SPIN STABILIZED SATELLITE'S ATTITUDE PROPAGATION WITH QUATERNIONS

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Abstract: This paper presents a study of a modeling scheme for the spin stabilized satellites attitude, entirely developed in terms of quaternion parametrization. The analysis includes numerical propagation of the rotational motion equation, considering the influence of the following torques: aerodynamic, gravity gradient, residual magnetic, eddy currents and the one due to the Lorentz force. Applications are developed considering the Brazilian Spin Stabilized Satellites SCD1 and SCD2, which are quite appropriated for verification and comparison of the theory with the real data generated and processed by the INPE's Satellite Control Center (SCC). The results show that for SCD1 and SCD2 the influence of the eddy current torque is bigger than the others ones, not only due to the orbit altitude, but also to other specific satellites characteristics. The influence of the torque due to Lorentz force is smaller than the others ones because of the dimension and the electrical charges of the SCD1 and SCD2. In all performed tests the errors remained within the dispersion range specified for the attitude determination system of INPE's SCC. The results show the feasibility of using the quaternion attitude parameterization for modeling the satellite dynamics of spin stabilized satellites.

Keywords: *attitude quaternion, spin velocity, external torques, numerical simulation, pointing deviation.*

1 Introduction

The objective of this paper is to analyze the attitude of spin stabilized satellites, entirely developed in terms of quaternion parametrization. The analysis includes numerical propagation of the rotational motion equation, considering the influence of the following torques: aerodynamic, gravity gradient, residual magnetic, eddy currents and the one due to the Lorentz force.

The gravity gradient torque is generated by the difference of the Earth gravity force direction and intensity actuating on each satellite mass element. This torque is inversely proportional to the cube of the satellite geocentric distance. The aerodynamic torque is created by the interactions of rarefied air particles with the satellite surface and it has the predominant orbit perturbation effect in LEO orbit satellites. In this paper TD-88 model is used to describe the atmospheric density. The residual magnetic torque results from the interaction between the spacecraft's residual magnetic moment and the Earth magnetic field and its main effect is to produce a spin axis orientation drift. On the other hand, the main effect of the eddy current torque is to produce a reduction in the satellite spin rate with time. The torque due to Lorentz force is associated with a rigid spacecraft equipped with an electrostatically charged protective shield, having an intrinsic magnetic moment. The main element of this shield is an electrostatically charged screen surrounding the protected volume of the spacecraft. This torque depends on the Earth's magnetic field, the form of the satellite shell, the satellite spin rate, the angular velocity of the diurnal rotation of the geomagnetic field altogether with the Earth and the electrical charge of the satellite. In this paper the dipole model is assumed for the Earth's magnetic field and the satellite is supposed to be in an elliptical orbit.

A mathematical model is presented for each considered torque in terms of quaternion attitude parametrization.

The numerical propagation of the equations of rotational motion shows the evolution of the components of the angular velocity vector and the four components of the attitude quaternion. The influences of the Earth oblateness in the orbital elements are taken into account. Applications are developed considering the Brazilian spin stabilized satellites SCD1 and SCD2, which are quite appropriated for verification and comparison of the theory with the real data generated and processed by the INPE's Satellite Control Center (INPE's SCC). A spherical coordinate system fixed in the satellite is used to locate the satellite spin axis in relation to the terrestrial equatorial system. The spin axis direction is specified by its right ascension and the declination angles. The time evolution of the spin axis right ascension and declination angles is gotten from the numerical results of the quaternion attitude propagation.

An initial approach is presented, in which the propagated attitude is daily updated with the help of real satellite data, supplied by INPE's SCC. A second approach is also presented, where daily updates of the attitude data has not been performed in the propagation process.

The results of this analysis can be useful for the Brazilian mission satellite.

