)^T}{-(L_g V)(L_g V)^T} & \text{if } -(L_g V)u_2 > 0 \text{ and } \|u_2\|_\infty < \varepsilon
\end{cases}
\] (19)

The controller of equation (19) has a variable structure and switches between three modes:

- The controller initially uses the same gains \( k_{p1}, k_{d1} \) as the PD benchmark controller \( u_i = k_{p1}q + k_{d1}\omega \) to avoid needlessly amplifying the torque initially. The tuning of the gain can then be such that the instantaneous value of the torque never exceeds the initial torque value (representing the saturation value minus a margin).

- The controller is then turned off “coasting phase” between two straight lines in the phase space domain (defined by \( L_g V = \omega + q = 0 \) and \( u_2 = k_{p2}q + k_{d2}\omega = 0 \)).

- When the controller is turned back on, higher gains \( (k_{p2},k_{d2}) \) are adopted because there are no torque saturation issues at that stage and rapidity is significantly enhanced, without increasing the overall torque because the controller was turned off for a significant amount of time. We shall demonstrate that the torque-rapidity trade-off is significantly enhanced compared to the PD law adopted as a benchmark to be improved.

5. NUMERICAL SIMULATIONS

The system parameters used for the simulations are:

\[ I_1 = 10 \ kg \ m^2, I_2 = 14 \ kg \ m^2, I_3 = 12 \ kg \ m^2 \]

The initial conditions are:

\[ q_i(0)=0 \ (i=1, 2, 3) , \ q_1(0) = 0.3062, q_2(0) = 0.1768, q_3(0) = 0.1768 , q_4(0) = 0.9186 \]

This corresponds to initial attitude errors of 30 degrees on all three axes for a 1-2-3 Euler rotation sequence. The benchmark controller gains are: \( k_p= 0.02, k_d= 0.5 \).
Fig. 1. Phase portrait of the standard minimum norm controller by varying the weighting factor $\gamma$

Figure (1) shows the effect of the parameter $\gamma$ (without varying the gains of the PD law $k(x)$, on the phase portrait of the system. The parameter $\gamma$ acts as a weighting factor between torque expenditure and rapidity. By decreasing the values of $\gamma$, the torque consumption is reduced and the response is slowed down because the controller is switched off at lower velocities.

Figure (2) shows the phase space trajectories of the system with PD, standard min-norm and gain scheduled min norm control, when all three controllers are tuned to respectively deliver an overall torque expenditure of 0.24 (Nms) with the first two controllers and 0.21 (Nms) with gain-scheduled min-norm control. The following analysis will show that the phase portrait of gain scheduled min-norm control is the most advantageous. In this case, the system will use a low gain mode during the acceleration phase before coasting at certain wheel speed values and activating a high gain deceleration mode, when torque saturation is no longer an issue.

On figure (3), the gain scheduled minimum norm controller (equation (19) with $\gamma=0.02$, $\varepsilon=0.04$) is shown to overcome an initial torque saturation issue occurring with the standard minimum norm control law of equation (14). The control torque without gain scheduling would indeed reach values up to 0.06Nm when a realistic torque saturation value for the microsatellite consideration would be 0.01Nm. Standard minimum norm optimisation is therefore infeasible with higher gains $k_{p2}=0.2$, $k_{d2}=1.5$, which were tuned to deliver the same overall torque expenditure of 0.24 Nms as the benchmark controller.

Gain scheduling brings the overall torque to an even smaller value of 0.21 Nms but rapidity is still significantly enhanced compared to PD benchmark (see figure 4). The controller of equation (19) starts with the gains $k_{p1}=0.02$, $k_{d1}=0.5$, is turned off, then turned back on with the higher gains $k_{p2}=0.2$, $k_{d2}=1.5$, producing a similar (in fact slightly smaller) overall torque than the PD law used as a benchmark.

With gain scheduled minimum norm control, the torque rapidity trade-off is therefore enhanced compared to the PD benchmark and the torque profile is feasible by only using higher gains at the appropriate phase space regions. Gains can in fact be increased until the second peak of the torque is equal to the initial torque (a margin away from the saturation torque).
Fig. 2. Phase portrait of the gain scheduled minimum norm and PD controllers

Fig. 3. Control torque with and without gain scheduling
6. AIR BEARING TABLE EXPERIMENTS

Following the simulation results described in the last section, attitude control experiments were conducted on the air bearing table facility of the Surrey Space Centre. The aims of the experiment were to demonstrate the feasibility of the approach by obtaining a stable response and to show the benefit, in terms of enhanced performance, of applying minimum norm optimisation to a PD benchmark.

The air bearing table allows for 3-axis rotational manoeuvres (up to 30 degrees off-pointing from horizontal orientation) of a disk shaped aluminium platform. The rotational motion is near frictionless when the bearing that supports the table is suspended on top of a pedestal structure by a flow of compressed air. It is balanced with a system of counterweights and sliding masses that can be automatically adjusted to have a stable centre of mass before the experiments. Different types of attitude sensors have been assembled to the table, namely a 3-axis gyroscope, two inclinometers and an Inertial Measurement Unit (IMU) providing full attitude and angular velocity measurements. Since the main general concepts of minimum norm optimisation can be established in a 2-axis system (coupling is present by minimum norm projection in a 2D space), the practical implementation consisted of 2-axis attitude control experiments (controlling roll and pitch angles). The IMU was used as the only sensor for our 2-axis attitude control experiments. Two reaction wheels (out of four available wheels) have been adjusted to lie orthogonally on the same plane (parallel to the horizontal table) to provide 2-axis control capability. Rotation about the X-wheel axis is a roll and rotation about the Y-wheel axis is a pitch.

