OPERATIONAL ORBITING STRATEGIES ABOUT MINOR BODIES

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ABSTRACT

A number of missions aimed at the characterisation and the return of samples from small bodies is currently in flight and under development. All of these missions require close proximity operations at the small body, such as hovering, orbiting and landing, and the highly perturbed associated dynamics make the definition of spacecraft trajectories a very complex task. Useful information can be derived from analytical results under restrictive assumptions, however, when real mission scenarios are considered, the interaction between the different perturbations can induce significant deviations from the theoretical case, especially for very small bodies. In the framework of future ESA mission PROBA-IP, the evolution of uncontrolled and controlled orbits at different asteroids has been investigated, under realistic assumptions, by means of a numerical propagator. Particular attention is paid to the search of intrinsically stable solutions, such as photo-gravitational stable orbits, which permit safe operations for large time spans without the need of active control. The results of these analyses are presented in this paper, with some sensitivity assessment with respect to orbit and asteroid characteristics.

1. INTRODUCTION

Currently, a number of missions have flown in the vicinity of a minor body actually performing in-orbit operations. Very successful examples of this are NASA's NEAR mission to Eros and JAXA's Hayabusa mission to Itokawa. NASA's Dawn and ESA's Rosetta missions are currently flying to respectively Vesta and Ceres asteroids and comet 67P/Churyumov-Gerasimenko.

When facing the problem of a spacecraft orbiting in close proximity of an asteroid a complex and highly perturbed dynamic environment must be taken into account [1]. Several studies have been carried out in the past to assess the feasibility of orbiting and motion, especially looking for stable solutions that would permit safe operations without the need of controlling the spacecraft.

The above problem becomes particularly relevant in case of minor bodies smaller than typically 1 km diameter [2]. Hayabusa actually flew Itokawa which having a maximum dimension of 540 m lies within this class. Eros with a minimum dimension of 11 km and 67P/Churyumov-Gerasimenko with a minimum dimension of about 3 km could be considered out of that class. Vesta and Ceres are actually large asteroids and would directly fall out of the category. This leaves Hayabusa as the only spacecraft having flown very close to a small minor body and thus having experienced the difficulties of such dynamic conditions.

In the above context, DEIMOS Space has performed analyses of small asteroid in-orbit operations in the frame of ESA's PROBA-IP study for an interplanetary mission aimed at close-up reconnaissance of a Near Earth Object (NEO) [3].

The main objective of the PROBA-IP mission is the in-orbit validation of autonomous onboard guidance, navigation and control technologies for interplanetary cruise and for the targeting and insertion in minor bodies, primarily using onboard optical systems technologies. Current design foresees a three-year mission launched with VEGA in 2015. Escape from Earth is achieved by means of an upper stage. A 2.5 years time span would be devoted to the required low-thrust transfer to the asteroid, and six months to in-orbit operations. Current estimate of spacecraft wet mass is in the range of 350 kg.

In the GNC field the PROBA-IP mission is meant to implement and validate the following technologies and functionalities:

1) Onboard GNC technology elements for autonomous spacecraft navigation, guidance and control for interplanetary cruise primarily using onboard optical systems;

2) Autonomous targeting of, and rendezvous with, a NEO;

3) Autonomous achievement and maintenance of a safe closed orbit around the target object.

In what regards the analysis of the in-orbit operations, different types of orbiting strategies are possible, such as uncontrolled orbiting and controlled orbiting. Stability of the different options must be analysed paying special attention to the features of the asteroid, which largely affect the dynamical environment, such as irregularity of its gravity field and rotation state. Those have to be considered in conjunction with the solar radiation pressure forces acting on the spacecraft, which become very relevant when orbiting these small bodies.

In the present paper, the evolution of the orbital solutions and their stability will be analysed by means of accurate numerical propagation under the dynamical environment considered. The obtained solutions can be classified in terms of the final conditions achieved after a given propagation time of 90 days: i.e. remaining in orbit, collision with the asteroid, escape from the asteroid influence. Some simplified scheme of orbit control will be analysed to assess the feasibility and the cost of a possible station keeping.

Such assessment will allow characterising the possible orbital solutions about the asteroids considered for the mission (currently Apophis and 1989 UQ with an option for 2001 CC21). In particular, the possibility to fly photo-gravitational stable orbits behind the asteroids and the conditions for their stability will be also analysed.

2. DYNAMIC ENVIRONMENT

The dynamic environment in proximity of a small body orbiting the Sun is particularly complex, due to perturbations deriving from Solar Radiation Pressure (SRP), from irregularities in the gravitational field, from the rotation state and from solar third body effect.

The action of SRP is modelled by considering the spacecraft as a flat surface always oriented towards the Sun. This assumption permits to capture the essence of the motion of a spacecraft subject to SRP force during preliminary design phases, when an accurate model of the spacecraft is still unavailable.

The SRP acceleration can be written as:

$$\mathbf{a}_{srp} = a_{srp}\hat{\mathbf{d}} \tag{1}$$

where $\mathbf{d} = d\hat{\mathbf{d}}$ is Sun-asteroid vector, while SRP magnitude is:

$$a_{srp} = G_1(1+\rho)\frac{S_{SC}}{m_{SC}}\frac{1}{d^2} \tag{2}$$

where G_1~$1\cdot10^8$ kg km^3/s^2/m^2 is a constant (SRP at 1AU multiplied per squared AU distance in km), ρ is the reflectance of the spacecraft and S_{SC} and m_{SC} are spacecraft equivalent surface and mass respectively.

