ANALYSIS OF GEOSTATIONARY TRANSFER ORBIT LONG TERM EVOLUTION AND LIFETIME

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Abstract: *The study presented in this paper deals with Geostationary Transfer Orbits (GTO) for which new French regulations (defined in the context of the French Space Act) will fully apply at the end of 2011. Geostationary Transfer Orbits are characterized by a low perigee (altitude of a few hundreds of kilometres) and a high apogee (altitude typically identical to that of geostationary satellites) among other features. The objective of the study is to analyze the dynamics of objects in geostationary transfer orbits in order to better understand what the lifetime (time during which the object remains in orbit) most depends on. Because of the high eccentricity, the orbit is strongly affected by the gravitational effects of the Sun and Moon. But because the perigee is low, drag has a strong impact too. The coupling of the two perturbations combined with the effects of the Earth potential (secular drifts mainly) makes the orbit's evolution particularly sensitive to initial conditions and modelling errors. One key element is the initial position of the Sun (and to a lesser extent the Moon) which changes the mean altitude of the perigee, which translates into more or less drag, hence more or less decrease rate of the semi-major axis at the beginning of the lifetime. But when the semi-major axis reaches a value of around 15000km, the perigee altitude may increase or decrease strongly because the angle between the Sun and the line of apsides is then nearly constant. The paper attempts to explain all these aspects and discusses the possibility of limiting the lifetime of objects in Geostationary Transfer Orbits.*

Keywords: *Geostationary Transfer Orbit, Long term propagation, Lifetime, French Space Act.*

1 Introduction

The amount of debris in orbit is a growing threat to operational satellites, as seen by the increasing number of avoidance manoeuvres performed each year. There is therefore a need for measures that will prevent this situation from becoming even worse. France has decided to take on an active role by implementing regulations (French Space Act), in line with IADC recommendations, in order to protect the most populated orbital regions (LEO, GEO...).

The study presented in this paper deals with Geostationary Transfer Orbits (GTO) for which French regulations will fully apply at the end of 2011. Geostationary Transfer Orbits are mainly characterized by a low perigee (altitude of a few hundreds of kilometres), a high apogee (altitude typically identical to that of geostationary satellites), and a low inclination.

At the end of their mission, objects in Geostationary Transfer Orbits will neither be allowed to cross the GEO region within one year nor to stay in orbit longer than 25 years if they cross the LEO region. No one will wait for that long to confirm that the actual lifetime has been less than 25 years. An reliable enough orbit prediction is of course necessary.

It had been noticed in previous studies conducted at CNES that the selection of potentially hazardous objects in elliptical orbits (for instance objects that may pose a risk to populations) was not obvious and that no simple rule seemed to exist to easily select the objects that would re-enter in the coming months or years.

Other analyses also conducted at CNES on GTO lifetime [1] have illustrated the complexity of the problem. Figure 1 shows the lifetime computed using DAS (*Debris Assessment Software*) as a function of day of year (x-axis) and time of day (y-axis). The red areas correspond to durations longer than 25 years, and the blue areas to durations shorter than 25 years.

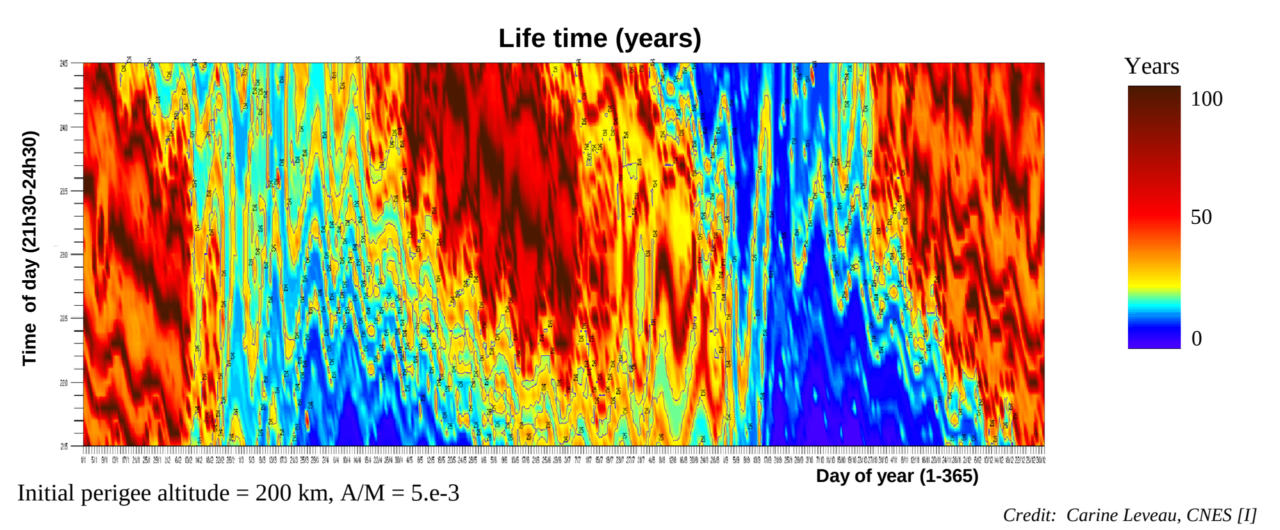


Figure 1: Lifetime results obtained with DAS

The objective of the study is therefore to analyze the dynamics of geostationary transfer orbits. The main goal is to better understand what GTO lifetime most depends on, and how it is sensitive to various factors.

In the first part of the paper, the main hypotheses of the study will be presented (model considered, initial orbital elements…).
The second part will be devoted to the effects of the main perturbations.
Finally we'll show a few lifetime results, analyse their sensitivity and their predictability, and discuss the possibility of reducing the lifetime of objects in Geostationary Transfer Orbits.

1 Hypotheses

In this part we'll give the main hypotheses used in the study, and detail a few elements regarding the perturbations and models used.
As the objective of the study is more focussed on sensitivity than prediction accuracy, the model has been chosen as simple as possible.

1.1 Orbit considered

The orbit is described by its Keplerian orbital elements. Unless otherwise mentioned, the initial parameters considered are the following:
Perigee altitude = 200 km (sometimes 250 km)
Apogee altitude = 36000 km
Inclination = 7 degrees
Argument of perigee = 180 degrees

1.2 Dynamics and perturbations

1.2.1 Third body perturbation and models used

The averaged effects (over one orbit) of the 3rd body perturbation can be approximated by the following equations (see [2]) :

$$\frac{da}{dt} = 0$$

$$\frac{de}{dt} = -\frac{15}{2}\frac{\mu}{n\,d^3}\,X\,Y\,e\sqrt{1-e^2}$$

$$\frac{di}{dt} = \frac{3}{2}\frac{\mu}{n\,d^3}\frac{Z}{\sqrt{1-e^2}}\left(\cos\omega\,(1+4e^2)\,X - \sin\omega\,(1-e^2)\,Y\right)$$

$$\frac{d\omega}{dt} = -\frac{3}{2}\frac{\mu}{n\,d^3}\left[\frac{Z}{\tan i\,\sqrt{1-e^2}}\left(\sin\omega\,(1+4e^2)\,X + \cos\omega\,(1-e^2)\,Y\right) + \sqrt{1-e^2}\,(Y^2 - 4\,X^2 + 1)\right]$$

$$\frac{d\Omega}{dt} = \frac{3}{2}\frac{\mu}{n\,d^3}\frac{Z}{\sin i\,\sqrt{1-e^2}}\left(\sin\omega\,(1+4e^2)\,X + \cos\omega\,(1-e^2)\,Y\right)$$

$$\frac{dM}{dt} = -\frac{1}{2}\frac{\mu}{n\,d^3}\left(6\,(3+2e^2)\,X^2 + 3\,(1-e^2)\,Y^2 - 3\,e^2 - 7\right)$$

