LONG TERM ORBIT PROPAGATION TECHNIQUES DEVELOPED IN THE FRAME OF THE FRENCH SPACE ACT

Hubert Fraysse(1), Vincent Morand(2), Clémence Le Fevre(1), Florent Deleflie(3), Sébastien Wailliez(3), Alain Lamy(1), Thierry Martin(1), Etienne Perot(2)

(1) CNES (French Space Center), Toulouse, France, 33561282152, hubert.fraysse@cnes.fr
(2) THALES C4I, 3 avenue de l'Europe 31400 Toulouse, 33561282981, vincent.morand@cnes.fr
(3) IMCCE, Observatoire de Paris, 33320524424, Florent.Deleflie@imcce.fr

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Abstract: *the paper describes the "good practices" applicable in the frame of the French Space Act, whose objective is to ensure that the technical risks associated with space activities are properly mitigated, to compute the long term evolution of LEO and quasi-GEO orbit in order to check the no-crossing of protected LEO and GEO regions defined by IADC. It presents the main computation rules, an overview of the dynamical model used by the reference CNES tool STELA and more precisely the "constant equivalent solar activity" approach that is taken into account in the drag force computation.*

1. Introduction

In the frame of the French Space Act, whose objective is to ensure that the technical risks associated with space activities are properly mitigated, CNES is providing technical expertise to government on regulations governing space operations, and will check compliance prior to delivery of authorizations submitted to the minister in charge of space for approval.
Then CNES is in charge of proposing and developing the technical methods to be recommended to cope with the law requirements.
Space debris mitigation is one objective of the French Space Act, in line with IADC (Inter-Agency Space Debris Coordination Committee) recommendations, through removal of non-operational objects from populated regions. At the end of their mission, space objects are to be placed on orbits that will minimize future hazard to space objects orbiting in the same region. The protected regions have been defined by IADC: region A for Low Earth Orbits and region B for Geostationary Earth Orbits.
The verification of these rules and criteria requires long term orbit propagation to evaluate orbit parameters evolution (up to more than 100 years).
It is well known that the long-term orbit evolution is very sensitive to key computation hypothesis as area, drag coefficient and solar activity prediction for LEO orbits, surface and reflectivity coefficient for GEO orbits. Then in the frame of the French Space Act, precise normalized methods or values have been recommended. A key problem for LEO is the fact that, due to solar activity, the disposal orbit lifetime is sensitive to the removal date that may not be well known when the spacecraft is designed or launched. Then a law-compliant disposal orbit, achievable with a dedicated amount of fuel for an expected end of mission date, may not be compliant anymore if this date shifts. To cope with this difficulty a "constant equivalent solar activity" approach (based on statistical approach using five past solar cycles) has been studied in order to determine constant equivalent solar activity value(s) that can be used in orbit lifetime computation.
An appropriate dynamical model (gravity field terms, third-body effects, drag and sun radiation pressure) has also been defined. All these "good practices" have been described in a dedicated chapter of the technical rules of the French Space Act. They are summarized in this paper.
Then a semi-analytical method, much better suited for long term extrapolation than numerical propagation, with non singular equations in eccentricity and inclination has been developed considering only the dynamical effects that are significant for each orbit type (LEO, GEO). The short periods have been removed from the evolution of orbital elements, allowing a large saving of computation time without losing precision on long term mean evolution.
They have been tuned and validated by comparison with numerical integration of the full dynamic equation.

A reference tool implementing these methods is now available (STELA). It is used to check the compliance of disposal orbits with the rules and can be also used to select the appropriate disposal orbits in the mission design phases.

2. French Space Act protected regions criteria

Two protected regions have been defined by IADC:

- region A : LEO protected region (altitude < 2000km)
- region B : GEO protected region (GEO–200km < altitude <GEO+200km; inclination<15deg).

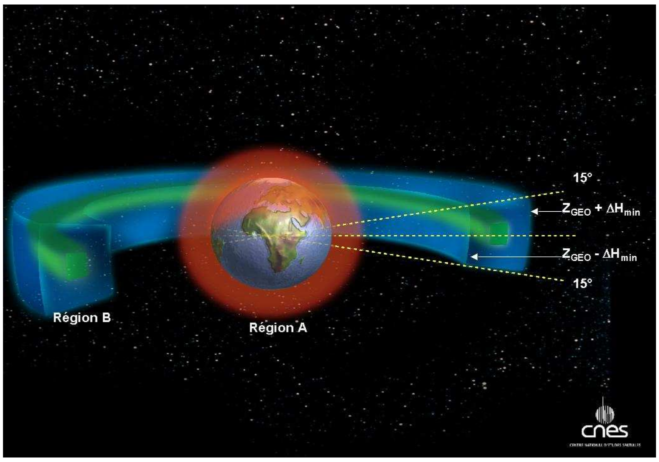

Figure 1 : IADC protected regions

Four applicable criteria have to be checked in the frame of the French Space Act:
- C1 criterion for LEO region: "Lifetime < 25 years". The C1 criterion is violated if the spacecraft lifetime on a disposal orbit crossing the LEO region exceeds 25 years.
- C2 criterion for LEO region : "No LEO crossing within 100 years". The C2 criterion is violated if disposal orbit above the LEO region crosses the LEO region during the first 100 extrapolation years.
- C3 criterion for GEO region : "No GEO crossing between 1 and 100 years". The C3 criterion is violated if a disposal GTO orbit crosses the GEO region between the first and the hundredth year.
- C4 criterion for GEO region : "No GEO crossing within 100 years"
The C4 criterion is violated if a disposal orbit above the GEO region crosses the GEO region during the first 100 extrapolation years.

3. Good practices for orbit propagation

This paragraph summarizes the applicable dynamical model (as a “minimum” model) and the hypotheses to be used for long term orbit propagation in order to be able to check the above criteria. ISO documents ref [2] and [3] have been used as guidelines.
As a simplified dynamical model is linked to an orbit range, three types (ranges) of typical (most common) orbits have been defined and studied:

- LEO type orbits: initial orbits being in the LEO protected region extended to 2200 km (in order to be able to cope with end of life orbits being slightly above the LEO region),
- GEO type orbits: initial orbits being in the GEO region extended by ±1000km in altitude and ± 20 deg in inclination (wrt to GEO orbit),
- GTO type orbits : to be defined.

3.1. Dynamical models

Table 1 gives the perturbations that have significant effects on orbital parameters in long term propagation:

Perturbation	LEO type orbits	GEO type orbits
Earth's gravity field	J2 to J4 zonal model*	Complete 4x4 model
Solar and Lunar gravity	yes**	yes
Atmospheric drag	yes	no
Solar radiation pressure (SRP)	no	yes

Table 1 : Dynamical models

* the effect on orbit eccentricity of zonal terms up to at least J10 and preferably J15 has to be considered when the inclination is close to the critical inclination (63.4 deg for prograde orbites)
** the luni-solar perturbation is significant for sun synchronous orbits or orbits with apogee altitude in the upper range of the LEO region

The §4 explains how these dynamical models have been implemented in STELA software and validated by comparison with numerical integration of the full dynamic equation.