Small bodies are usually irregularly shaped, the gravitational field thus being highly inhomogeneous. However the major effects on the orbit of a spacecraft are given by the second order terms, in particular oblateness (J_2) an ellipticity (C_{22}). Gravity field will be then represented as a second order expansion of the gravity potential, expressed in terms of the moments of inertia of the body [4]:

$$V = \frac{\mu}{r} + \frac{G}{2r^3}\left[\left(3x^2-1\right)J_{xx} + \left(3y^2-1\right)J_{yy} + \left(3z^2-1\right)J_{zz} + 6\left(xyJ_{xy} + xzJ_{xz} + yzJ_{yz}\right)\right] \tag{3}$$

where μ is asteroid gravitational parameter, G is universal gravity constant, r is spacecraft distance from the centre of the body, and the gravitational coefficients are related to the terms of inertia tensor by:

$$\begin{aligned} &J_{xx} = \frac{1}{2}\left(I_{yy} + I_{zz} - I_{xx}\right),\ J_{yy} = \frac{1}{2}\left(I_{zz} + I_{xx} - I_{yy}\right),\ J_{zz} = \frac{1}{2}\left(I_{xx} + I_{yy} - I_{zz}\right) \\ &J_{xy} = I_{xy},\ J_{yz} = I_{yz},\ J_{zx} = I_{zx} \end{aligned} \tag{4}$$

The last perturbation taken into account is the one deriving from solar gravity, and it can be expressed as:

$$\mathbf{a}_{Sun} = -\frac{\mu_{Sun}}{d_{sc}^3}\hat{\mathbf{d}}_{sc} + \frac{\mu_{Sun}}{d_{ast}^3}\hat{\mathbf{d}}_{ast} \tag{5}$$

where μ_{Sun} is solar gravitational constant, $\mathbf{d}_{sc} = d_{sc}\hat{\mathbf{d}}_{sc}$ is orbiter distance from the Sun and $\mathbf{d}_{Ast} = d_{Ast}\hat{\mathbf{d}}_{Ast}$ is asteroid distance from the Sun. The equations of motion with the considered accelerations, written in an inertial reference system centred in the minor body, are:

$$\frac{d^2\mathbf{r}}{dt^2} = \frac{\partial V}{\partial \mathbf{r}} + \mathbf{a}_{srp} + \mathbf{a}_{Sun} \tag{6}$$

Finally, the motion of the asteroid around the Sun is considered purely keplerian, with no perturbations from the planets.

Several studies treated to analyse separately the contributions of the different perturbations. In [5] and [6], an elegant analytical formulation is presented for the evolution of averaged orbital parameters in presence of SRP. Assumptions to derive this closed solution permit to find two classes of frozen orbits, the first one in asteroid orbital plane and the other one in asteroid terminator plane. On the other hand, in [7] the problem of gravitational perturbations is isolated, in the simplified assumption of ellipsoid-shaped body. The existence of equilibrium points in the body frame is shown and a classification of minor bodies is made on the basis of the stability of these points. Solar third body perturbation does not seem to have a relevant weight unless large orbits at big sized asteroids are considered [8].

In real mission scenarios, however, the effect of combined perturbations must be considered. In particular, when orbiting very small size minor bodies, SRP and gravitational perturbations are in the same order of magnitude. Some generic results, under restrictive assumptions, can be obtained with analytic formulations as in [9], but in general the existence of stable solutions must be verified through accurate numerical simulations, as it will be shown in section 4.

3. ASTEROIDS CHARACTERISATION

Before any possible investigation of close proximity operations at asteroids, a characterization of their dynamical environment is needed. A brief analysis has been conducted to estimate physical properties of PROBA-IP possible target asteroids: Apophis, 1989 UQ and 2001 CC21. Due to the limited availability of fundamental physical parameters such as albedo and density, in some cases coarse assumptions have been necessary to obtain quantitative results.

The size of the bodies has been estimated by assuming a prolate spheroid shape and by calculating the projected maximum area with the usual equation that relates diameter, absolute magnitude and albedo:

$$A = \frac{\pi}{4}\left(\frac{10^{-H/5}\ 1329000}{\sqrt{p_v}}\right)^2 \tag{7}$$

where A is area in m^2, H is absolute magnitude and p_v is geometric albedo.

The asteroids have equal minor axes and rotate around one of their maximum inertia axes. From the amplitude of the lightcurve it is possible to derive the minimum value of elongation, in the hypothesis of equatorial line of sight:

$$r = 2.512^{\Delta M} \tag{8}$$

where ΔM is lightcurve amplitude. The maximum value of axis ratio has been assumed to be 2.8 for all asteroids, corresponding to the maximum elongation known for solar system bodies. In the cases of 1989 UQ and 2001 CC21, where no information is available about density, a rough value of 2 g/cm^3 has been assumed. In Table 1 the physical properties for the three asteroids are reported, based on the data collected from references [11] to [16]. Obtained mass and size are reported in Table 2.

Table 1: General physical properties of PROBA-IP asteroids

PARAMETER	Apophis	1989UQ	2001 CC21
General Properties			
Magnitude	19.7	19.3	18.5
Uncertainty in Magnitude	0.2	0.25	0.2
Geometric Albedo	0.33	0.06	0.04 / 0.20
Uncertainty in geometric Albedo	0.02	0.02	?
Rotational Period [h]	30.5376	7.733	5.017
Density [g/cm^3]	3.2	2	2
Lightcurve amplitude	0.951	0.27	0.81
Spectral type	SQ	B	L

4. NUMERICAL SIMULATIONS FOR PROBA-IP SCENARIO

An extensive campaign of numerical simulations has been run to acquire a better understanding of the dynamics of a spacecraft orbiting a very small body. It has already been mentioned that SRP and gravity perturbations are in the same order of magnitude and can play a relevant role in combination. To study this difficult problem a dedicated propagator for orbits around small bodies has been developed, where equations of motion are integrated in inertial Mean Earth Equator 2000 reference frame, taking into account all the perturbations cited in section 2. The propagator has also been endowed with orbit maintenance capability, to make possible a preliminary estimation of the ΔV cost of a station keeping strategy.