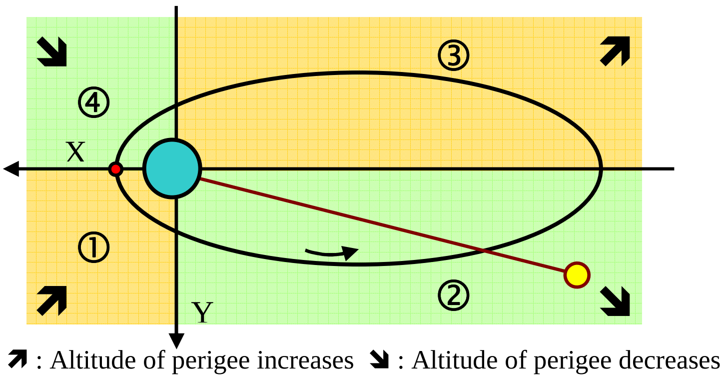
With:

- a: semi-major axis, e: eccentricity, i: inclination, ω: argument of perigee, Ω: RAAN, M: mean anomaly
- X,Y,Z: components of the unit vector directed from the centre of the Earth to the celestial body, in the (P,Q,W) frame, where: P=direction from the centre of the Earth to the perigee, Q=W^P, W=orbit angular momentum.
- d: distance between the centre of the Earth and the body.
- μ : gravitational constant of the body.
- n: orbit mean motion.

dx/dt actually means: $\frac{1}{2\pi}\int_0^{2\pi}\frac{dx}{dt}\,dM$, with x=a, e, i, etc…

The assumption here is that body and orbit are "fixed" during the time of one revolution of the satellite in its orbit, which is acceptable in our case.

From these equations, we can deduce the directions of the celestial body (with respect to the orbit plane) that make the perigee altitude increase or decrease. The result, shown in Figure 2, is not very intuitive.



The perigee altitude increases if the projection of the direction of celestial body onto the orbit plane is in quadrants 1 or 3. It decreases otherwise.
The altitude is stable if the projection of the body is aligned with the x-axis (line of apsides) or y-axis (perpendicular to the line of apsides).

Figure 2: Mean gravitational effect of a celestial body on the perigee altitude

The decrease or increase rate also depends on the elevation of the body with respect to the orbit plane (function of $\cos^2$(elevation)). The decrease or increase rate is higher when the body is in the plane.

1.2.2 Atmospheric drag

Drag is an essential factor as it makes the semi-major axis decrease until it becomes small enough for the satellite to burn in the atmosphere. But drag is also hardly predictable because of the effect of solar activity on atmospheric density (particularly over long periods of time).
In this paper we'll use a simple enough model in order to avoid part of that complexity. We'll care about the most important effects: change of density with altitude only.
The following hypotheses are assumed:

- Atmospheric model: *us76*
- Aerodynamic coefficient: 2.2
- Area to Mass ratio (A/M): 10^{-2} m^2/kg

We'll compute the mean effect of drag over one orbit by averaging the derivatives da/dt and de/dt as given by the Gauss equations:

$\bar{x} = \frac{1}{2\pi}\int_0^{2\pi} \dot{x}\, dM = \frac{1}{2\pi}\int_0^{2\pi} \dot{x}\, \frac{r^2}{a^2\sqrt{1-e^2}}\, dv$, where M is the mean anomaly, v is the true anomaly, and x is any orbital element.

In practice, the integral is evaluated using spline functions and a limited number of points in the orbit equally distributed in true anomaly.

Figure 3 shows the mean effect of drag over one orbit as a function of altitude of apogee for different choices of the altitude of the perigee.

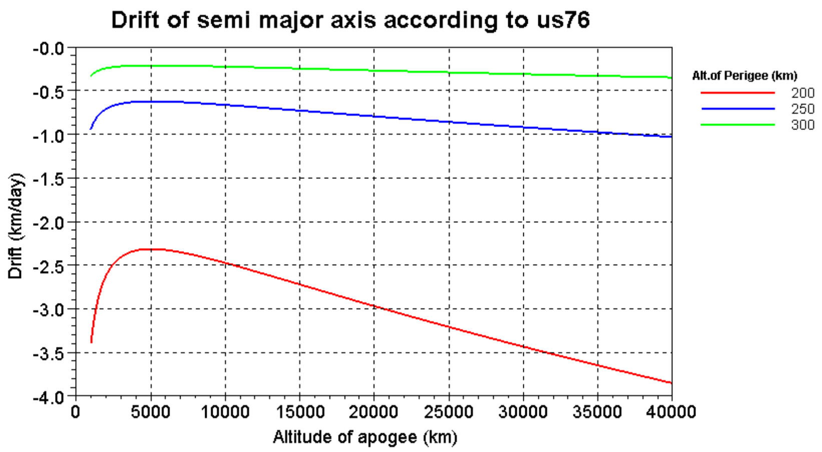


Figure 3: Drag effect on the semi-major axis

We note that there is nearly a factor 10 between the perigee altitudes 200 km and 300 km (drift of semi-major axis around 3 km/day and 0.3 km/day respectively).
The decrease rate of the altitude of the perigee is relatively small, about 10 times smaller than that of the altitude of the apogee. The main results are summarized in the table below:

hp/ha (km)	da/dt (km/day)	dhp/dt (km/day)	dha/dt (km/day)
200/36000	-3.7	-0.6	-7.4
300/36000	-0.33	-0.08	-0.67

(hp: altitude of perigee, ha: altitude of apogee)

1.2.3 Earth potential

We'll consider the secular effects due to the zonal terms of the potential only. As we are interested in explaining the phenomena rather than predicting the orbit accurately, we'll limit the number of zonal terms to 1 (J2).

For a GTO orbit, the terms J_2^2 and J_4 can generate a drift on the line of apsides of about 0.5 degree per year. It means that these terms should be taken into account in accurate comparisons with other results.
Resonances originating in tesseral harmonics are considered to have negligible impacts as the long-term oscillations induced on the orbital elements soon vanish as the semi-major decreases.
Another point that is neglected is the effect of the short term perturbations (mainly caused by J2) on the orbit. Their amplitude is small enough (~4 km on the perigee altitude for a GTO) but increases as the semi-major decreases. The result is an underestimation of drag which is considered acceptable.

1.2.4 Other forces

Solar pressure has little impact (compared to the other forces) and will be neglected.

1.3 Integration of the motion

The motion of the satellite is obtained by integrating the averaged derivatives of the orbital elements.
The integration method is a bit original as it is iterative (this had initially been designed for efficiency reasons, given the software used: Scilab).
To compute the solution (i.e. the state vector X) over a given time range (say one year), the algorithm iterates over: $X_{n+1}(T) = X(T_1) + \int_{T1}^{T} \dot{X}(t, X_n(t))\, dt,\ T \in [T_1, T_2]$ starting from an initial guess (effect of J2 only), until X_{n+1} is close enough to X_n. Once the process has converged, the solution at the last instant is used to initialize the process for the next time range.