2. External Torques Model

2.1 Gravity Gradient Torque

The gravity gradient torque [9,12] for a spacecraft can be modeled by:

$$\vec{M} = 3\frac{\mu}{r'^3}\left[a_{21}a_{31}\left(I_z - I_y\right)\hat{e}_x + a_{11}a_{31}\left(I_x - I_z\right)\hat{e}_y + a_{11}a_{21}\left(I_y - I_{xx}\right)\hat{e}_z \right], \qquad (1)$$

where μ (3.986×10^{14} m³/s²) is the Earth gravitational parameter, r' is the satellite geocentric distance, a_{11}, a_{21} and a_{31} are the direction cosines which relate the orbital system and the satellite fixed system (the latter being associated with the principal moments of inertia axes of the satellite), I_x, I_y, I_z are the Principal Moments of Inertia of the satellite and $\hat{e}_x, \hat{e}_y, \hat{e}_z$ units vectors of the satellite fixed system. The elements a_{11}, a_{21} and a_{31} depend on the orbital elements (orbit inclination, true anomaly, longitude of the ascending node and argument of the perigee) and the attitude quaternion [12,14]. In this study the z-axis corresponds to the long axis of cylinder. Equation (1) shows that this torque decreases with the cube of the altitude and depends on the shape, dimension and mass distribution of the satellite. If the satellite has a uniform mass distribution and the principal moments of inertia are equal, this torque vanishes. The gravity gradient torque magnitude has short oscillations due to the satellite rotational motion [12] when the influence of rotational motion is included in the direction cosines a_{11}, a_{21} and a_{31}.

2.2 Aerodynamic Torque

When the satellites move in the tenuous layers of the upper atmosphere, the interactions of the molecular stream with a satellite's surface produces a torque about the center of mass. For spacecraft below approximately 400 km, the aerodynamic torque is the dominant environmental disturbance torque [10, 12]. The end of a spacecraft mission often occurs when the aerodynamic torques becomes so great as the spacecraft reenters the Earth's atmosphere that the attitude control systems ceases to become effective and the spacecraft tumbles.

In this paper we will adopt to represent the aerodynamic torque the following model [5]:

$$\overrightarrow{N_A} = \overrightarrow{m_e} \times \vec{D}, \tag{2}$$

Where $\overrightarrow{\boldsymbol{m_e}}$ is the position vector between the center of pressure and the center of mass, the $\vec{\boldsymbol{D}}$ is the drag force (in this paper the influence of the lift force in the aerodynamic torque is negligenciable) and in the satellite fixed system it is given by:

$$\vec{D} = D_x \hat{e}_x + D_y \hat{e}_y + D_z \hat{e}_z, \tag{3}$$

with

$$D_x = -D[a_{11} \cos(\gamma_S) + a_{12} sen(\gamma_S)], \tag{4}$$

$$D_y = -D[a_{21} \cos(\gamma_S) + a_{22} sen(\gamma_S)], \tag{5}$$

$$D_z = -D[a_{31} \cos(\gamma_S) + a_{32} sen(\gamma_S)], \tag{6}$$

$$D = \frac{1}{2} \rho v^2 S C_D \quad . \tag{7}$$

where ρ is the local density, v represents the magnitude of the satellite's velocity relative to the atmosphere, S is a reference section area of the satellite, C_D is the Drag Coefficient, γ_S is the angle between the position vector and the orbital velocity vector and a_{ij} , i=1,2,3, j=1,2, are the direction cosines which relate the orbital system and the satellite fixed system and depend on the orbital elements and the attitude quaternion [14]. An analysis concerning the uncertainties and usual values of some of these parameters can be found in[5].

Then by substituting the Eq. 3 in Eq. 2, the aerodynamic torque in the satellite fixed system is given by:

$$\vec{N}_A = [D_z me_y - D_y me_z]\hat{e}_x + [D_x me_z - D_z me_x]\hat{e}_y + [D_x \, me_z - D_z \, me_x]\hat{e}_z \; . \tag{8}$$

In order to estimate the influence of the aerodynamic torque magnitude in the rotational motion, in this paper some simplifications are done and the thermosphere model TD-88 is used for the atmospheric density [7]. The velocity v is assumed equal to the orbit velocity and the drag coefficient is fixed. The thermosphere model TD-88 is defined for the height range of 150 – 750 km. According to [7], the atmospheric density on a surface of constant altitude can be described by the expression

$$\rho = f_x f_0 k_0 \sum_{n=1}^{7} h_n g_n \tag{9}$$

where

$$h_n = K_{n,0} + \sum_{j=1}^{3} K_{n,j} \, exp\left[\frac{(120-(r'-r_t))}{29j}\right]. \tag{10}$$

Here, r_t is Earth's Equatorial radius; $K_{n,j}$ are numerical constants; f_x and f_0 depend on the solar flux and k_0 depends on the geomagnetic index. The terms g_n (describing mean density, individual dependence on the mean solar flux, North-South asymmetry, anual, semi-anual, diurnal and semidiurnal variations) are functions of the day count of the year, of the local time, of the latitude, and numerical constants which are summarized in tables [7].