Table 2: Calculated mass and size for different PROBA-IP asteroids

PARAMETER	Apophis	1989UQ	2001 CC21
NOMINAL VALUES FOR SPHERE			
Minimum diameter [m]	235.23	578.07	592.94
Maximum diameter [m]	300.50	1029.19	1325.85
Minimum mass [kg]	2.18E+10	2.02E+11	2.18E+11
Maximum mass [kg]	4.55E+10	1.14E+12	2.44E+12
VALUES FOR PROLATE SPHEROID			
Minimum light curve axis ratio a/b	2.401	1.282	2.109
Maximum axis ratio a/b	2.8	2.8	2.8
Small asteroid			
Smaller semi-axis for low ratio a/b [m]	75.90	255.24	186.20
Larger semi-axis for low ratio a/b [m]	182.25	327.31	392.63
Smaller semi-axis for high ratio a/b [m]	70.29	172.73	161.58
Larger semi-axis for high ratio a/b [m]	196.81	483.65	452.44
Maximum mass [kg]	1.41E+10	1.79E+11	1.14E+11
Minimum mass [kg]	1.30E+10	1.21E+11	9.90E+10
Large asteroid			
Smaller semi-axis for low ratio a/b [m]	96.96	454.43	500.56
Larger semi-axis for low ratio a/b [m]	232.82	582.73	1055.54
Smaller semi-axis for high ratio a/b [m]	89.79	307.53	434.40
Larger semi-axis for high ratio a/b [m]	251.42	861.08	1216.31
Maximum mass [kg]	2.93E+10	1.01E+12	2.22E+12
Minimum mass [kg]	2.72E+10	6.82E+11	1.92E+12

All the simulations referred to the PROBA-IP mission have been repeated for the three asteroids under consideration. To obtain some parametric assessment over different asteroid sizes with a reduced number of cases, the following reference scenarios have been selected from Table 2, with a variation of two orders of magnitude in asteroid mass:

Table 3: Asteroid properties for numerical simulations

Property	Apophis	1989 UQ	2001 CC21
Smaller semi-axis [m]	70.29	172.73	434.4
Larger semi-axis [m]	196.81	483.65	1216.31
Mean radius [m]	99.07	243.46	612.27
Mass [kg]	1.30E+10	1.21E+11	1.92E+12
Gravitational constant [$km^3/(kg*s^2)$]	8.6964E-10	8.0665E-09	1.2830E-07

In all the cases the highest possible value for the axis ratio has been chosen, being the worst-case in terms of gravitational perturbations. As the perturbing accelerations also depend on spacecraft parameters like the mass or the exposed area, Table 4 summarises spacecraft properties used for the simulations. Properties of the spacecraft electric propulsion (EP) subsystem are also included.

Table 4: Spacecraft properties for numerical simulations

Mass [kg]	Area [m^2]	Reflectivity	EP thrust [mN]	EP specific impulse [s]
300	8	0.1	15	2500

4.1 Analysis of Uncontrolled Polar Orbits

When considering missions to small bodies, such as asteroids or comets, a relevant role from the scientific point of view is played by polar orbits, where the term “polar” in this case refers to the pole of the asteroid orbit. In fact they permit a complete observation and mapping of the whole small body, at least in a quarter of revolution around the Sun, for any direction of asteroid spin axis, with proper illumination conditions and the possibility to avoid eclipses. However these orbits result highly unstable due to perturbations from SRP and inhomogeneous gravity field. Some numerical analyses have been made to investigate how long can a spacecraft, initially placed into a polar orbit, continue orbiting before a fatal event occurs. Maximum propagation time is set to 90 days, but simulations are stopped when the spacecraft crashes on the asteroid, escapes from it, reaches a dangerous eccentricity or enters into eclipse.

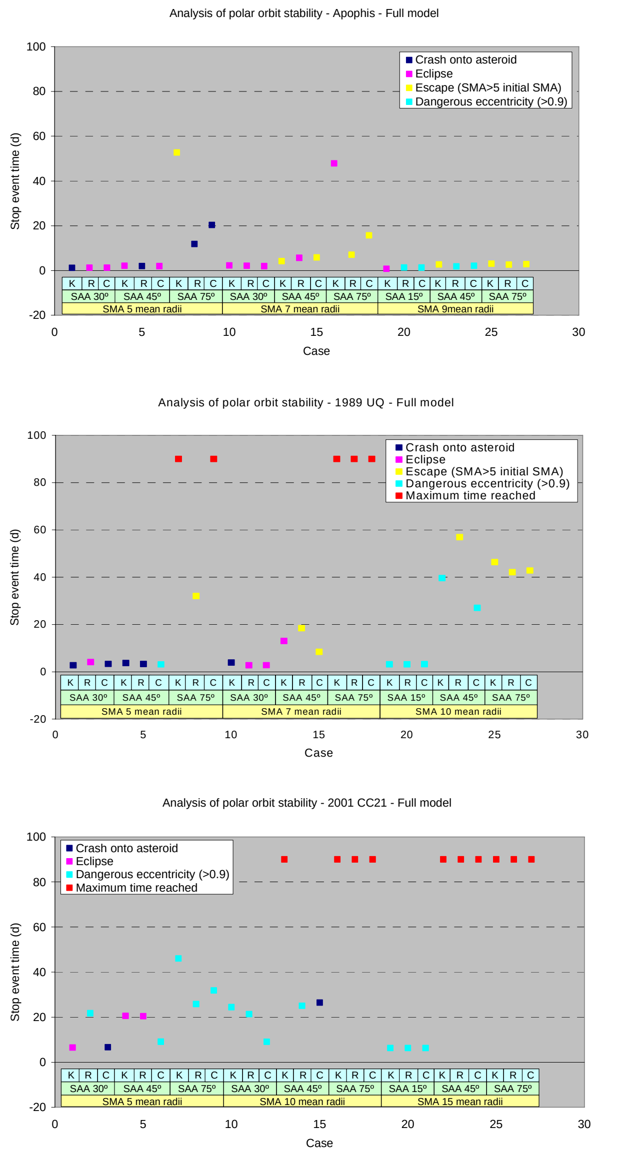

Fig. 1: Propagation time and final event for different initial orbits at Apophis (top), 1989 UQ (mid) and 2001CC21 (bottom)