2 Analysis of the effects of the main perturbations

2.1 Sun/Moon perturbation (effect on the mean perigee altitude)

In this part, we'll analyze the orbit's evolution in absence of drag.

2.1.1 Simulation results of the influence of the RAAN local time and day of year

When integrating the equations defined in 1.2, considering the Sun, Moon and the secular effects due to J2 only, varying the initial ascending node mean local time (MLT), we obtain the graph shown in Figure 4.

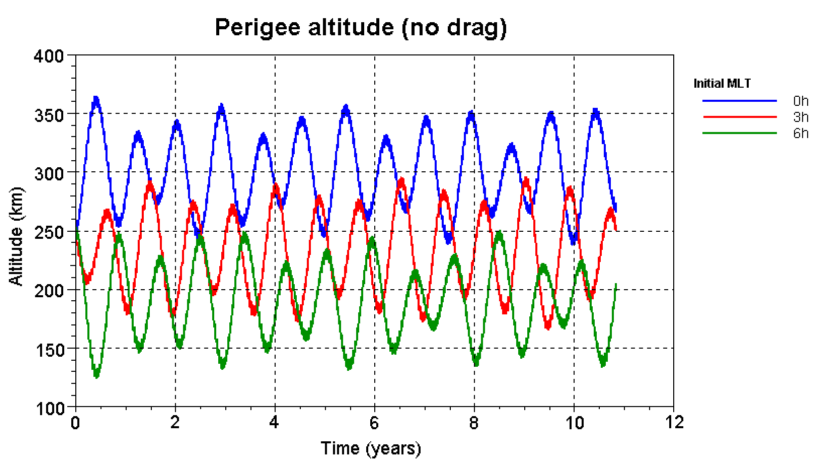


Figure 4: Evolution of the perigee altitude without drag

The large amplitude oscillations are related to the motion of the perigee with respect to the Sun ($\dot{\omega} + \dot{\Omega} - \omega_{sun} \approx -0.6$ deg/day), with a modulation mostly related to the declination of the Sun.

Note: The initial mean local time of the ascending node is considered even if the “real” (and better defined) quantity that matters is the mean local time of the perigee (= RAAN mean local time + 12h as the argument of the perigee is (initially) 180 degrees).

Eccentricity appears to be stable, as is inclination, as shown in Figure 5 (simulation over 100 years). Also note that the mean semi-major axis is not affected by the perturbations.

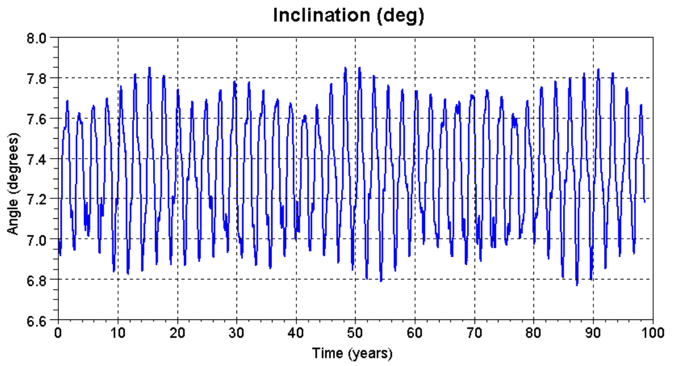
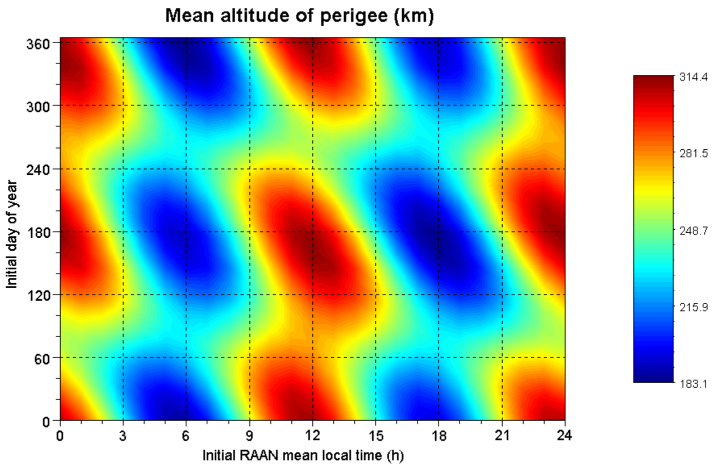


Figure 5: Evolution of inclination without drag

If the altitude of the perigee is averaged over a long period of time, we obtain a mean value that depends on the initial mean local time of the ascending node and the day of year.

Figure 6 shows the result for a particular period of time. There may be some variations between years because of the orbit of the Moon that changes slightly from one year to the next.

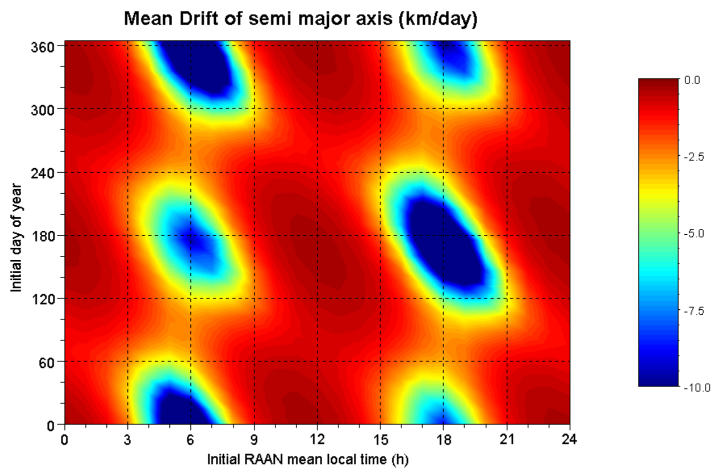


The initial perigee altitude is 250 km
Inclination is 7 degrees
Model used : J2 + Sun + Moon
Averaging is done over 4 years from 2008

Figure 6: Mean altitude of perigee as function of RAAN local time and day of year

Even if drag in not included in the propagation model, it is still possible to evaluate its potential effect on the orbit.

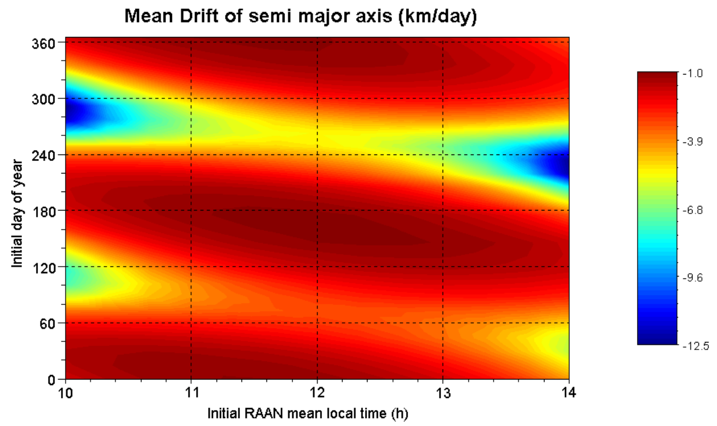
Figure 7 shows the (virtual) mean drift of the semi-major axis as evaluated from the trajectory computed without drag.
Higher values of the mean altitude of the perigee in Figure 6 correspond to lower values of the decrease rate of the semi-major axis in Figure 7 (so possibly to shorter lifetimes).