2.3 Magnetic Torques

Magnetic disturbance torques result from the interaction between the spacecraft's residual magnetic field and the Earth´s magnetic field. In this paper it is assumed that the spacecraft is manufactured from material such that the primary sources of magnetic torques are the spacecraft magnetic moments and eddy currents with other sources negligible. The spacecraft's magnetic moment is usually the dominant source between the disturbances torques [9]. If $\vec{m}$ is the magnetic moment of the spacecraft and $\vec{B}$ is the geocentric magnetic flux density, the residual magnetic torques is given by [4, 11, 12]:

$$\vec{N}_r = \vec{m} \times \vec{B}. \tag{11}$$

The torque induced by eddy currents is caused by the spacecraft spinning motion. It is known [4,11] that the eddy currents produce a torque which causes the precession in the spin axis and causes an exponential decay of the spin rate. If $\vec{W}$ is the spacecraft's angular velocity vector and p is a constant coefficient which depends on the spacecraft geometry and conductivity, this torque is given by:

$$\vec{N}_i = p\ \vec{B} \times (\ \vec{B} \times \vec{W}\ ). \tag{12}$$

Here the magnetic torque is developed only for a spin-stabilized satellite. In this case, the spacecraft's angular velocity vector and the satellite magnetic moment are along z-axis and the residual magnetic torque and induced eddy currents can the expressed in the satellite fixed system by [4]:

$$\vec{N}_r = -m B_y \hat{e}_x + m B_x \hat{e}_y\ , \tag{13}$$

$$\vec{N}_i = p\, W \left(B_x B_z \hat{e}_x + B_y B_z \hat{e}_y - (B_x{}^2 + B_y{}^2)\, \hat{e}_z \right), \tag{14}$$

where B_x, B_y, B_z are the components of the geomagnetic field in the satellite fixed system [13]. These components are obtained in terms of the geocentric inertial components of the geomagnetic field [8] and the attitude quaternion of the satellite [13]. In order to describe the geomagnetic field the dipole vector model [8,11] is used.

2.4. Torque due Lorentz force

In this paper we will adopt the following model to represent the torque due Lorentz force [1]:

$$\overrightarrow{M_L} = \overrightarrow{W} \times \overrightarrow{\beta_S} + \overrightarrow{w_g} \times \overrightarrow{\beta_S}, \tag{15}$$

where $\overrightarrow{W}$ is the spin velocity vector of the satellite; $\overrightarrow{w_g}$ is the vector of the spin velocity of the diurnal rotational of the geomagnetic field together with the Earth, with the direction given by the unit vector $\hat{\beta}$ and

$$\overrightarrow{\beta_S} = S\hat{\beta}\ , \qquad S = \begin{pmatrix} 4Dd^2 & 0 & 0 \\ 0 & 4Dd^2 & 0 \\ 0 & 0 & 4Dh^2 \end{pmatrix}, \qquad \text{and} \qquad D = \frac{QB}{4r'^3}, \tag{16}$$

where d and h are the diameter and height of the cylindrical charged manta around the satellite, B is the magnitude of the geomagnetic Field, r' is the satellite geocentric distance and Q is the satellite's electrical charge. The geomagnetic field is described by the dipole vector model [8,11].

After the development of the Eq. 15, the components of the torque due Lorentz force in the satellite fixed satellite is [3]:

$$\vec{M}_L = M_{Lx}\hat{e}_x + M_{Ly}\hat{e}_y + M_{Lz}\hat{e}_z\ , \tag{17}$$

with

$$M_{Lx} = Dh^2\beta_{3P}(q + w_g\beta_{2P}) - Dd^2\beta_{2P}(r + w_g\beta_{3P}), \tag{18}$$

$$M_{Ly} = Dd^2\beta_{1P}(r + w_g\beta_{3P}) - Dh^2\beta_{3P}(p + w_g\beta_{1P}), \tag{19}$$

$$M_{Lz} = Dd^2(p\beta_{2P} - q\beta_{1P}), \tag{20}$$

where p, q, r are the components of the satellite spin velocity $\overrightarrow{W}$ in the satellite fixed system and β_{1P}, β_{2P}, β_{3P} depend on the components of the $\hat{\beta}$ in the equatorial system and the attitude quaternion.