The following assumptions are made:

- Initial distances of 5, 7 and 9 mean radii for Apophis
- Initial distances of 5, 7 and 10 mean radii for 1989UQ
- Initial distances of 5, 10 and 15 mean radii for 2001CC21
- Initial orbit is circular with a 90º inclination on asteroid orbital plane
- The initial nodes (RAAN) of the orbits are 105º, 135º and 150º with respect to the asteroid orbital reference frame. These angles correspond to observation angles of 75º, 45º and 30º with respect to Sun direction. In the case of larger orbits a minimum angle of 15º has been allowed, corresponding to a node of 165º. These orbits, in absence of perturbations, would avoid eclipse condition for the whole duration of the simulation.
- Three possible directions for spin axis are considered, oriented as the orbital reference axes: radial (R), circumferential (C) and polar (K).
- All perturbations are taken into account

In all cases initial simulation time is close to perihelion passage (worst case for SRP).

Fig. 1 illustrates the simulation duration in abscissa and the terminal event for all the considered cases as a colour code.

It is evident from the plots that the polar orbits are in general highly unstable, especially in the case of small and slow rotating asteroids as Apophis, where none of the proposed orbits can reach the nominal simulation time if uncontrolled. The main cause of this behaviour can be identified in the effect of SRP perturbation (as verified by considering every single perturbation at once), that produces an increase of orbit eccentricity until a crash or escape condition is reached. Moreover, the combined effect of gravity and solar perturbations can induce a rapid change in orbital plane node and inclination until an eclipse condition is reached.

The cases of 1989 UQ and 2001 CC21 result to be more benign with some orbits reaching the final propagation time. Especially in the case of 15º view angle, the orbit is relatively close to a terminator plane orbit and shows oscillations with respect to a mean orbit in the terminator plane. These orbits then, although more stable, are less useful from a scientific point of view due to the poor illumination conditions. The 45º orbits present in 2001 CC21, although reaching the maximum simulation time, cannot be considered safe in absolute terms as the motion still results particularly chaotic.

From these considerations it is possible to conclude that an active orbit control system is needed to maintain a spacecraft into a polar orbit, except the case of terminator plane orbits, which will be treated with more detail in section 4.3.

4.2 Analysis of Controlled Polar Orbits

The results obtained for the stability analysis of uncontrolled polar orbits show the necessity of some kind of orbital control to ensure the safety of in-orbit operations. In a first instance, the performance of a simple altitude control, based on a dead-band scheme with spherical boxes, is evaluated, to prevent at least the two most dangerous events of escape and crash onto asteroid surface and maintain a quasi-circular orbit shape.

Lower and upper bounds for spacecraft altitude are defined, and an impulsive manoeuvre is performed when the spacecraft exits from the confined region. A purely radial manoeuvre has shown to be too inefficient in terms of ΔV and frequency of impulses, hence the following scheme has been adopted:

- The new velocity module is computed as the one that would nominally put the spacecraft in the middle of the band after half revolution
- The new velocity lies in the same orbital plane as the incoming velocity
- The direction of the new velocity is rotated a user-specified angle α with respect to the plane tangent to the bounding sphere, as shown in Fig. 2:

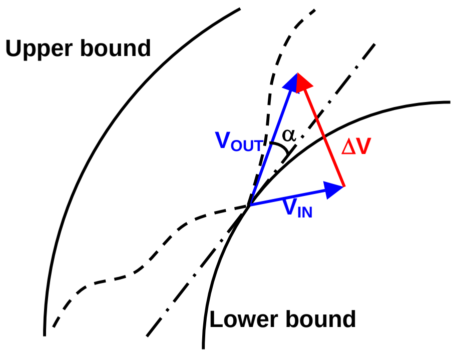

Fig. 2: Orbit altitude control scheme

- The manoeuvre ΔV is then calculated by difference.

The angle α and the deadband width are the parameters that mostly influence the performance of the control. It can be shown that a lower value of α corresponds to lower ΔV but higher number of manoeuvres. As the optimal setting of α is case-dependent, a compromise value of 5º has been chosen for all the simulations. Regarding the dead-band width, a narrow dead-band permits ΔV savings while a larger one reduces the frequency of the impulses. Also in this case a compromise value of 10% of nominal altitude is chosen.

A set of simulations has been run for circular polar orbits with different initial altitudes and initial value of RAAN set to 135º. Three possible directions for spin axis are considered, oriented as the orbital reference axes: radial (R), circumferential (C) and polar (K).

As the perturbing accelerations are approximately two orders of magnitude lower than the propulsive capability of PROBA-IP spacecraft [3], electric propulsion seems to be a viable solution for the orbit maintenance during proximity operations. To verify this hypothesis, the time needed to perform the largest manoeuvre with electric propulsion (EP), without considering gravity losses, is divided by the mean time between manoeuvres, and a performance index called *EP-time ratio* is obtained, approximating the ratio between the time needed and the time available to perform the manoeuvre. Assumptions for spacecraft EP system can be found in Table 4.

The results for a simulation time of 90 days, reported in Table 5, show that the considered control scheme, although suboptimal, is viable in terms of ΔV demand, which does not exceed 6 m/s in the worst case, and time spacing between manoeuvres.