Initial altitude is 250 km
Inclination is 7 degrees
Model used : J2 + Sun + Moon
over 4 years from 2008

Figure 7: Evaluation of semi-major axis mean drift

The most favourable conditions for a short lifetime (or at least a strong initial decrease rate of the semi-major axis) are at the solstices with the orbit nearly perpendicular to the Sun direction (RAAN local time = 6h or 18h).



Initial altitude here is 200 km
Inclination is 7 degrees
Model used : J2 + Sun + Moon
over 4 years from 2008

Figure 8: Evaluation of semi-major axis mean drift

In the RAAN local time range [10h-14h] (see Figure 8), the equinoxes are more favourable, and the autumn equinox is a little more. This conclusion is consistent with the results shown in Figure 1.

2.1.2 <u>Explanations through simple modelling</u>

If we assume that inclination and drifts of argument of perigee and ascending node are constant over time, the equation describing the evolution of eccentricity can be integrated analytically.

For simplification reasons, we’ll consider here that the orbits of the celestial body and the satellite are in the equatorial plane.

We can then write:

$$\frac{de}{dt} = K\, e\, \sqrt{1-e^2}\, \frac{\sin(2\phi)}{2}$$

where $K = -\frac{15}{2}\frac{\mu}{n\, d^3}$ (supposed constant). ϕ is the angle between the direction of the perigee and the body. It can be written: $\phi = \omega\, t + \phi_0$ (ω constant).

The exact solution is then given by: $e = \frac{2\,z}{1+z^2}$,

with: $z = z_0 \exp(-\frac{K}{4\omega}(\cos(2\omega\, t + 2\phi) - \cos(2\phi_0)))$ and: $z_0 = \frac{1-\sqrt{1-e_0^2}}{e_0}$

The expression can be simplified as $\frac{K}{4\omega}$ is small (see values below).

This enables the calculation of the mean (over an infinite period of time) value $\bar{e}$ (first order expansion in $\frac{K}{4\omega}$): $\bar{e} - e_0 = e_0 \frac{K}{4\omega} \frac{e_0 - z_0}{z_0} \cos(2\phi_0)$

Numerically, we have:

$\left|\frac{K}{4\omega}\right|$ = 3.8e-3 for the Sun, and 4.2e-4 for the Moon (about 10 times smaller).

$\frac{e_0\,(e_0 - z_0)}{z_0}$ is about 0.5, so that the maximum values found for $a\,(\bar{e} - e_0)$ are close to: 46km for the Sun and 5.1 km for the Moon.

Figure 9 shows the average value of the perigee altitude (as computed by the formula above) as a function of the initial phase (ϕ_0) for the Sun.

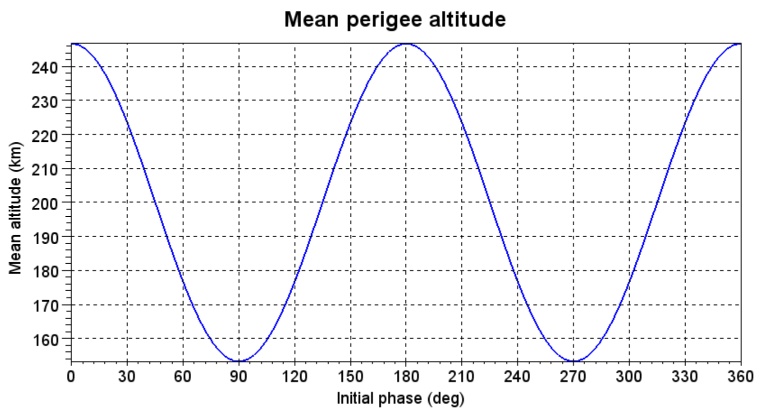


Figure 9: Mean altitude of perigee as function of initial phase (simplified model)

Similarly, the initial position of the Moon changes the mean value of the altitude of the perigee. Under the same simplified hypotheses as above, we can derive the impacts the Sun and Moon simultaneously have on the perigee altitude depending on their initial positions. This is illustrated in Figure 10.

The maximum effect coming from the Moon on the mean altitude of the perigee is nearly the same whatever the initial phase of the Sun: around 5 km.

The maximum altitude is obtained when the 2 bodies are aligned and for the initial phases = 0 or 180 degrees (dark read areas).

The minimum altitude is obtained when the 2 bodies are aligned and for the initial phases = 90 or 270 degrees (dark blue areas).

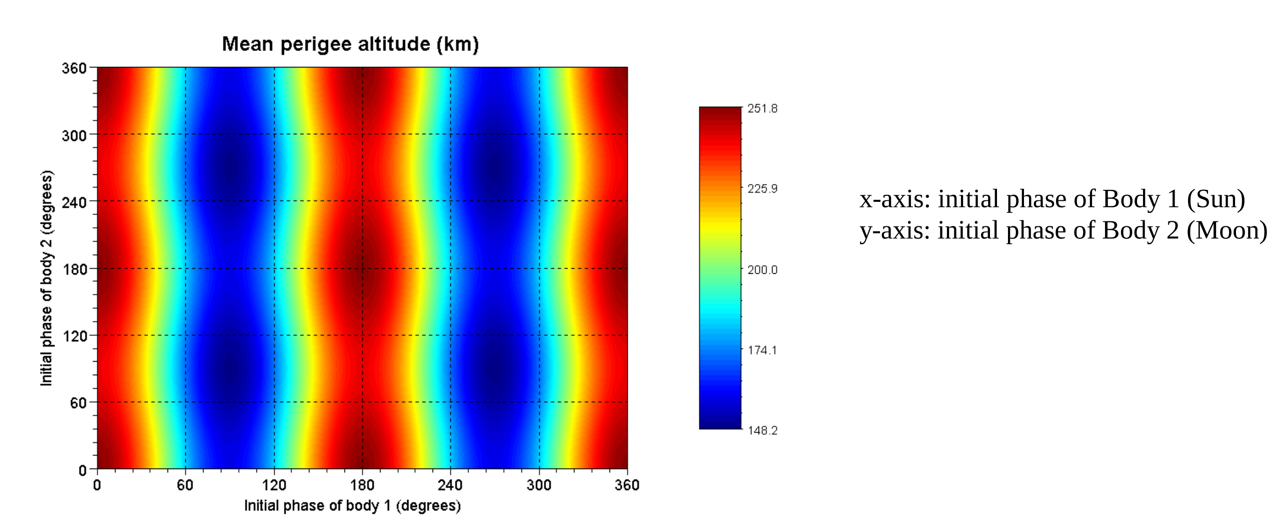


Figure 10: Impact of initial positions of Sun and Moon (simplified model)

The equations are now made slightly more complex by considering a celestial body (the Sun) in a 30 degree, circular, inclined orbit. The satellite's orbit is still in the equator.

The mean altitude of the perigee that is obtained is plotted in Figure 11.