3.2 Computation of the Solar Pressure force

The solar pressure force is to be computed following Eq. 1:

$$\vec{Fp} = C_R \, P_0 \, S \left(\frac{d_0}{d} \right)^2 \vec{u} \qquad (1)$$

with :

- u unit vector of the sun-spacecraft direction
- d sun-spacecraft distance
- d_0 Earth-Sun mean distance = 1AU
- P_0 solar radiation pressure at d0
- S cross sectional area (projected area on a plane perpendicular to u vector)
- C_R reflectivity coefficient (between 1 and 2).

The minimum allowed C_R value is 1.5 (in line with ref [1]).
Except in case of an equilibrium attitude (spin mode for example, to be demonstrated by analysis), the area S can be computed as a mean cross sectional area considering a tumbling attitude of the spacecraft (mean value of cross sectional areas observed from any direction). It gives a S_{mean} value.

3.3 Computation of the atmospheric drag force

The atmospheric drag force is to be computed following Eq. 2:

$$\vec{Fa} = -\frac{1}{2} \rho \, S \, C_d \, V_r \, \vec{V_r} \qquad (2)$$

with:
ρ : atmosphere density
S : cross sectional area (projection on a plane perpendicular to Vr vector)
Cd : drag coefficient
Vr : spacecraft velocity relative to the atmosphere.

Atmospheric model:

The NRMLMSISE-00 model was chosen because the results were "centered" when comparing the results of computing reentry durations obtained with different atmospheric models.

Area:

Except in case of an equilibrium attitude (aerodynamic equilibrium, gravity gradient stabilization for example; to be demonstrated by analysis) the area S can be computed as a mean cross sectional area considering a tumbling attitude of the spacecraft (mean value of cross sectional areas observed from any direction). It gives a S_{mean} value.

Cd:

For the drag coefficient, a reference Cd = f (altitude) law has been established in line with the mean cross sectional area hypothesis. It is based on the value of the drag coefficient of a plate in tumbling mode.

In the considered altitude range, the drag coefficient of a plate can be computed as follows (see ref [4],[5], [6]):

$$C_d = C_d^a + C_d^r$$

$$C_d^a(\theta) = \frac{2}{\sqrt{\pi}s}\exp\left(-s^2\sin^2\theta\right) + \sin\theta\left(2+\frac{1}{s^2}\right)erf(s\sin\theta)$$

$$C_d^r(\theta) = \frac{\sin\theta}{2s^2}\left[\exp\left(-s^2\sin^2\theta\right) + \sqrt{\pi}s\sin\theta\ erf(s\sin\theta)\right]\sqrt{1-\alpha}(s-1)$$

$$+ \frac{\sqrt{\pi}\sin^2\theta}{2s}\left(2\sqrt{\frac{T_w}{T}} + \sqrt{1-\alpha}\left(s+1-2\sqrt{\frac{T_w}{T}}\right)\right)$$

$$\alpha = \frac{k\mu}{(1+\mu)^2}, \quad with \quad \mu = \frac{M}{MO} \le 1$$

$$\alpha = 1 \qquad T_f = T_r = T_w$$

$$\alpha = 0 \qquad T_f = \frac{V^2}{2r} = s^2 T, \quad T_r = T$$

$$0 < \alpha < 1 \qquad \begin{cases} \sqrt{T_f} = \sqrt{T_w} + \sqrt{1-\alpha}\left(s\sqrt{T} - \sqrt{T_w}\right) \\ \sqrt{T_r} = \sqrt{T_w} + \sqrt{1-\alpha}\left(\sqrt{T} - \sqrt{T_w}\right) \end{cases}$$

$$s = \frac{V}{v_m} \quad with \quad v_m = \sqrt{2rT} \quad and \quad r = \frac{\Re}{M}$$

$$erf(x) = \frac{2}{\sqrt{\pi}}\int_0^x \exp(-\xi^2)d\xi$$

C_d^a : *absorption coefficient*

C_d^r : *re – emission coefficient*

V : *plate velocity vs atmosphere*

θ : *incidence angle* (*from velocity vector to the plate*)

T : *temperature of the gas* (*atmosphere*)

T_w : *wall temperature of the plate*

T_f : *gas temperature at the front surface of the plate*

T_r : *gas temperature at the rear surface of the plate*

$\Re$: *perfect gas c*onstante (8.314 472 $J\cdot mol^{-1}\cdot K^{-1}$)

M : *mean molar mass of the gas*

MO : *molar mass of oxygen atom* ($16.10^{-3}\,kg$)

k : accomodation constante, *from* 2 *to* 4, *recommanded value from* 3.6 *to* 4

The mean drag coefficient of a tumbling plate is then computed as follows:

$$S_{mean} C_{d\,mean} = (S_{plate}.C_d)_{mean} \Rightarrow C_{d\,mean} = \frac{(S_{plate}.C_d)_{mean}}{S_{mean}} \tag{3}$$

$$S_{mean} = \frac{\int_0^{2\pi} \int_0^{\pi/2} S_{plate} \sin\theta \cos\theta \, d\theta \, d\varphi}{2\pi} = \frac{S_{plate}}{2} \int_0^{\pi/2} \sin 2\theta \, d\theta = \frac{S_{plate}}{2}$$

$$(S_{plate}.C_d)_{mean} = S_{plate} \frac{\int_0^{2\pi} \int_{-\pi/2}^{\pi/2} C_x(\theta) \cos\theta \, d\theta \, d\varphi}{4\pi} = S_{plate} \int_0^{\pi/2} C_x(\theta) \cos\theta \, d\theta$$

$$\Rightarrow \quad C_{d\,mean} = 2\int_0^{\pi/2} C_d(\theta) \cos\theta \, d\theta$$

which is the drag coefficient of a sphere ($\pi C_{d\,sphere} = \int_0^{\pi/2} C_d(\theta).2\pi \cos\theta \, d\theta$).