3. Equations of rotational motion in terms of the quaternion

The dynamic equations of the satellite's rotational motion are described by the Euler equations and the kinematic equations for the attitude quaternion. The Euler equations give the tax of variation of the components of the satellite's spin velocity and depend on the components of the external torques in the satellite fixed system [6,10,13]:

$$\begin{aligned} \dot{p} &= q\,r\,(I_y - I_z)/I_x + N_x/I_x, \\ \dot{q} &= p\,r\,(I_z - I_x)/I_y + N_y/I_y, \\ \dot{r} &= p\,q\,(I_x - I_y)/I_z + N_z/I_z. \end{aligned} \tag{21}$$

In these equations I_x, I_y, I_z are the Principal Moments of Inertia of the satellite, p,q,r and N_x, N_y, N_z are the components of the spin velocity and the external torques in the satellite fixed system, respectively.
In this paper the kinematic equations are described in terms of the attitude quaternion q. The quaternion q is a vector 4x1 given by:

$$q = [q_1 \quad q_2 \quad q_3 \quad q_4]^t = [\vec{q} \quad q_4]^t\ , \tag{22}$$

where t represents the transposed of the matrix. It is usual to call the vector $\vec{q}$ as the vector component of the quaternion and q_4 the scalar component of the quaternion. They can be expressed in function of the rotation angle (ϕ) and of the axis of rotation $\vec{n}$ in agreement it consists below:

$$\vec{q} = [q_1 \quad q_2 \quad q_3]^t = sin(\phi/2)\,\vec{n} \quad and \quad q_4 = cos(\phi/2). \tag{23}$$

It is ease to prove that the module of the quaternion is 1, since $\vec{n}$ is a unitary vector in the direction of the spin velocity vector. The matrix of attitude in terms of the quaternion is presented in [6]. The kinematic equations that describe the tax of variation of the components of the quaternion of attitude, due to rotation of the satellite, are given by [6,13]:

$$\begin{aligned} \dot{q}_1 &= \frac{1}{2}[p\,q_4 - q\,q_3 + r\,q_2], & \dot{q}_2 &= \frac{1}{2}[q q_4 - r q_1 + p q_3], \\ \dot{q}_3 &= \frac{1}{2}[r q_4 - p q_2 + q q_1], & \dot{q}_4 &= -\frac{1}{2}[p q_1 + q\,q_2 + r q_3]. \end{aligned} \tag{24}$$

As it can be observed, the kinematic equations in terms of the attitude quaternion are linked and depend on the components (*p, q, r*) of the spin velocity, given for the Eq. 21 making it difficult to get an analytical solution. Then in this paper a numerical propagation for the Eq. 21 and Eq. 24 is realized, using *Runge Kutta* method and the language FORTRAN.

4. Applications for spin stabilized satellite

Spin stabilized satellites has the spin axis along the biggest principal moment of inertia's axis. A spherical coordinates system fixed in the satellite is used to locate the spin axis of the satellite in relation to the equatorial system. The direction of the spin axis is specified by its right ascension (α) and the declination (δ) which are represented in Fig. 1.This spherical coordinate can be obtained using the attitude quaternion *q* and the components of the spin velocity *W*. If the units vectors are associated with the equatorial system and the units vectors $\hat{e}_x, \hat{e}_y, \hat{e}_z$ of the satellite fixed system then the spin velocity vector can be expressed by:

and (25)

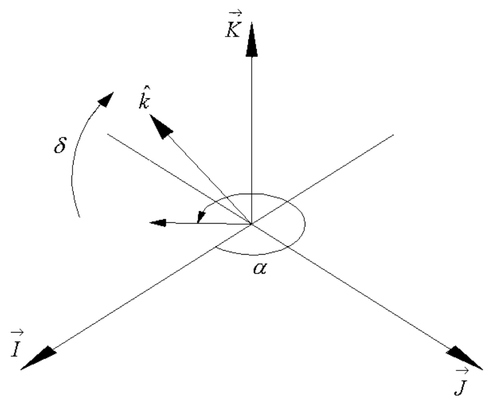

Figure 1 –Equatorial System ($\hat{I}, \hat{J}, \hat{K}$), spin axis orientation ($\hat{k}$), right ascension (α) and declination (δ) of the spin axis.