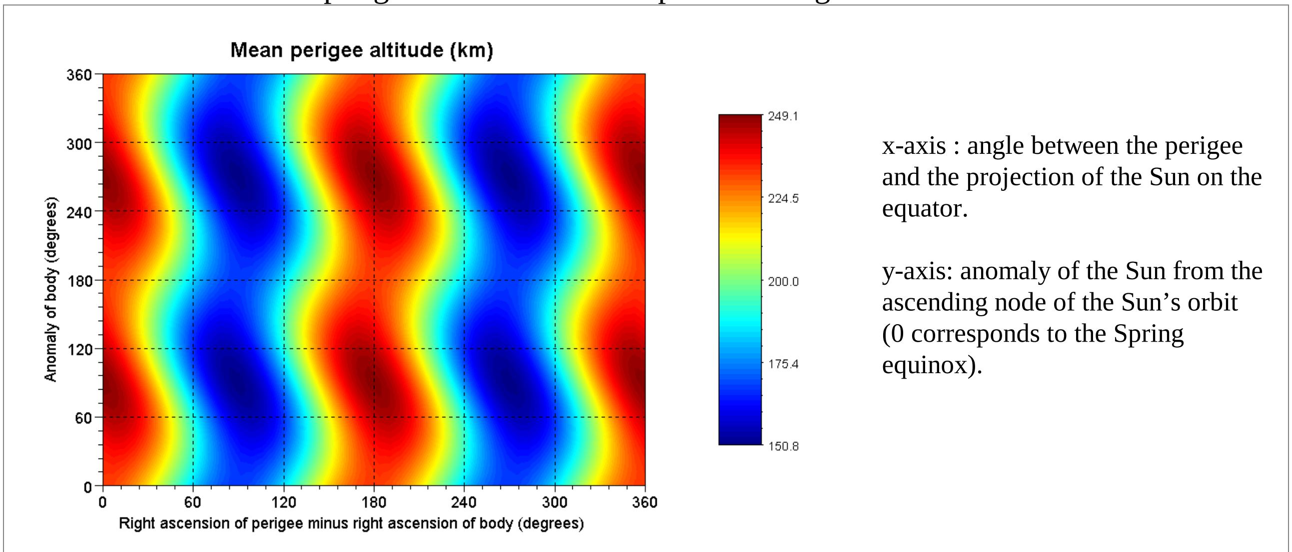


Figure 11: Impact of the Sun declination (simplified model)

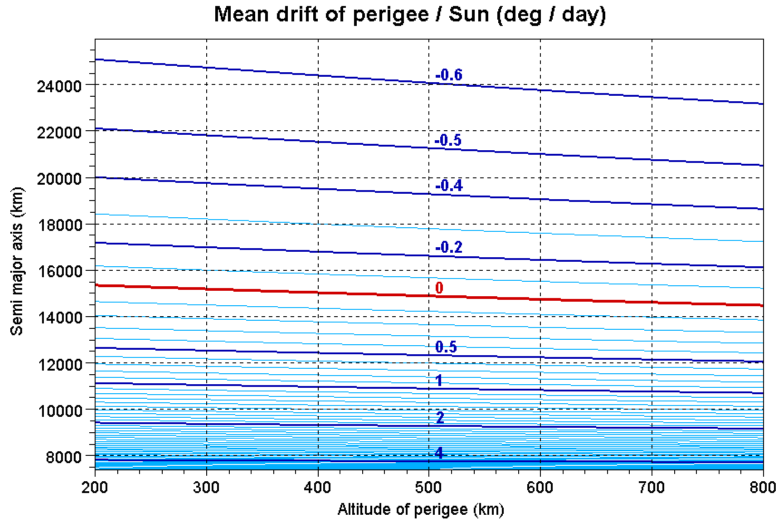
We observe that these results are similar to those shown in Figure 6, except for a shift along the y-axis due to different choices for the origin of the axes: day of year in Figure 6 and Spring equinox in Figure 11.
The same kind of effect would be obtained for the Moon, with variations from year to year coming from the changes in the Moon's orbital elements.

2.2 Influence of drag and coupled J2 / Sun perturbations effects

The effect of the atmosphere on the orbit causes the semi-major axis to decrease, with a decrease rate depending on the perigee altitude (the perigee altitude decreases too, but more gently). We have seen in 2.1 (Figure 6 and Figure 7) how the mean value of the perigee altitude changes because of the effect of the Sun and Moon, which impacts drag.

As the semi-major axis decreases (the eccentricity decreasing too), the mean local time of the perigee varies too: $\dot{\omega} + \dot{\Omega} - \omega_{sun}$ starts at –0.6 deg/day, increases to 0 and changes sign.

Figure 12 shows $\dot{\omega}+\dot{\Omega}-\omega_{sun}$ computed using J2 only, as a function of altitude of perigee and semi-major axis.

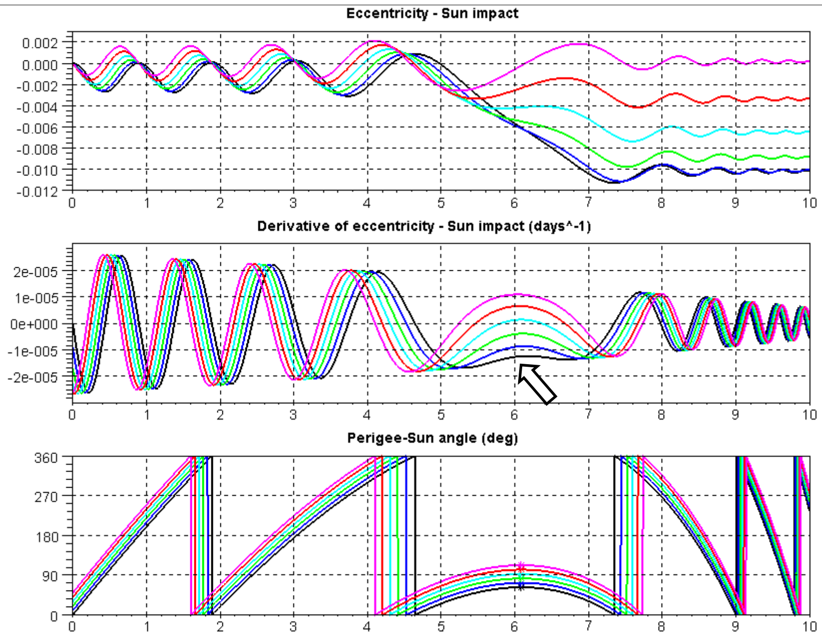


The inclination considered here is 7 degrees.

Figure 12: Mean drift of the angle between the Sun and the perigee

When $\dot{\omega}+\dot{\Omega}-\omega_{sun}$ approaches 0 (the semi-major is then about 15000km), the gravitational effect of the Sun intensifies because the Sun is then nearly fixed with respect to the orbit plane. We have a situation close to that illustrated in Figure 2, with the perigee increasing or decreasing at a nearly constant rate.

We will illustrate the effect of the Sun on eccentricity with a simple (theoretical) example.
Here the inclination is 0 and the Sun's orbit is in the equator. The orbit's semi-major axis is supposed to decrease at a constant rate, and the altitude of the perigee is constant (in order to simplify the problem). The gravitational effect of the Sun is then evaluated on this "reference" orbit (but the Sun has no effect on the orbit).