Then:

$$C_{dmean} = C_d^f + C_d^r \quad with:$$

$$C_d^f = 2\left(1 + \frac{1}{s^2} - \frac{1}{4s^4}\right) erf(s) + \frac{2s^2+1}{\sqrt{\pi} s^3} \exp\left(-s^2\right)$$

$$C_d^r = \frac{\sqrt{\pi}}{3s}\left(2\sqrt{\frac{T_w}{T}} + \sqrt{1-\alpha}\left(s + 1 - 2\sqrt{\frac{T_w}{T}} + (s-1)\left[erf(s) + \frac{1 + (2s^2-1)\exp\left(-s^2\right)}{2\sqrt{\pi} s^3}\right]\right)\right) \tag{4}$$

The following values have been considered (after a sensitivity analysis) to establish the reference law:

- T_w (not very sensitive, the higher this value the higher the Cd): 300°K
- atmosphere temperature T and mean molecular mass M: obtained through a vertical density profile (NRMLMSISE-00 model) above an equatorial reference point at equinox epoch
- local time at the reference point: 10H30 (local time that gives a "mean" curve)
- solar activity (the higher the solar activity the lower the Cd) : mean values ($F_{10.7}$ = 145 sfu, AP = 15) see Fig. 15.
- V (not very sensitive, the higher the velocity the lower the Cd): 8 km/s.
- k (the higher this constant the lower the Cd): 4

Finally, the following Cd = f (alt) law was obtained (alt = geodetic altitude):

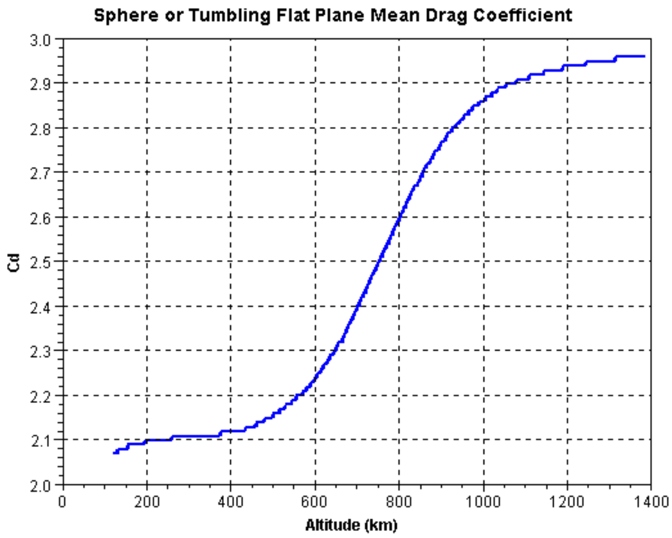

Figure 2 : Drag coefficient value versus altitude

Nb: Cd is kept constant above 1320 km.

Solar activity

Principle:

The result of a reentry duration computation strongly depends on the solar activity hypothesis. Indeed, the level (high, medium, low) and the duration of the next 3 or 4 solar cycles are key parameters in the orbit lifetime computation. The long term (more than several years) prediction of solar activity is very uncertain and does not allow the computation of a reliable and robust reentry duration. Another difficulty in a Space Act process is to deal with the fact that the spacecraft end of mission date may shift during the spacecraft development process, and that this initial date (vs the solar cycle) is a sensitive parameter of the reentry duration.
Then a normalization approach of the solar activity hypothesis has been developed, based on an constant equivalent solar activity. Following this approach the solar activity is supposed to be constant versus time so that the reentry duration computation no longer depends on the end of mission date. Both the solar flux ($F_{10.7}$) and the geomagnetic index (Ap) are set constant. Their values have been tuned to ensure that this constant solar activity is equivalent to the possible future solar activities. More precisely, the equivalency (validity) is from a 25 years lifetime point of view. It means that if a space object, with its specific ballistic coefficient, is placed on an orbit that has a 25 years lifetime computed by using the constant equivalent solar activity, then the reentry duration of this space object computed by using a large number of possible future solar activities will be 25 years with a 50% probability level (see Fig. 3)
Let us note that the method used to tune the equivalent constant solar activity allows to find the solar activity equivalent to z% of the possible future solar activities whatever z is (not only 50%).

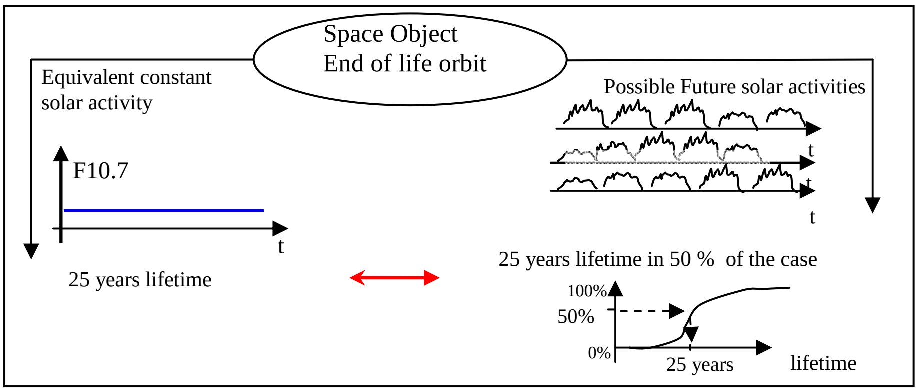

Figure 3 : Equivalency between constant and variable solar activity

The equivalent constant solar activity has been tuned using the following algorithm.

1. Choice of an initial orbit and a ballistic coefficient
2. Computation of *n* random possible future solar activities
3. Run of the n reentry duration computation using the *n* random solar activities. If we call OLT_50% the orbit lifetime in 50% of the case, then we don't have yet OLT_50% = 25 years.

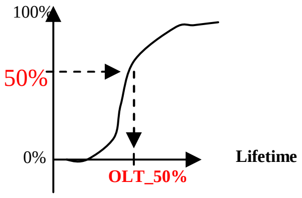

Figure 4 : Lifetime cumulative distribution function

4. Iteration on the initial orbit (by changing the perigee altitude) until OLT_50% = 25 years. At every step *n* reentry duration computation using *n* new random solar activities are performed. The orbit that has an OLT_50% of 25 years is called the end of life orbit.

5. Computation of the reentry duration of the end of life orbit by using a constant solar activity. Iteration on the value of the constant solar activity so that the reentry duration is 25 years. Calling $F_{10.7}$_50% the constant equivalent solar flux :

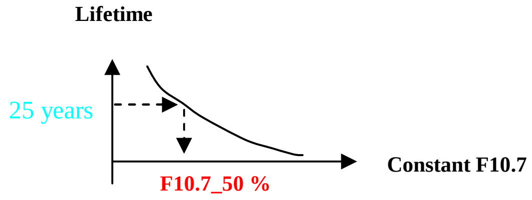

Figure 5 : $F_{10.7}$_50% value computation

6. Study of the sensitivity of the $F_{10.7}$_50% value to initial parameters (ballistic coefficient, initial orbit).

<u>Description of the algorithm :</u>

1. <u>Initial orbit and ballistic coefficient</u>

The results presented below have been computed for a space object with an area to mass ratio of 0.01, a constant drag coefficient of 2.2 and an initial sun synchronous orbit at 800 km (as an example).

2. <u>Random Solar activity</u>

The solar activity data (daily Flux and 3H Geomagnetic index) from the last decades are used (solar flux data from http://www.spaceweather.gc.ca/sx-11-eng.php and geomagnetic index data from http://www.ngdc.noaa.gov/stp/geomag/kp_ap.html). The data are split into 5 solar cycles (the spliting dates correspond to solar flux minima).