It can be noted that larger asteroids require larger ΔV for the closer orbits, as the gravitational perturbations, and also motion typical velocities, grow with asteroid size. In particular, ΔV results in the order of 1.3-2.2 m/s for Apophis, 2.5-3.4 m/s for 1989 UQ and 5.6-6.0 m/s for 2001 CC21. At higher altitudes differences between asteroids seems to reduce, due to the fact that major perturbation is now SRP, which is in the same order of magnitude for all the three asteroids. ΔV results in all cases around 0.8 m/s.

Average time spacing between manoeuvres is in the order of the tens of hours for all cases, in particular 9-27 h for Apophis, 11-31 h for 1989 UQ and 13-68 h for 2001 CC21. It must be remarked that these values are strongly dependent on the tuning of the altitude control algorithm, in particular an optimal value could be found for α parameter, depending on the asteroid and the altitude of the orbit. All solutions illustrated then could show better figures of merit, but here a representative compromise value has been adopted for all simulations to obtain a preliminary assessment of the station keeping cost.

Table 5: Analysis of polar orbits with dead-band altitude control

Asteroid	Altitude (km)	Axis	Man #	ΔV tot (m/s)	ΔV max (m/s)	ΔT mean (h)	EP Max Tprop (h)	EP time ratio
Apophis	0.5	K	154	1.261	0.0161	13.935	0.089	0.0064
	0.5	R	237	2.225	0.0177	9.076	0.098	0.0108
	0.5	C	219	2.091	0.0163	9.818	0.091	0.0092
	1	K	83	0.480	0.0140	25.714	0.078	0.0030
	1	R	83	0.478	0.0134	25.714	0.074	0.0029
	1	C	78	0.456	0.0132	27.342	0.073	0.0027
	1.5	K	184	0.778	0.0148	11.676	0.082	0.0070
	1.5	R	184	0.778	0.0150	11.676	0.083	0.0071
	1.5	C	185	0.782	0.0149	11.613	0.083	0.0071
1989 UQ	1	K	170	2.550	0.0242	12.632	0.134	0.0106
	1	R	170	3.091	0.0331	12.632	0.184	0.0146
	1	C	195	3.411	0.0312	11.020	0.173	0.0157
	2.5	K	69	1.014	0.0273	30.857	0.152	0.0049
	2.5	R	70	1.022	0.0271	30.423	0.151	0.0049
	2.5	C	69	0.991	0.0270	30.857	0.150	0.0049
	4	K	91	0.773	0.0229	23.478	0.127	0.0054
	4	R	93	0.798	0.0228	22.979	0.127	0.0055
	4	C	91	0.773	0.0228	23.478	0.127	0.0054
2001 CC21	3	K	169	5.607	0.0446	12.706	0.248	0.0195
	3	R	158	5.808	0.0508	13.585	0.282	0.0208
	3	C	162	5.969	0.0501	13.252	0.278	0.0210
	10	K	32	0.925	0.0428	65.455	0.238	0.0036
	10	R	33	0.930	0.0420	63.529	0.233	0.0037
	10	C	31	0.904	0.0408	67.500	0.227	0.0034
	17	K	36	0.781	0.0453	58.378	0.252	0.0043
	17	R	36	0.781	0.0450	58.378	0.250	0.0043
	17	C	36	0.782	0.0449	58.378	0.249	0.0043

Last column shows that an electric propulsion system is capable of providing the necessary thrust level to counteract SRP and gravitational perturbations in a time that is, in the worst case, the 2.1% of the average time span available. Beyond the complexity due to the multiple switching of the electric thrusters, electric propulsion results to be a feasible alternative to chemical propulsion also for in-orbit operations.

The control scheme proposed, although simple in implementation and efficient in maintaining the spacecraft at a safe distance from the asteroid, does not permit any direct control over inclination and node of the orbit, which evolve in an unpredictable way under the combined effect of SRP and gravitational perturbations, especially for smaller orbits. This situation is undesirable and may lead the spacecraft to fall into asteroid shadow cone, therefore a simplified model for a full orbital control has been implemented and evaluated. This results more demanding in terms of autonomous navigation requirements, as the complete knowledge of the spacecraft state is now necessary, but enables complete control over the orbital plane.

The simplified control algorithm used for the simulations can be summarised as follows:

- Altitude control scheme is the same as that described above, with a deadband amplitude of ±10% of the nominal altitude.
- Inclination is checked at every crossing of asteroid orbital plane (node), and if its value falls out the prescribed band a manoeuvre is performed to restore the nominal value. A narrow band of ±2º is considered, as large inclination oscillations may be present between two subsequent passages for the nodes.
- Right ascension of ascending node is checked at every apex of the orbits, when the z - component of velocity changes in sign, and a manoeuvre is performed if its value is out of the control band. Also in this case a narrow control band of ±2º has been chosen to allow the orbital plane to follow the revolution of the asteroid around the Sun, as a fixed observation angle with respect to sun-asteroid direction is desired.

The same parametric analysis as in the altitude control case has been run, with a RAAN value of 135º and a simulation time of 90 days. The results are reported in Table 6.

It is possible to observe that in general both the cost of maintenance and the number of manoeuvres increase when orbital plane control is performed. For smaller orbits, however, full control prevents the drift of the orbit plane to unfavourable orientations, caused by the interaction between gravity and SRP perturbations, showing then in some cases an advantage in terms of ΔV with respect to altitude control only. The total cost for a complete orbit maintenance does not show a relevant variation, at least in the order of magnitude, for the three asteroids and it is in the range 1.3-2.5 m/s for Apophis, 1.6-3.6 m/s for 1989 UQ and 1.3-4.9 m/s for 2001 CC21. Mean time between manoeuvres reduces to 6-11 h for Apophis, 7-18 h for 1989 UQ and 10-47 h for 2001 CC21, due to the additional plane control manoeuvres. Also in this case electric propulsion results feasible with a maximum EP time ratio of 2.7%.