If the components of the attitude quaternion and components of the satellite spin velocity (*p, q, r*) are known, the vectors and are related by [13]:

$$
\begin{aligned}
P &= (q_1^2 - q_2^2 - q_3^2 + q_4^2)p + 2q(q_2 q_1 - q_4 q_3) + 2r(q_3 q_1 + q_4 q_2) \\
Q &= 2p(q_1 q_2 + q_4 q_3) + (-q_1^2 + q_2^2 - q_3^2 + q_4^2)q + 2r(q_3 q_2 - q_4 q_1) \\
R &= 2p(q_3 q_1 - q_4 q_2) + 2q(q_3 q_2 + q_4 q_1) + (-q_1^2 - q_2^2 + q_3^2 + q_4^2)r
\end{aligned}
\tag{26}
$$

and the magnitude of the spin velocity is given by:

$$
W = (p^2 + q^2 + r^2)^{\frac{1}{2}}. \tag{27}
$$

According to Fig. 1, the components of spin velocity P. Q, R can be obtained by:

$$
P = W \cos\ \delta\ \cos\ \alpha, \quad Q = W \cos\ \delta\ \sin\ \alpha, \quad R = W \sin \delta. \tag{28}
$$

Then the right ascension (α) and declination (δ) of the spin velocity can be computed by:

$$
\sin \delta = \frac{R}{W}, \quad \cos \propto = \frac{P}{W \cos \delta}, \quad \sin \alpha = \frac{Q}{W \cos \delta} \tag{29}
$$

with $0 \leq \delta \leq 90°$ and $0 \leq \alpha \leq 360°$.

In this paper the right ascension and declination of the spin velocity will be computed using the numerical results obtained for components of the spin velocity and attitude quaternion by the numerical simulation of the Eq. 21 and Eq. 24.

5. Pointing Deviation

For the tests it is important to observe the deviation between the actual attitude data supplied by INPE and the computed attitude, for each satellite. Here this deviation is called pointing deviation and given by the angle θ between the actual spin axis $\hat{k}$ and the computed spin axis $\widehat{k_c}$. It can be computed by [4]:

$$
\cos \theta = \hat{k} \cdot \widehat{k_c}, \tag{30}
$$

where ($\cdot$) indicates the scalar product between actual spin axis $\hat{k}$ and computed spin axis $\widehat{k_c}$.
The unit vectors $\hat{k}$ and $\widehat{k_c}$ can be obtained using the right ascension and declination of the spin axis as:

$$
\hat{k} = \cos \alpha \cos \delta\ \hat{I} + \sin \alpha \cos\ \delta\ \hat{J} + \sin \delta\ \hat{K}, \tag{31}
$$

$$
\widehat{k_c} = cos\ \alpha_c\ cos\ \delta_C \hat{I} + sin\ \alpha_c\ cos\ \delta_C\ \hat{J} + sin\ \delta_C\ \hat{K}, \tag{32}
$$

with α and δ supplied by INPE's SCC and α_c and δ_c computed by the presented theory.

6. Numerical Applications

The theory developed has been applied to the spin stabilized Brazilian Satellite (SCD1 and SCD2) for verification and comparison of the theory against data generated by the INPE's SCC. The 8th Runge Kutta method is used to determine the numerical solution for Eq. 21 and Eq. 24.
The numerical solutions give the components of the attitude quaternion and of the spin velocity, which are used to compute the spin velocity, right ascension and declination of the spin axis by using Eq. 27 and Eq. 29. Then these computed values are compared with real data supplied by INPE's SCC in order to check the precision of the presented theory. Also important to observe is the deviation between the actual spin axis and the computed spin axis, that is, the pointing deviation computed by Eq. 30.

Two approaches are presented. In the first one the propagated attitude is daily updated with the help of actual satellite data, supplied by INPE's SCC. In the second approach the daily updates of the attitude data has not been performed in the propagation process. In both approaches the orbital elements are updated, taking into account the main influences of the Earth oblateness.

Initial conditions for the attitude were taken from INPE supplied data [4,9,12].