These five past solar cycles are plotted in Fig. 6

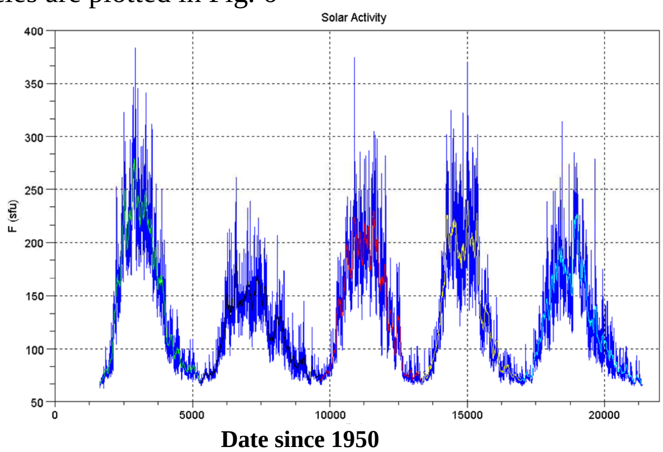

Figure 6 : Solar activity data

These five solar cycles are basic pieces in the building of the random solar activity. To perform a reentry duration computation four solar cycles are needed : three to cover the reentry duration (25 years) and one to allow the initial date to be at the beginning, in the middle or at the end of a solar cycle. Indeed, the solar activity level at the initial date can change the orbit lifetime from more or less 5 years. Using the five solar cycles we are able to create $5^4 = 625$ possibilities of four-cycles sequences. Then, the random realization of the initial date within the first cycle allows us to generate several random solar activities using the same four-cycles sequence. Finally n=1250 random solar activities were used when computing the OLT_50%: 625 four-cycles sequences have been generated using the five basic solar cycles and the initial date is a random realization within the first solar cycle, twice for each sequence. It was considered as representative samples of the possible future solar activities.

3. <u>Run of the n reentry duration computation using the *n* random solar activities.</u>

The 1250 reentry duration computations are performed using a semi-analytical propagator validated against full numerical propagation. The atmospheric density in computed with the NRMLMSISE-00 model.

The first guess for the perigee altitude that would lead to a OLT_50% of 25 years was 508 km.
The orbit lifetimes versus the extrapolation number (each extrapolation using a different random solar activity) are plotted Fig. 7 with the associated cumulative distribution function that gives the OLT_50%.

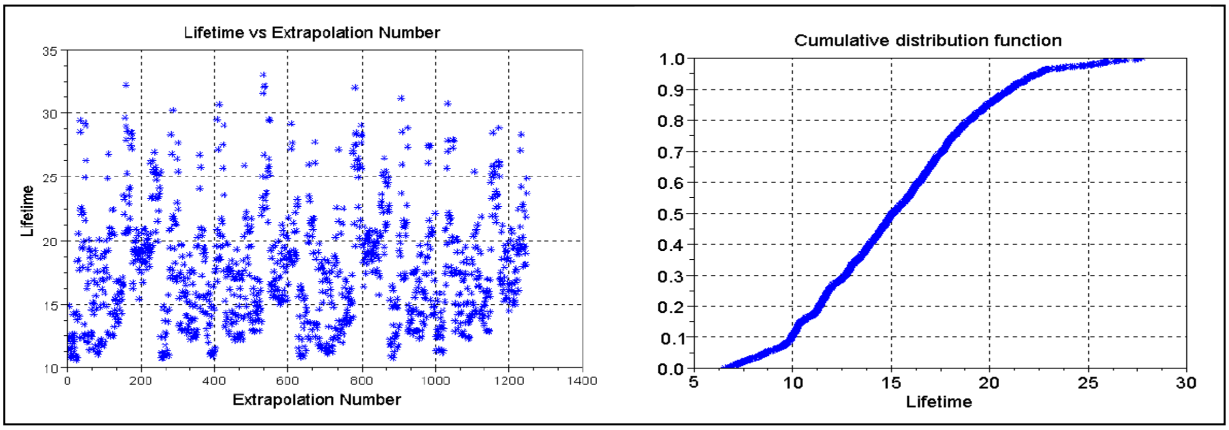

Figure 7 : Orbit lifetime variation and cumulative distribution function

Here we can see that the change in solar activity leads to orbit lifetime variations of more than 20 years. We also see that the OLT_50% is 15 years for this perigee altitude, it means that the first guess gives a too low perigee altitude.

4. <u>Iteration on the initial orbit (by changing the perigee altitude) until OLT_50% = 25 years.</u>

The degree of freedom is the perigee altitude Zp (the apogee altitude remains constant). We can define the function F that gives the difference between the actual OLT_50% and the target one (ie 25 years) for the current orbit (ie the current perigee altitude Zp). We have : F(Zp)= OLT_50% - 25 years. In our example we already have F(508km) = 15 – 25 = -10.
We used a classic Brent algorithm to find the zero of the function F. The Brent algorithm gives the next perigee altitude to be evaluated until the F value is small enough. At every step we perform the 1250 reentry duration computations by using 1250 randomly realized solar activities.
In our example the convergence is obtained after 5 iterations, see Fig. 8. The OLT_50% of 25 years is reached for a perigee altitude of 561km.

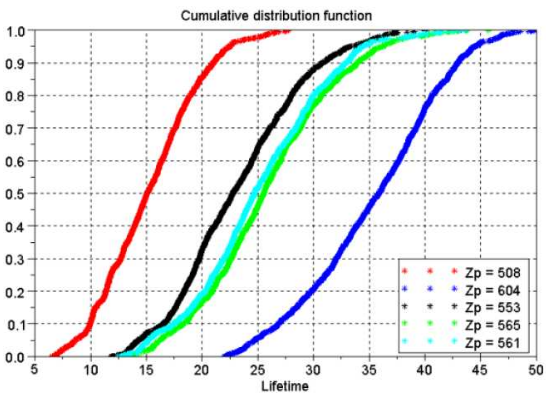

Figure 8 : Iterations on perigee altitude for an OLT_50% of 25 years

5. <u>Computation of the reentry duration of the end of life orbit by using a constant solar activity.</u>

In our example the end of life orbit (Zp,Za) = (561 km, 800 km) has a 25 years lifetime in 50% of the cases of random solar activity. We are now looking for the constant equivalent solar activity that also leads to a 25 years reentry duration. The initial date and local time of the ascending node remain constant for now (the day of the year has no significant influence on the orbit lifetime and we will see later the sensitivity to local time). The solar flux $F_{10.7}$ has a stronger influence in the computation of the atmospheric density than the geomagnetic index Ap and we need only one degree of freedom for the computation of the constant equivalent solar activity (the use of two degrees of freedom brings no additional advantage). That is why the geomagnetic index has been set to the mean value 15 for the constant equivalent solar activity. Only the solar flux value $F_{10.7}$_50% is then tuned.