It is interesting to notice that when asteroid rotation axis is oriented as the pole of its orbit, gravitational perturbations have a smaller impact and station keeping cost is lower. When the size of the orbit grows, SRP perturbation becomes dominant and the cost of orbit maintenance becomes relatively insensitive to the orientation of asteroid spin axis.

Table 6: Analysis of polar orbits with dead-band complete orbital control

Asteroid	Altitude (km)	Axis	Man #	ΔV tot (m/s)	ΔV max (m/s)	ΔT mean (h)	EP Max Tprop (h)	EP time ratio
Apophis	0.5	K	284	2.256	0.0155	7.579	0.086	0.0114
	0.5	R	351	2.368	0.0155	6.136	0.086	0.0140
	0.5	C	368	2.476	0.0149	5.854	0.083	0.0141
	1	K	233	1.710	0.0178	9.231	0.099	0.0107
	1	R	245	1.736	0.0173	8.780	0.096	0.0109
	1	C	239	1.734	0.0161	9.000	0.089	0.0099
	1.5	K	196	1.353	0.0190	10.964	0.106	0.0096
	1.5	R	197	1.342	0.0194	10.909	0.108	0.0099
	1.5	C	196	1.342	0.0195	10.964	0.108	0.0099
1989 UQ	1	K	207	2.676	0.0297	10.385	0.165	0.0159
	1	R	306	3.355	0.0295	7.036	0.164	0.0233
	1	C	309	3.556	0.0360	6.968	0.200	0.0287
	2.5	K	157	2.102	0.0327	13.671	0.182	0.0133
	2.5	R	157	2.081	0.0333	13.671	0.185	0.0135
	2.5	C	163	2.117	0.0331	13.171	0.184	0.0140
	4	K	121	1.557	0.0357	17.705	0.198	0.0112
	4	R	123	1.594	0.0327	17.419	0.182	0.0104
	4	C	123	1.600	0.0307	17.419	0.171	0.0098
2001 CC21	3	K	132	3.304	0.0506	16.241	0.281	0.0173
	3	R	203	4.198	0.0482	10.588	0.268	0.0253
	3	C	218	4.751	0.0495	9.863	0.275	0.0279
	10	K	73	1.759	0.0516	29.189	0.287	0.0098
	10	R	73	1.761	0.0515	29.189	0.286	0.0098
	10	C	67	1.677	0.0518	31.765	0.288	0.0091
	17	K	45	1.278	0.0515	46.957	0.286	0.0061
	17	R	46	1.301	0.0512	45.957	0.284	0.0062
	17	C	46	1.302	0.0515	45.957	0.286	0.0062

4.3 Photo-Gravitational Stable Orbits

In [6] the existence of self-stabilising terminator plane orbits is shown, in the restrictive hypothesis of spherical central body with Solar Radiation Pressure (SRP) perturbation, with an analytical formulation describing the evolution of the orbital elements, averaged on a single revolution around the asteroid. These orbits, which can be called Photo-Gravitational Stable Orbits (PGSO), require a non-null eccentricity, with the line of apsides parallel to the asteroid orbit pole, in order to synchronise the rotation of the line of nodes induced by SRP with the revolution of the asteroid around the sun. The optimal eccentricity value is given by:

$$e = \cos\Lambda \tag{9}$$

$$\tan\Lambda = \frac{3\beta}{2}\sqrt{\frac{a}{\mu_{Ast}\mu_{Sun}A\left(1-E^2\right)}} \tag{10}$$

$$\beta = a_{srp}d^2 \tag{11}$$

where μ_{Ast} and μ_{Sun} are asteroid and Sun gravitational constants, a is spacecraft orbit semi-major axis, A and E are asteroid semi-major axis and eccentricity, a_{srp} is SRP acceleration magnitude and d is the distance of the asteroid from the Sun. The value of eccentricity decreases when the importance of SRP acceleration grows, which is the typical situation for very small asteroids, while, on the contrary, small size orbits around larger asteroids require a higher value of eccentricity.

Rigorously speaking, this class of orbits is frozen only on average, meaning that over a single revolution important oscillations of orbital parameters, particularly inclination and RAAN, are present. In fact, mean orbital plane is shifted by SRP effect in the direction opposite to the Sun, and due to the intrinsic eccentricity, the distance of the orbiter from the asteroid varies during one revolution, producing also an oscillation along the x axis. The orbit assumes a fully 3D shape, as illustrated in Fig. 3 for a 5 km amplitude orbit at asteroid 2001 CC21, resembling the appearance of halo orbits at libration points. It must be noted however that the scale along x-axis has been augmented for illustrative purposes, and the oscillations along the x-direction are very small compared with the orbit size in y-z plane.