The cumulative effect on eccentricity is around 10^{-2} in the "worst" case (black curve).
That corresponds to about 150 km on the perigee altitude (for a semi-major axis = 15000km).

Drift of the semi-major axis considered: -4km/day.
Initial perigee-Sun angles: 0 to 50 degrees (every 10 degrees).

Figure 13: Illustration of the effect of the Sun on eccentricity

Various initial conditions (angles between the perigee and the Sun) are chosen. We see the cumulative effect on eccentricity (upper curve): the period of the oscillations increases until the drift of the perigee-Sun angle changes sign.

The "worst" case (that makes eccentricity vary the most) is obtained for a situation where the derivative of eccentricity remains nearly constant at its maximum elongation (see arrow in middle graph). This corresponds to the perigee-Sun angle (Φ) remaining close to the middle of a quadrant (45 degrees in the example, see lower graph).
Thus, what particularly (but not only) matters is the value of the perigee-Sun angle when its derivative changes sign.
For a given value of the perigee altitude, the perigee-Sun angle changes sign for a well defined value of the semi-major axis. But the value of the perigee-Sun angle at that moment depends on many factors and particularly drag. A rough evaluation (based on a constant decrease rate of the semi-major axis and a constant perigee altitude) gives the following rule:

$$\Delta\Phi_{\text{degrees}} \approx -1.3\,(\frac{\Delta\dot{a}}{\dot{a}})_{\%}\;T_{\text{years}}$$

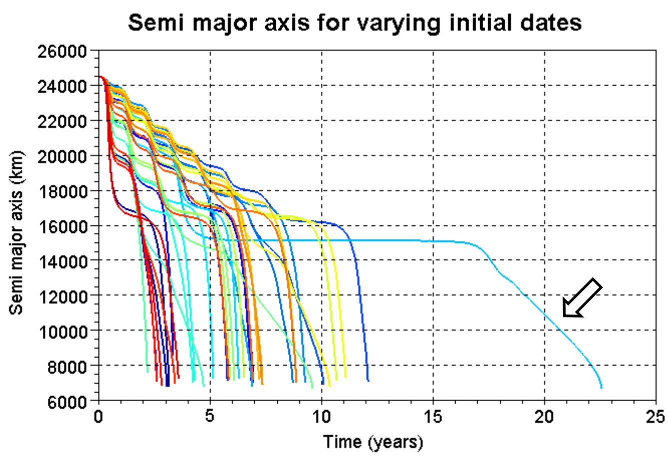
where $\Delta\Phi_{\text{degrees}}$: error in degrees on the perigee-Sun angle, $(\frac{\Delta\dot{a}}{\dot{a}})_{\%}$: relative error (%) on $\dot{a}$ (or on the drag coefficient), T_{years}: duration (years) between initial time and time when the Sun-synchronism condition is met (i.e. $\dot{\Phi} = 0$). For example only ±5% error on drag for 10 years gives an error on the angle of about ±60 degrees. The Sun gravitational effect is then completely changed.

The previous example is very theoretical as the Sun gravitational effect doesn't affect the orbit: the semi-major axis decreases at the same rate whatever the changes in eccentricity.
In reality, these changes will make the drag suddenly increase or decrease, so that the orbit's evolution from that moment will be completely modified. This will be shown later.

3 GTO Lifetime

3.1 Simulation of lifetime

The effect described in 2.2 is clearly visible in the following simulation results.
The simulations include the gravitation effects of the Sun and Moon, the secular effect of J2 and drag.
The initial time of the simulation varies by steps of 10 days.



Initial year: 2011
Initial perigee/apogee altitudes: 250/36000 km
Inclination: 7 degrees
RAAN local time : 6h

Figure 14: Simulated evolution of semi-major axis

We note that the lifetime is, in most cases, related to the initial mean drift rate of the semi-major axis. There are however a few exceptions for which the semi-major axis remains constant at a value close to 15000km for a considerable amount of time (see arrow).

The evolutions of the perigee altitude and mean local time corresponding to the “exotic” case are shown below.

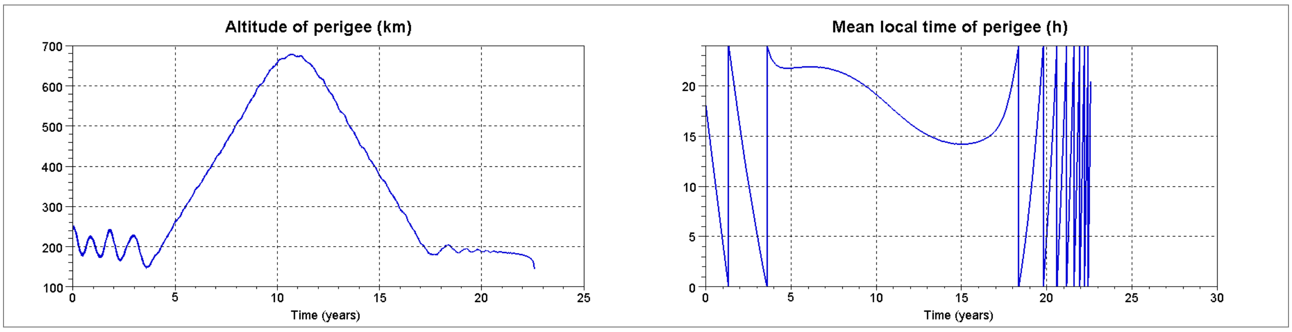


Figure 15: Evolution of perigee altitude and mean local time

The altitude of the perigee jumps from 200 km to 700 km, much more than in Figure 13 (corresponding to a semi-major axis decreasing at a constant rate).

Here is what is happening (see also 4 for more details):

- The semi-major axis decreases for about 4 years until is reaches a value of about 150000km (we can see in Figure 15 that the perigee altitude decreases too but moderately). $\dot{\omega}+\dot{\Omega}-\omega_{sun}$ is then close to 0 and is about to become positive (see Figure 12).
- As the angle between the perigee and the Sun is then nearly constant (local time of perigee between 18 and 24h), the eccentricity decreases, the semi-major axis being approximately constant (as drag vanishes). This causes the perigee altitude to increase. This situation is close to that represented in Figure 2, the Sun being in quadrant 3.
- The increase of eccentricity impacts the drift of the angle between the perigee and the Sun. The value of $\dot{\omega}+\dot{\Omega}-\omega_{sun}$ becomes negative again (see Figure 12). This extends the impact of the Sun as the angle remains nearly constant for a longer time. Eccentricity then continues to decrease (and the perigee altitude to increase).
- As the angle between the perigee and the Sun travels through quadrant 3 and enters quadrant 4, the drift on eccentricity decreases in intensity and becomes positive. This causes the perigee altitude to start decreasing.

This “resonant” case is also very sensitive to drag, as could be anticipated.
The results below show how small variations in the drag coefficient (which can also reflect variations in solar activity that could be encountered in practice) impact the lifetime.