The main difference from satellite attitude manoeuvres is the presence of a significant gravity torque on the air bearing table. It was therefore anticipated that the nature of performance enhancement would differ from that of satellite rest to rest attitude manoeuvres, having different dynamics. Indeed, for rest to rest manoeuvres, the satellite behaves as a double integrator, while the air bearing table behaves as a 3D pendulum. However, the same principles apply and the experimental results, described in the following, showed that a level of performance enhancement (in terms of better attitude response for a level of torque expenditure) is obtained by gain scheduled minimum norm optimisation compared to a PD controller used as a benchmark.

The PD and the gain-scheduled pointwise min-norm controller (GS-PMN) have been compared for a similar experimental setup. Before activating both controllers, open loop excitation was used to produce roll and pitch oscillatory motion on the air bearing table, by applying a chirp signal to the
X-axis and Y-axis reactions wheels, lying parallel to the plane of the table and orthogonal to each other.

At t=25 seconds, the stabilising controller (PD in experiment 1 and GS-PMN in experiments 2) is activated to damp the roll and pitch oscillations and bring the table to point horizontally. The gains of both controllers were adjusted to conduct a fair comparison. The overall torque defined as

\[ \int_0^t \sqrt{u(\tau)u(\tau)^T} \, d\tau \]

has been calculated as 3.82 Nms with the GS-PMN controller, against 4.53 Nms with the PD law used for comparison.

The PD law was tuned to deliver sufficient damping of the oscillations, without causing wheel speed saturation. Increasing the derivative gain or both gains was found to quickly lead to torque saturation in this case. The GS-PMN controller was tuned to switch between two sets of gains (a low gain mode and a high gain mode) and a zero torque ‘coasting’ mode, as described in equation (19). The gains of the low gain mode are the same as those of the PD law used for comparison. The gains of the high gain mode are larger but limited to reduce the level of steady state oscillations, which are amplified by higher gains.

The wheel speeds recorded during both experiments (PD and gain scheduled PMN controller) are shown on figure (5). With standard PD control, the wheel momentum builds up to compensate for the gravity torque. It is observed that the GS-PMN controller stabilises the system with a smaller momentum remaining at the end of the experiment because switching the controller is turned off when the system proceeds to the equilibrium. This advantage is however not the focus of our analysis because modifications to the PD law such as filtering could reduce the momentum build-up. The fairness of the following rapidity comparison is justified by the fact that the GS-PMN controller was tuned to deliver even lower overall torque than the PD law.

Figures (6), (7) and (8) show that the GS-PMN controller achieves better damping of the oscillations than the PD law used as a benchmark. The main oscillation after t=25sec (time of activating the controller) is significantly smaller with the GS-PMN controller. Both controllers achieve steady state accuracy within ±1deg, with a slight increase of residual steady state oscillations on the air bearing table by GS-PMN control. Note that this phenomenon (which can be mitigated by reducing the higher set of gains) would not occur in the case satellite rest to rest manoeuvres, having double integrator (not pendulum) dynamics. The fact that oscillations are damped faster with GS-PMN control shows that, as predicted in theory, the system proceeds on a more direct path towards the origin (in the phase space domain) with GS-PMN control.

The rapidity enhancement should be even higher in the case of satellite rest to rest attitude manoeuvres because, as shown on figure (1), the system would proceed even more directly towards the origin with the double integrator dynamics of a zero momentum satellite.
Fig.5. Wheel speeds during air bearing table experiments 1 and 2

Fig. 6. Phase space diagram of the PD controller experiment 1
Fig. 7. Phase space diagram of the GS-PMN experiment 2

Fig. 8. Roll and pitch angles during 2-axis control experiments 1 and 2
7. Conclusion

Optimal trade-offs between attitude control rapidity and torque expenditure, have been achieved by minimising the norm of the control torque and imposing constraints on the convergence rate of a Lyapunov function. The overall attitude control performance of a classical stabilising benchmark controller (a PD law) has been significantly enhanced by a novel gain-scheduled minimum norm optimisation technique. A comparison of the optimal and benchmark controllers, for identical torque expenditure and the same manoeuvre, shows significant improvement in terms of rapidity for satellite rest to rest manoeuvres with the optimal minimum norm approach.

The performance enhancement has been experimentally demonstrated on an air bearing table, where the improvement was in terms of better damping of the oscillations with similar torque expenditure. A gain-scheduled version of the minimum norm controller has been proposed to avoid torque saturation in the phase space regions where it is likely to occur. Based on this technique, gains can be increased in certain regions of the phase space with the minimum norm approach. The approach is generic and can be adopted to achieve similar performance improvement of any stabilising controller other than the PD law under consideration.

The proposed optimization approach has the practical advantages of guaranteed global stability and low implementation complexity.

8. ACKNOWLEDGEMENT

This work was supported by the European Union funded Marie Curie Astrodynamics research network “Astronet”.

9. REFERENCES