6.1 First approach: daily updated data

Simulations for the SDC1 were made for 17 days. The results for the deviation between the computed values and actual values for right ascension, declination and spin velocity and the pointing deviation are shown in Tab.1 for the case that considers all torques actuating together. In the Fig. 2, Fig. 3, Fig. 4 and Fig. 5 are shown the results for the deviation between the computed values and actual values of the right ascension, declination and spin velocity and for the pointing deviation when it is considered each torque individually and all torques actuating together. In the Tab. 2 are shown the mean and standard deviation for each simulation.

Table 1 – Deviation between computed values and actual values when all torques actuating together for SCD1, with the daily updated data.

Day	Deviation in Right Ascension (degrees)	Deviation in Declination (degrees)	Deviation in Spin Velocity (rpm)	Pointing Deviation (degrees)
17/8/1993	0.000	0.000	0.000	0.000
18/8/1993	0.870	-0.347	-0.021	0.373
19/8/1993	0.662	-0.350	-0.016	0.366
20/8/1993	0.385	-0.358	-0.026	0.364
21/8/1993	0.154	-0.361	-0.033	0.362
22/8/1993	-0.079	-0.338	-0.034	0.338
23/8/1993	-0.232	-0.328	-0.061	0.33
24/8/1993	0.654	-0.172	-0.057	0.21
25/8/1993	-0.657	-0.308	0.010	0.332
26/8/1993	-0.759	-0.281	0.010	0.317
27/8/1993	-1.596	-0.031	-0.078	0.314
28/8/1993	-0.192	-0.452	0.057	0.454
29/8/1993	-1.041	-0.200	-0.020	0.292
30/8/1993	-1.068	-0.184	-0.027	0.288
31/8/1993	-1.110	-0.147	-0.033	0.276
1/9/1993	-1.163	-0.119	-0.022	0.274
2/9/1993	-1.178	-0.090	-0.025	0.268

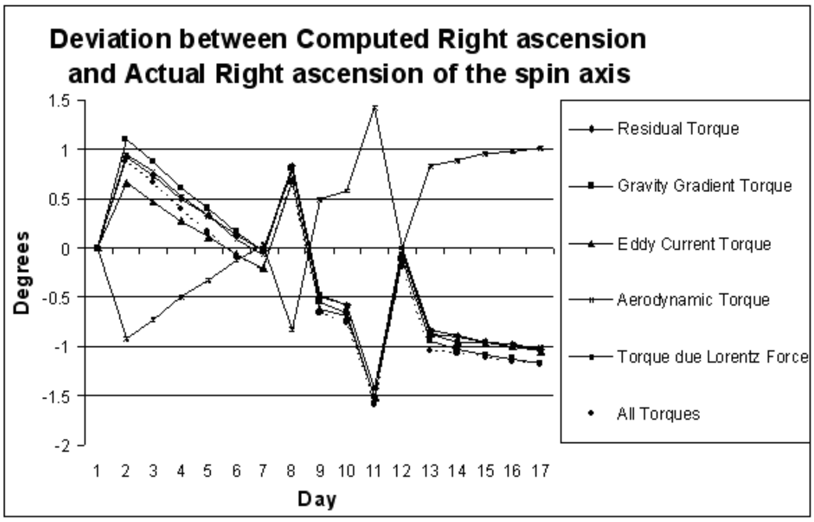

Figure 2 –Temporal variation for the deviation between the computed and actual right ascension of the spin axis for SCD1 and with the daily updated data

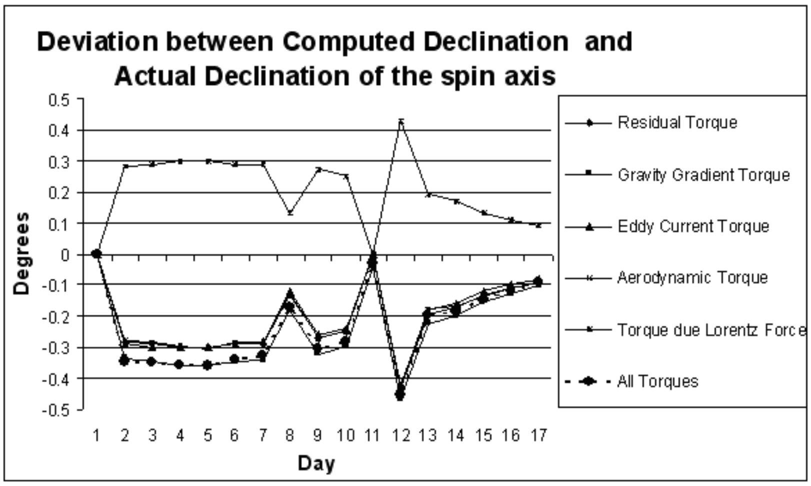

Figure 3 - Temporal variation for the deviation between the computed and actual declination of the spin axis for SCD1 and with the daily updated data