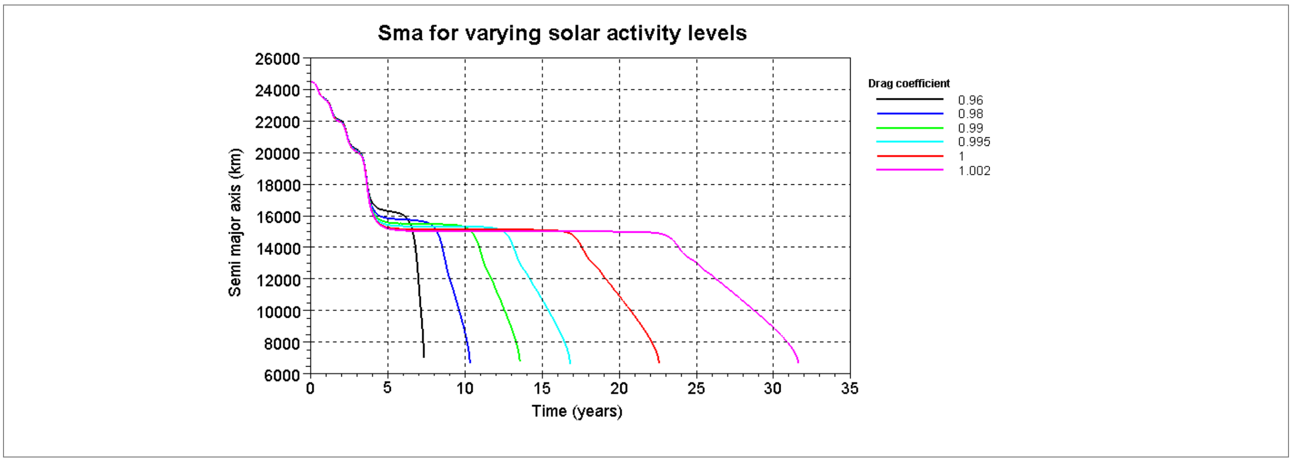


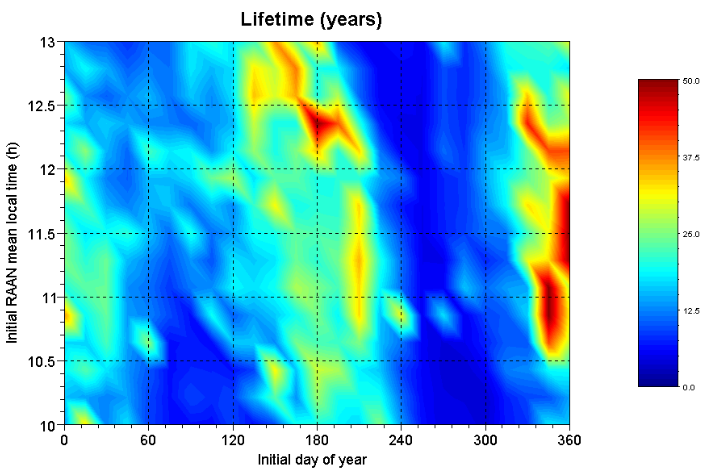
Figure 16: Sensitivity of lifetime to drag

A 4% change in the drag coefficient can make the lifetime change from 7 years to 32 years.
Even a 0.2% change has an noticeable influence. But the impact may originate in the computation of drag itself (averaging over one orbit), which is not guaranteed more precise than a few 0.1%.

But the conclusion holds whatever the reason: even the most accurate model would fail to predict the lifetime accurately (in some particular cases at least), for the main reason that solar activity cannot be predicted with a sufficient accuracy.

3.2 Comparison with other results

We now use the same propagation model as in 3.1 to compute lifetime as a function of RAAN mean local time (in the range [10h-13h] and day of year.
These results can be compared with those obtained using DAS (see introduction), although the hypotheses are not exactly the same (not mentioning the propagation model).
We note that some similarities exist between the 2 graphs: the shortest lifetime is obtained for the Autumn equinox in both cases.



Cx.A/M = 2.2e-2
Initial altitude = 200 km

Around 300 computation points to generate the plot.

Figure 17: Lifetime as function of day of year and RAAN mean local time

4 Lifetime predictability

The previous paragraphs have shown that the lifetime can be very sensitive to modelling errors and initial conditions.
In this paragraph we'll try to be a bit more precise.

We'll assume again a simple model ("plane" model as in 2.1.2) where ϕ is the angle between the perigee and the Sun.
If we neglect the effect of drag (which can be justified because we're interested in situations where the perigee increases, so that drag vanishes), the evolution of eccentricity is mainly governed by the combined effects of the Sun and J2 which can be described by the following equations:

$$\begin{cases} \dot{e} = K\ e\sqrt{1-e^2}\ \dfrac{\sin(2\phi)}{2} \\ \dot{\phi} = \omega_{sun} - \dot{\omega} - \dot{\Omega} = \omega_{sun} - \dfrac{3}{2}\ \dfrac{R_{eq}^2}{a^2}\ \dfrac{n\ J_2}{(1-e^2)^2} \end{cases}$$

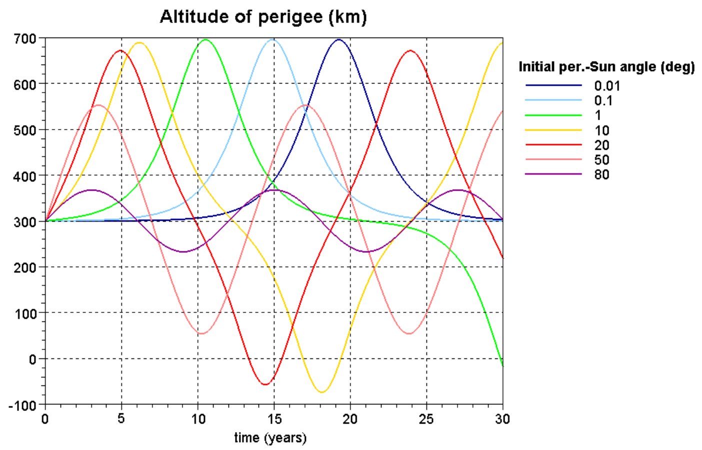
In the above equations, *K* is constant, as is *a* (semi-major axis) and *n* (mean motion).

We simulate (i.e. integrate) the system above using a Runge-Kutta method, starting from initial conditions defined by:

- altitude of perigee = 300km
- $\dot{\phi} = 0$ (Sun-synchronism condition)

The two above conditions are equivalent to defining the initial values for the eccentricity and the semi-major axis. The value of ϕ at initial time (ϕ_0) can be chosen arbitrarily.

The evolution of eccentricity obtained for various values of ϕ_0 is shown in Figure 18.



All the curves are periodic, but with a period and amplitude depending on the initial angle between the perigee and the Sun.

Figure 18: Simulation of resonance

We note that the perigee altitude can go up to around 700km, which is consistent which previous results (see Figure 15). We also note that it comes back to the initial value after a few years.

The values of the perigee-Sun angle in the range [0.01, 20] degrees nearly give the same maximum value for the perigee altitude. But one major difference between the cases is the time at which the maximum value is reached : at initial time + 19 years for 0.01, at initial time + 15 years for 0.1, etc…

A initial value of 0 would keep the perigee altitude constant. A slightly negative value would invert the evolution (the perigee would start by decreasing).