We can then define the function G that gives the difference between the actual orbit lifetime OLT and the target one (ie 25 years) versus the constant solar flux value used for the computation : $G(F_{10.7}\_50\%) = OLT-25$ years. We are looking for the zero of G which is easily computable through a brute force fashion or a Brent algorithm.

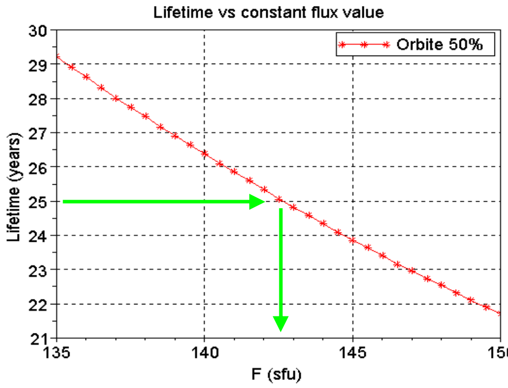

Figure 9 : $F_{10.7}$_50% value computation

In our example the solar flux value that leads to an OLT of 25 years is about 142 sfu, so the constant equivalent solar activity is :

$$\begin{cases} F_{10.7}\_50\% = 142 \text{ sfu} \\ Ap = 15 \end{cases}$$

6. Study of the sensitivity of the $F_{10.7}$_50% value to initial parameters

We have studied the influence of :

- the inclination of the initial orbit,
- the apogee altitude of the initial orbit (the perigee altitude is the degree of freedom of the algorithm),
- the ballistic coefficient of the space object,
- the local time of the ascending node.

Starting from our example we will see how these parameters can change the solar flux value that we have previously found.

We can see the influence of the constant solar flux value on the perigee altitude to reach to achieve a 25 years reentry, see Fig. 10. For an area to mass ratio of 0.01 the perigee altitude to reach will change by 2 km when the constant solar flux value change by 1 sfu. Let’s keep this value in mind to decide whether the change in the solar flux value is significant or not. For small variations of the solar flux we sometimes decided to adopt a worst case approach which led us to take into account the lower values of the solar flux. Indeed, using a lower solar activity leads to a lower perigee altitude to achieve a 25 years lifetime ie a larger manoeuvre cost.

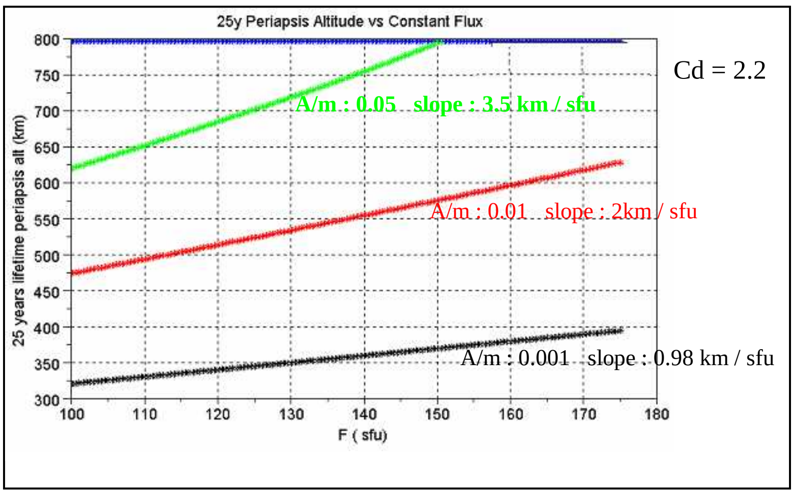

Figure 10 : Perigee altitude sensitivity to constant solar flux value

- *Inclination*

The very same computation of the constant equivalent solar flux value was performed but with different initial values of the orbit's inclination. The results are plotted Fig. 11.

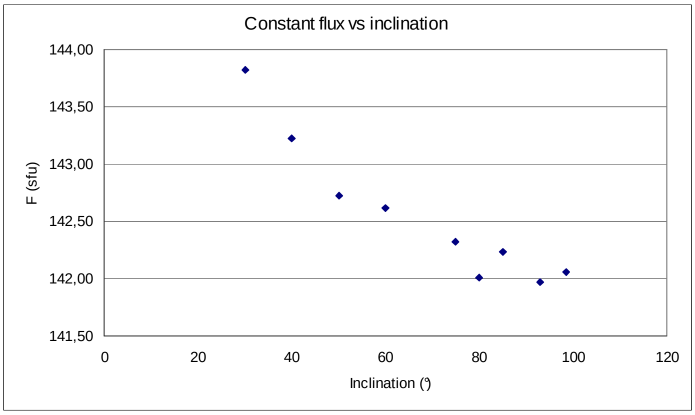

Figure 11 : $F_{10.7}$_50% sensitivity to inclination

It appears that the inclination does not have a strong influence on the $F_{10.7}$_50% values : less than 2 sfu. Moreover most of the space objects in Low Earth Orbits are situated in the inclination range [60°; 100°]. That is why it was decided to focus on quasi-polar orbits, which is equivalent to a worse case approach since it gives us lower value of the solar flux. Using the equivalent constant solar activity for non polar orbit will lead to a end of life orbit with a lifetime of 25 years in a little bit more than 50% of the cases of future solar activities.

- *Apogee altitude*

The very same computation of the constant equivalent solar flux value was performed but with different initial values of the apogee altitude, up to the LEO altitude limit. The results are plotted Fig. 12.

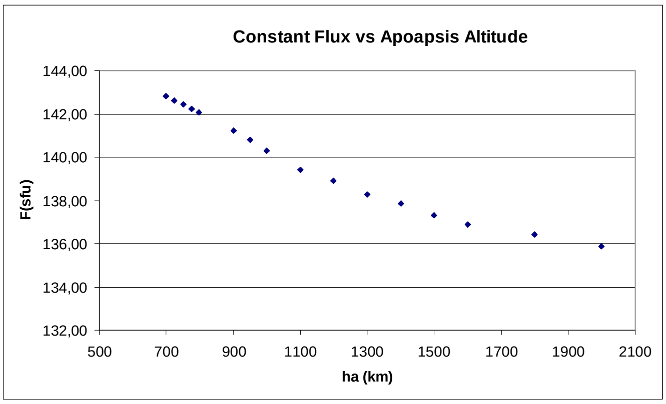

Figure 12 : $F_{10.7}$_50% sensitivity to apogee altitude

The solar flux value changes with the initial apogee altitude. The dependency of $F_{10.7}$_50% versus the apogee altitude follows a log-function with a coefficient of determination R^2> 99%.