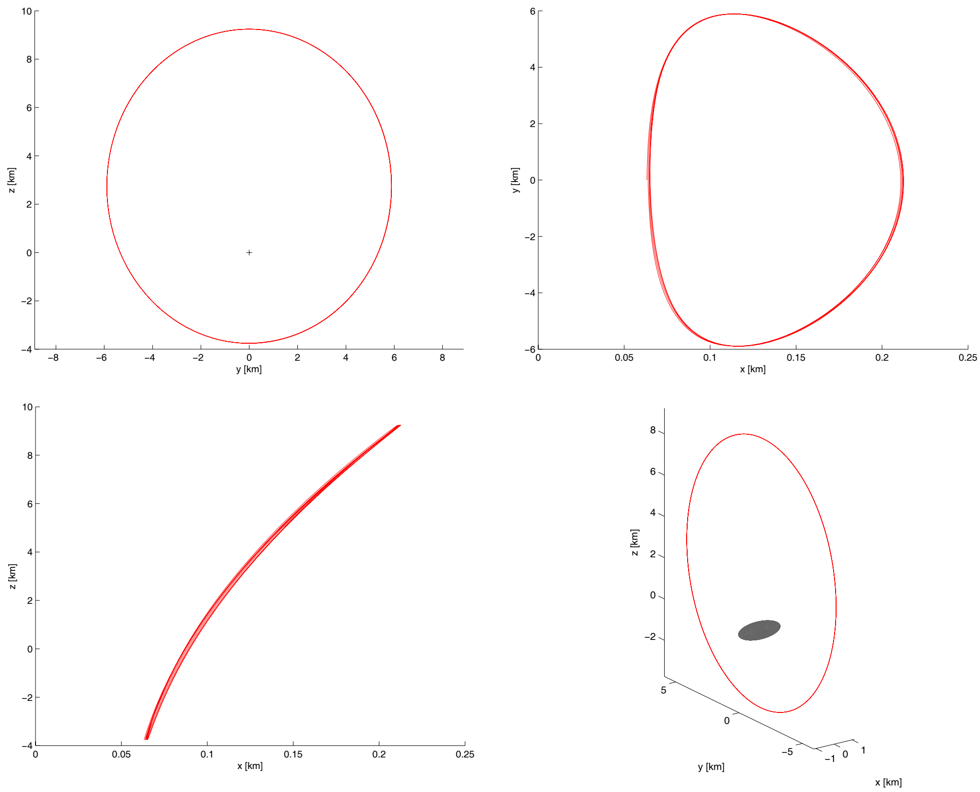

Fig. 3: 3D representation of a 5 km photo-gravitational stable orbit at 2001 CC21

Terminator plane orbits are particularly appealing for missions around small sized asteroids as they represent a possibility to maintain the spacecraft in a safe orbit, in a highly perturbed dynamic environment, without the need of station keeping manoeuvres. This can be particularly useful for a Radio-Science Experiment (RSE), where the noise introduced by orbit control can have a negative impact on the accuracy of the estimations.

In a real mission scenario however the effect of gravitational perturbations can deviate considerably the behaviour of PGSO from the ideal case, and in some situations even completely destabilise the orbit. A parametric analysis for different orbit altitudes and different directions of asteroid spin axis has been made to study the effective stability of PGSO.

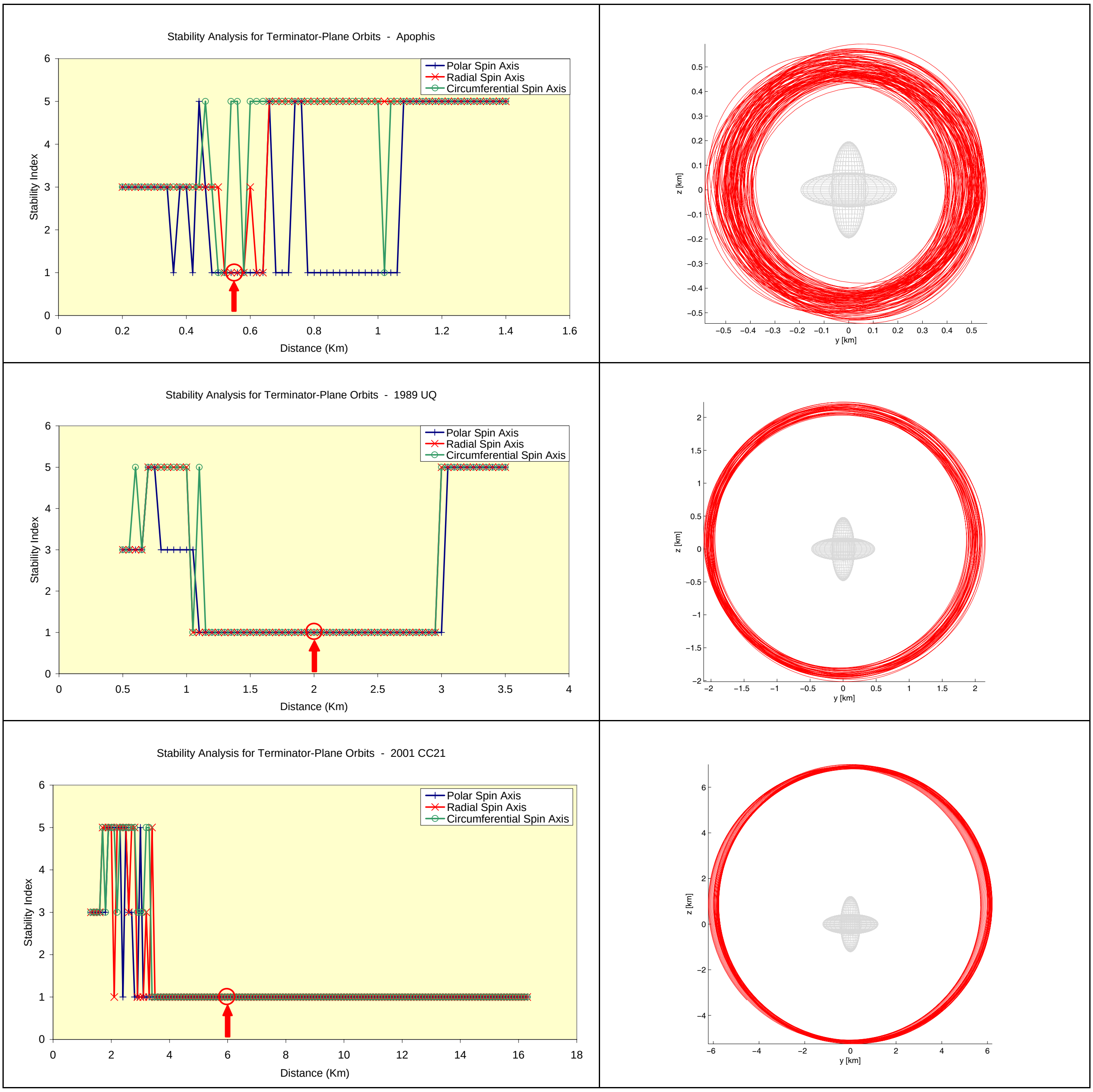

Fig. 4: Parametric study of PGSO for Apophis (top), 1989UQ (mid) and 2001CC21 (bottom), for different altitudes and asteroid spin axes. Right plots represent selected orbits in radial spin-axis case.