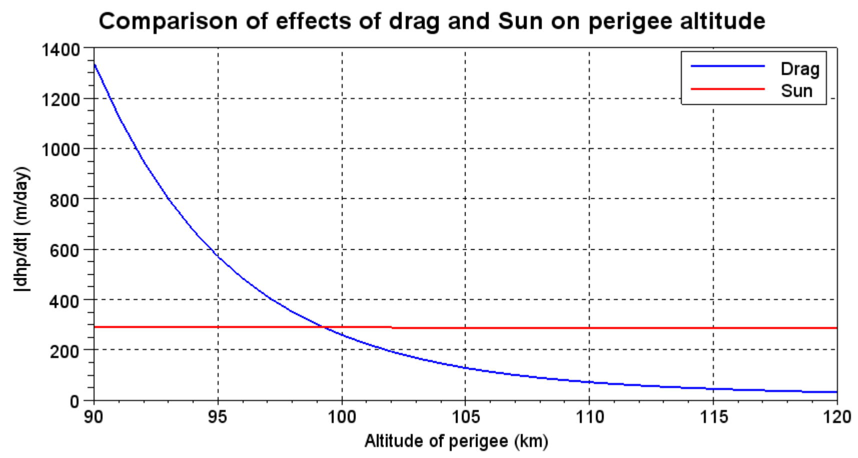
Thus this situation is very instable (and therefore sensitive to initial conditions). Very little changes in the angle between the perigee and the Sun can make the altitude of the perigee remain constant, increase by several hundreds of km, or decrease by the same amount.

This simple demonstration proves that it may be impossible predict GTO lifetime accurately, at least in some particular cases. The key parameters are ϕ_0 and the uncertainty on ϕ_0. If ϕ_0 is well chosen (so that it is in the right quadrant) and the uncertainty on ϕ_0 small enough, it's possible to guarantee a fast re-entry by avoiding the perigee to increase as in Figure 18. Another key parameter is the perigee altitude: if the perigee is low enough, drag will finally succeed in making the semi-major axis decrease; the effect of the Sun can only delay the re-entry by 10-20 (or so) years in the worst cases.

A sufficient (although strict) condition to limit the lifetime would be to guarantee that the effect of drag on the perigee altitude would always be greater than the maximum gravitational effect due to the Sun.

The evaluation is done for orbits such that the perigee does not drift with respect to the Sun (Sun-synchronism condition).

The maximum effect on the perigee altitude due to the gravitational effect of the Sun is given by $\left|\frac{d h_p}{dt}\right|_{\max} = \frac{15}{4} \frac{\mu a}{n d^3}$ and the effect due to drag is computed numerically.

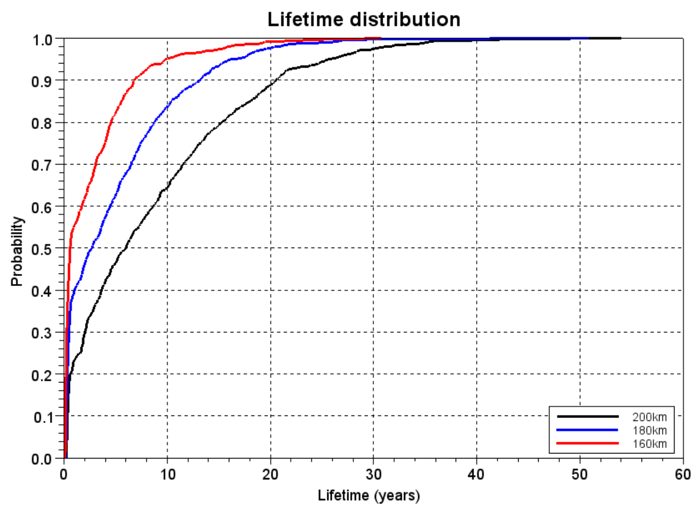


Inclination is here 7 degrees.

Figure 19: Drag compared to Sun gravitational effect

Thus, the perigee altitude has to be very low (less than ~100 km) for drag to be stronger than the Sun gravitational effect when the Sun-synchronism condition is met (i.e. when the semi major axis is around 15000 km). The condition on initial altitude is not obvious at this point and should be finely analysed (effect of drag on the perigee, Sun/Moon perturbations causing oscillations...). However, the initial altitude is likely to be very low too.

Less strict conditions may exist though to guarantee that the lifetime will not exceed some predefined threshold.
The graph below gives the lifetime distribution for 3 different initial perigee altitudes (160km, 180km, 200km).



Model used: Sun, Moon, J2 and Drag.

1000 draws for each perigee altitude case.

Uniformly distributed RAAN MLT (0-24h), initial time (interval of 18years), drag coefficient (maximum dispersion is 5%) and even altitude of perigee (maximum dispersion is 500m).

Figure 20: Lifetime distribution

The probability to have the lifetime smaller than some threshold increases as the initial perigee altitude decreases, as shown in the table below.

Initial perigee altitude (km)	160 km	180 km	200 km
Probability (lifetime < 25 years)	0.995	0.99	0.94
Probability (lifetime < 35 years)	~1	0.995	0.99

But an error of only 5% was taken into account on the drag coefficient. The dispersion for real cases (considering a variable solar activity) could be much bigger.

5 Conclusion

Thus, the paper has shown different results relative to geostationary transfer orbits long term evolution and lifetime.

The models used in the study were simple enough, yet interesting results have been obtained.

First, it was shown how some initial conditions (ascending node mean local time, day of year) affect the decrease rate of the semi-major axis through a change in the mean altitude of the perigee.
But the lifetime is also strongly dependent on the gravitational effects due to the Sun at the time the Sun becomes nearly fixed with respect to the orbit's line of apsides. This situation may lead to a rapid increase of the perigee altitude (by several hundreds of kilometres), thus reducing the drag and increasing the lifetime. The sensitivity to initial conditions and perturbations (and drag in particular) is high: a change in atmospheric density of only a few percent can lead to variations in the lifetime of tens of years.

This has led to the conclusion that GTO long-term evolution may be hard to predict in practice, at least in some particular situations, as solar activity will never be predictable with sufficient accuracy. It is possible to find strategies that minimize the uncertainty on lifetime, but the uncertainty on drag has to be small enough, probably smaller than can be achieved in practice. An evaluation of a "safe" altitude has been done in the paper but leads to a very low value. However, less strict conditions may exist on the perigee altitude to guarantee that lifetime won't exceed some threshold; more remains to be done on this point.

Statistical analyses as done in [3] for LEO are probably worthwhile for GTO as well. They have led to defining a "constant equivalent" solar activity level such that, if considered in calculations, the satellite has a 50% chance of re-entering the atmosphere in less than 25 years. This was considered acceptable because the dispersion on lifetime (when varying solar activity) is not excessive for LEO. For GTO, a low solar activity level would possibly lead to big lifetime values. So that the exact conditions that should be satisfied in order to meet the French Space Act regulations have to be defined adequately.

6 References

[1] Carine Leveau et al (CNES-DLA/SDT/SPC), Analysis of Ariane5 third stage reentry (CNES internal study).

[2] A. Lamy, G. Azema, Selection and optimization of a constellation of satellites for continuous zonal coverage, ISSFD2009, Toulouse, France

[3] Hubert Fraysse, Vincent Morand, Clémence Le Fevre, Florent Deleflie, Sébastien Wailliez, Thierry Martin, Alain Lamy, Long term orbit propagation techniques developed in the frame of the French space act, ISSFD2011, Sao Jose Dos Campos, Brazil

[4] Alan B. Jenkin et al, Lifetime reduction for highly inclined, highly eccentric disposal orbits by changing inclination, AIAA 2008-7376.