- *Ballistic coefficient*

The very same computation of the constant equivalent solar flux value was performed but with different ballistic coefficient : the drag coefficient is constant (this choice has been done to decouple the solar flux computation from a specific law Cd = f(altitude) with a C_d value of 2.2 and the area to mass ratio goes from

10^{-3} to 3.10^{-2}, that covers most of the spacecrafts in LEO. The computations are done for two different apogee altitudes : 800 km and 1100 km. The results are plotted Fig. 13.

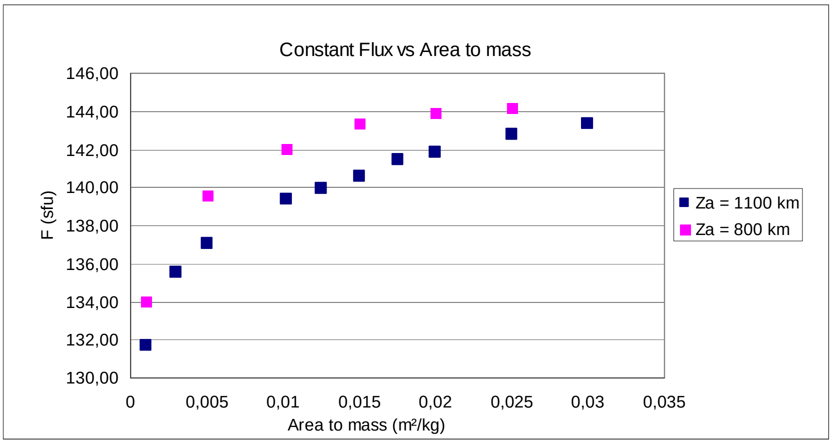

Figure 13 : $F_{10.7}$_50% sensitivity to ballistic coefficient

The solar flux value changes with the ballistic coefficient. The dependency of $F_{10.7}$_50% versus the ballistic coefficient follows a log-function with a coefficient of determination R^2>99%.

- *Local time*

The sensitivity of the solar flux values to the local time of the ascending node of the end of life orbit was studied. Let us notice that most of the end of life orbit reached by a space object are not purely sun synchronous since the disposal manoeuvres change the orbital parameters. However we have to take into account the cases without disposal manoeuvres (for example when using braking airbags) that let the object on its operational orbit which can be sun synchronous.
Fig. 14 shows the constant equivalent solar flux variations versus the initial mean local time for quasi SSO and SSO.

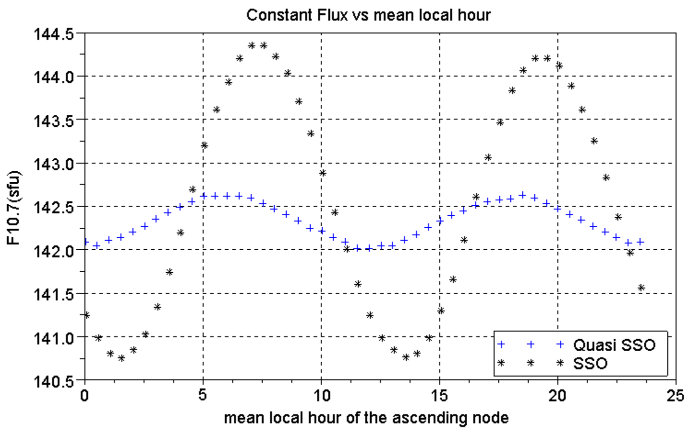

Figure 14 : $F_{10.7}$_50% sensitivity to mean local time

It appears that the $F_{10.7}$_50% variations are about 1 sfu for quasi SSO and 4 sfu for SSO.
It was decided to compute the solar flux values only for initial sun synchronous orbits (that leads to quasi sun synchronous disposal orbits) and to adopt a worst case approach by performing the $F_{10.7}$_50% computation with several local times and keeping the minimun value. By using the constant equivalent solar activity we then have :

- for quasi sun synchronous end of life orbit, a conservative result : the orbit lifetime will be 25 years in more than 50% of the cases of future solar activity whatever the local time is,
- for sun synchronous end of life orbit, a mean result : the orbit lifetime will be 25 years in about 50% of the cases of future solar activity depending on the initial local time.

Summary for the use of the "constant equivalent solar activity"

Finally, the constant equivalent solar activity is defined as follows :

$$\boxed{\begin{aligned} & AP = 15 \\ & F_{10.7} = 201 + 3.25\ \ln(\frac{SC_d}{m}) - 7\ln(Z_a) \end{aligned}} \qquad (5)$$

- $F_{10.7}$ in sfu
- SC_d/m : ballistic coefficient (m^2/kg)
- Z_a : apogee radius (mean parameter) minus Earth radius (km)

Note that for extrapolation using a variable drag coefficient vs altitude a constant value of C_d has to be used to compute the constant equivalent solar activity. In this case a C_d value of 2.2 has been chosen as a "good practice".

Fig. 15 shows a plot of $F_{10.7}$ values for several ballistic coefficients and apogee altitudes.

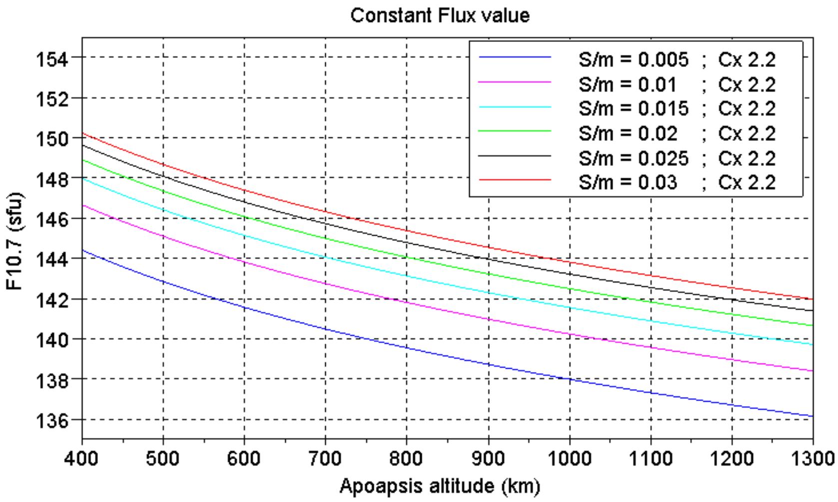

Figure 15 : Constant equivalent flux values

The use of the constant equivalent solar activity approach to compute the 25-years lifetime end of life orbit :

- removes the sensitivity of the disposal maneuver cost to the level and the length of future solar cycles. The end of mission date can shift without questioning the disposal orbit and maneuver strategy,
- brings the information of a mean 25 years reentry duration for the whole space objects using this method.

4. Implementation of the dynamical models

A reference implementation of the dynamical models in a freely downloadable tool has been developed: STELA (Semi-analytic Tool for End of Life Analysis software).