In Fig. 4 the results of the analysis are reported in the form of synthetic plots, where a stability index is associated to the orbit depending on the event which terminated the simulation (90 days time span): (1) if the maximum propagation time is reached (stable), (2) if maximum allowed distance is exceeded, (3) if spacecraft crashes onto the asteroid, (4) if it enters eclipse and (5) if maximum semi-major axis is violated. In right plots selected orbits are illustrated for the radial spin axes case.

For all of the three asteroids, lower altitude orbits show a difficultly predictable evolution due to the combined effect of SRP and gravity perturbations. In these areas dynamics show a highly non-linear behaviour and the minimum change in initial conditions can induce a completely different evolution of the orbit. These regions should then be avoided for in-orbit operations.

The case of Apophis in this sense seems to be particularly problematic, as all the possible stability range is close to the resonance radius (643 m). In the case of polar rotation axis this seems to have a limited impact on orbit stability, with wide stable ranges between 0.48 km and 0.64 km, and between 0.78 km and 1.06 km. However, if the spin axis is not polar only few altitudes permit orbits that satisfy this coarse stability criterion, approximately in the range between 0.45 km and 0.6 km. In any case, these orbits present a really chaotic evolution of orbital parameters, so that they cannot be considered safe in absolute terms. For altitudes greater than 0.6 km the typical terminal condition is escape, caused by resonance phenomena induced by gravity perturbations.

For asteroids 1989 UQ and 2001 CC21, on the contrary, a wide range of options for safe orbit altitude is available, with all the three spin axis directions considered. In the case of 1989 UQ stable orbits can be found in the range between 1.15 km and 2.95 km, while in the case of 2001 CC21 all the range between 3.5 km and 16 km presents feasible orbits. Resonance radii for these asteroids are 541 m and 1020 m respectively, so it results that the upper limit of instability is, in both cases, larger than 1.5 times the resonance radius, as recommended in [9] and [10].

As in Apophis case it was impossible to find a stability range valid for all the possible orientation of the spin axis, some form of orbital control is required. A rough estimate of the cost to maintain a PGSO has been made for two different altitudes with the full control algorithm previously described, with larger dead-bands to allow a more natural evolution of the dynamics, in particular orbital eccentricity. The chosen band amplitudes are ±50% for altitude and ±10º for inclination and node. The results, reported in Table 7, show that maintenance cost is very low with a reduced number of manoeuvres and the maximum time between manoeuvres still permits the execution of radio-science experiments.

Table 7: Orbit maintenance cost for terminator plane orbits at Apophis

Altitude (km)	Axis	Man #	ΔV tot (m/s)	ΔT max (d)
0.60	K	0	0.0000	90.00
	R	15	0.1050	19.53
	C	13	0.0857	19.43
0.85	K	0	0.0000	90.00
	R	17	0.0853	49.90
	C	8	0.0478	48.43

4.4 Analysis of Retrograde Equatorial Orbits

Several past studies demonstrated that retrograde orbits in asteroid equatorial plane are intrinsically stable, even when the shape of the body is highly irregular. However, when the asteroids are very small, SRP effect can destabilise the orbit up to the point that the increase in eccentricity can drive the spacecraft to crash onto the asteroid [2]. In order to complete the study of possible stable orbits for PROBA-IP mission, an analysis on the stability of retrograde orbits has been performed, by considering circular orbits at different altitudes, with different orientations of the spin axis in the asteroid orbital frame: polar (K), circumferential (C) and generically inclined (45º right ascension, 45º declination). The maximum propagation time has been set to 90 days, and the initial epoch corresponds to pericentre passage, worst-case for SRP intensity. The results are reported in Table 8.

Table 8: Stability limit for retrograde equatorial orbits

Asteroid	Axis	Max Altitude [km]
2001 CC21	K	2.19
	45º, 45º	2.39
	C	2.25
Apophis	No stable solution found	
1989 UQ	No stable solution found	

It is interesting to observe that for smaller asteroids no stable solution has been found for retrograde orbits. SRP effect in all cases destabilise the orbit in a time that never exceeds 2-3 days. For 2001 CC21 stable solutions have been found, and a stability limit has been calculated.

5. CONCLUSIONS

The problem of defining a safe orbiting environment for close proximity operations at small asteroids in the 1 km range and below, has been investigated in this paper, in the framework of ESA PROBA-IP mission study. The complex interaction between SRP and gravity perturbations do not allow relying on analytical methods, which require too restrictive assumptions, and the problem has been tackled with accurate numerical propagations. Different asteroids have been modelled as prolate spheroids rotating around their principal inertia axes. Uncontrolled and controlled orbits have been studied, showing that uncontrolled orbits are generally unstable, with the exception of terminator-plane photo-gravitational stable orbits. The dynamic environment resulted particularly unstable in the case of Apophis, a very small asteroid in slow rotation state, where the existence of PGSO is not ensured for all possible spin axis directions. Also equatorial retrograde orbits are rapidly destabilised, in the case of smaller asteroids, by SRP. Controlled orbits simulations presented affordable ΔV costs and showed that electric propulsion could be a viable option for station keeping manoeuvres.

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