The main requirements for this software were:

- to be able to propagate the orbit in an efficient (from a computation time point of view) but precise enough way : semi-analytical models adapted to each orbit type have been developed,
- to allow a flexible use of the tool: as a reference and official tool to check the criteria of the French Space Act but also usable more widely in mission design and analysis studies,
- to be easily usable on different operating systems (-> use of java language and java-based COTS).

The semi-analytical theory models:

This paragraph describes the main points of the dynamic modeling that is on the basis on the equations of motion that are propagated within the STELA s/w. Further detail can be found in (Deleflie, 2010a and 2010b), and two further publications (devoted to LEO and GEO orbits, respectively, and describing the whole dynamical modeling which are in preparation). To ensure reasonable CPU integration times, even over very long time scales of the order of 200 years, the long time scale analysis is based on the numerical integration of equations of motion, where the short periodic terms have been removed by means of an analytical averaging. This allows to use a very large integration step size, reducing significantly the total time of computation. The numerical integration algorithm is a Runge Kutta of order 6; we have shown, thanks to comparisons with classical numerical integrations in various dynamical configurations that an integration step size of the order of 24 hours is consistent with the requirement of the STELA s/w. The averaging approach follows methods developed in the theory of mean orbital motion (Metris et al., 1995), and derived for orbits with very small eccentricities, removing all divisions by the eccentricity in the mean equations of motion (Deleflie, 2005). The orbital modeling accounts for all significant perturbations, but, again, to ensure reasonable computation times, only the significant perturbations have been built in the STELA s/w: zonal terms of the geopotential (J2, J3, J4), tesseral resonant parameters up to degree and order 4, luni-solar perturbations (up to degree 4 for geostationary orbits), solar radiation pressure and atmospheric drag. Let us mention that an additional term gathering up the influence of even and odd zonal parameters from degree 5 to degree 15 (Ref [15]) have been inserted in the equations of motion governing the eccentricity vector, to model properly the dynamical properties of the motion not far from the critical inclination: the secular effect on the argument of perigee vanishes, the effects of odd zonal parameters become preponderant, leading to an eccentricity growth that makes the satellite cross regions of high atmospheric density around the perigee, reducing significantly the lifetime.

For the LEO type orbits, studied first, the equations have been formulated through the following set of orbital elements : $E = (L = \sqrt{\mu a}, C = e\cos\omega, H = L\sqrt{1-e^2}\cos i, \lambda = \omega + M, S = e\sin\omega, h = \Omega)$. It leads to a singularity for equatorial orbits.

For the GEO type orbits all the equations have been formulated through a set or orbital elements that is suitable to describe the motion for orbits with a small inclination and/or a small eccentricity: $E = (a, \Omega + \omega + M, e\cos(\Omega + \omega), e\sin(\Omega + \omega), \sin {}^{i}\!/_{2}\cos\Omega, \sin {}^{i}\!/_{2}\sin\Omega)$.

The corresponding perturbation equations have been written, namely the Planetary Lagrange equations (for perturbations deriving from a potential: internal gravity field, luni-solar perturbations, solar radiation pressure), and the Gauss equations (for the atmospheric drag). In both cases, the averaged forces over the rapid variable is inserted into the equations of motion, which are, consequently, mean equations of motion. The mean potential $\overline{U}$ are computed once for all in an analytical way, from the expression of the osculating potential: $\overline{U} = \frac{1}{2\pi}\int_0^{2\pi} U dM$. The mean drag effect on orbital parameters is evaluated through a Simpson quadrature method, considering *n* constants intervals in true anomaly along one orbit, the first and the last one being at perigee.

The integration Frame is the “Mean Equator and Equinox of Date” seen as inertial. The Earth true equator is considered as merged with the mean equator (that is, nutation and polar motion have been neglected). The Sun and Moon coordinates are estimated following the simplified analytical model described by MEUS (ref [7]).

Let us mention, furthermore, that an explicit analytical transformation from osculating to mean elements, and conversely, has been expressed, through the set of orbital elements E, and containing all short periodic terms in J2. This first-order analytical transformation will be the subject of a further and dedicated publication, and was required for two main points of the validation of the STELA s/w: first, to deduce mean initial conditions for STELA from osculating initial conditions used for the numerical integration (the transformation between osculating to mean elements is used at this step) ; secondly to compute the altitude when evaluating the drag effect at each point of the Simpson quadrature; thirdly to compute the altitude which is required to check the C1 to C4 criteria; fourthly to provide STELA outputs that can be directly compared to osculating elements deduced from numerical integration: short periodic terms in J2 have to be added to the mean elements.

Validation

The implementation of the dynamical model was validated by comparison with CNES reference numerical propagators (PSIMU, ZOOM) which take into account a complete model of perturbations (except tidal effects which are negligible).

The validation domain is the following:

- any LEO type orbit (see definition above), including orbits at critical inclination,
- any GEO type orbits (see definition above).

Tests cases covering the whole possible orbital parameter range were run and the evolution of orbital parameters were compared. For LEO orbits key parameters were the perigee altitude evolution vs time and the reentry duration. For GEO orbits key parameters were the perigee and apogee altitudes and the inclination evolution vs time.

STELA User's guide (ref [10]) gives more details about the validation range (orbit parameters, propagation duration, $S.C_d/m$ and $S.C_R/m$).

For LEO orbit, the 25 years reentry duration is obtained with a precision of about 1% by comparison with a full model.

For GEO orbits the minimum and maximum altitudes are obtained with a precision of less than 5 km over 100 years by comparison with a full model. A large part of this error is linked to the current short period model which can be improved.

The following curves show typical LEO and GEO results.

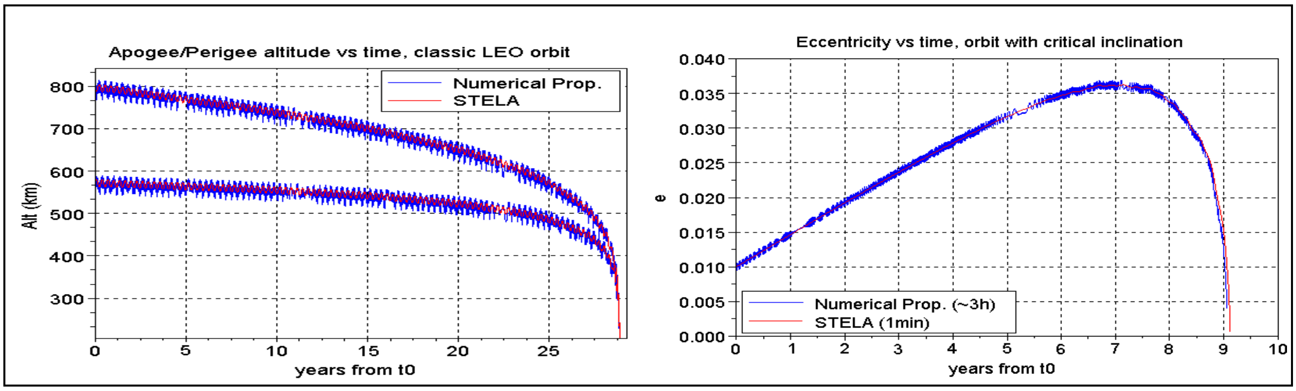

Figure 16 : Apogee/perigee evolution of a classic LEO orbit, eccentricity evolution for a LEO orbit at critical inclination

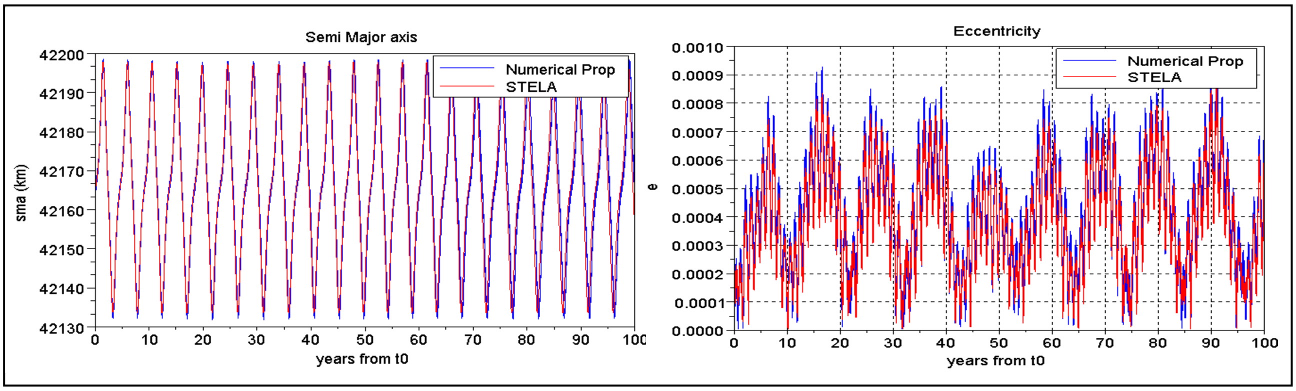

Figure 17 : Semi major axis and eccentricity evolution of a GEO orbit

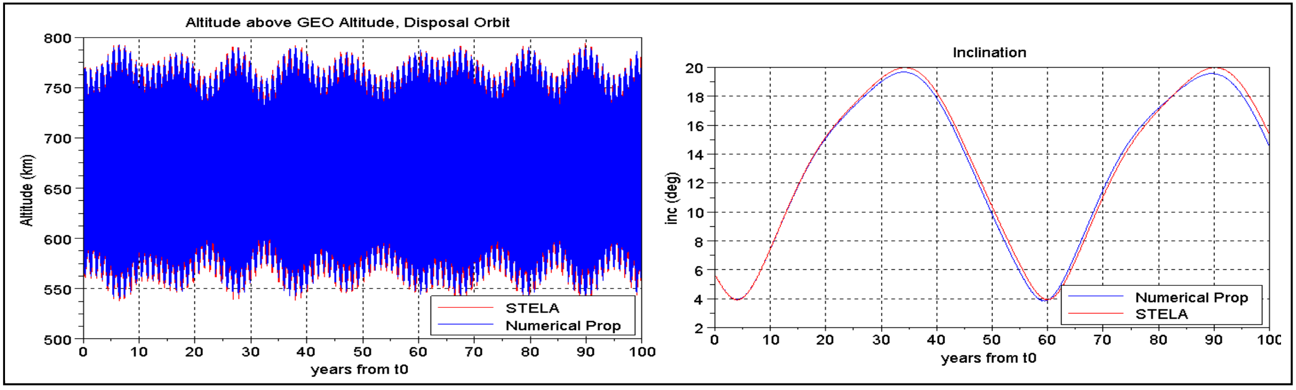

Figure 18 : Altitude and inclination evolution on a disposal orbit above GEO altitude

<u>STELA functionalities:</u>

STELA allows in its current version to propagate LEO types and GEO types orbits.
The C1 to C4 criteria are checked to give an official status in the frame of the French Space Act.
STELA software also includes:

- an iterative computation mode adjusting the initial orbit to achieve a given atmospheric reentry duration, or to avoid GEO region crossing during a given period,
- a tool that computes the mean cross sectional area of a spacecraft.

It is usable in GUI mode, batch mode and as a library.

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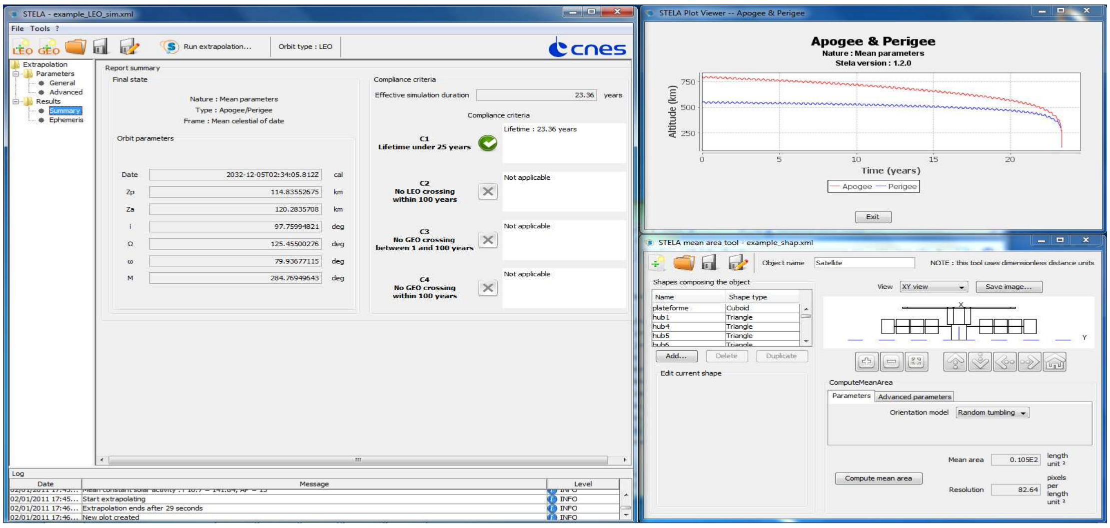

Figure 19 : STELA's GUI

STELA can be downloaded on CNES website: http://logiciels.cnes.fr/STELA/

5. Next step

The next step (on going activities) is to define good practices to deal with Geostationary Transfer Orbits and to develop and implement in STELA a semi analytical model adapted to this orbit type. A well known difficulty to develop the semi analytic model is the high eccentricity value that does not allow classical approach based on series depending on this parameter. An other difficulty for GTO crossing the LEO region is the high sensitivity of reentry duration to initial conditions due to Sun-Moon perturbation (see ref [9]).

6. Acknowledgement

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