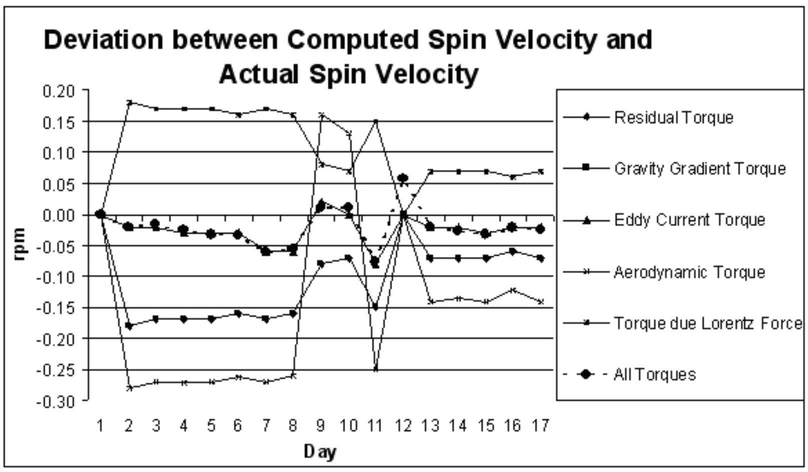

Figure 4 - Temporal variation for the deviation between the computed and actual spin velocity for SCD1 and with the daily updated data.

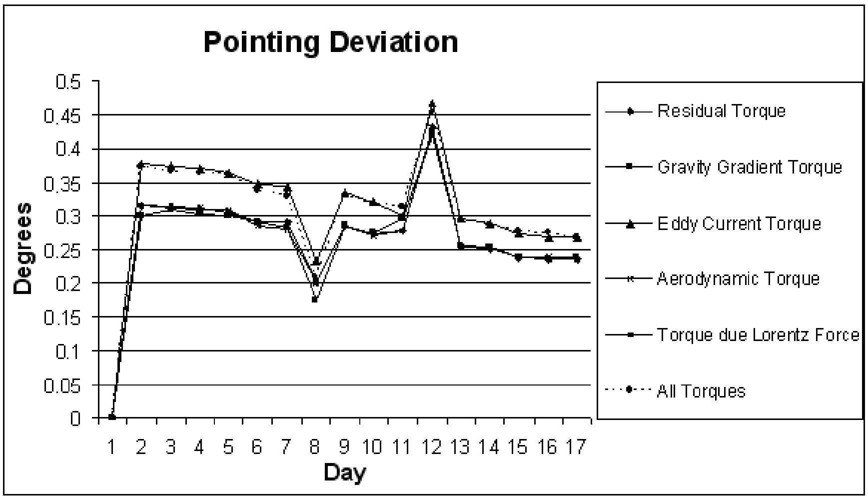

Figure 5 - Temporal variation for the pointing deviation for SCD1 and with the daily updated data.

For the test period of 17 days, the results shown that mean deviation error in right ascension, declination, spin velocity and pointing deviation are within the dispersion range of the attitude determination system performance of INPE’s control center, which is 0.5^o for the angles and 0.5rpm for the spin velocity.

The residual torque acts in the opposite direction of the torque due Lorentz force. The biggest influence in spin velocity and in the declination of the spin axis is given by eddy currents torque, with the mean deviation equal -0.025rpm and -0.202^o respectively.

Table 2 – Mean values for SCD1 simulations with the daily updated data.

Included Torques	Residual	Gravity Gradient	Eddy current	Aerod.	Due Lorentz force	All torques
Right Ascension Deviation (degrees)	-0.221	-0.251	-0.341	-0.225	0.221	-0.374
Declination Deviation (degrees)	-0.207	-0.247	-0.202	-0.205	0.207	-0.239
Spin Velocity Deviation (rpm)	-0.107	-0.107	-0.025	-0.148	0.107	-0.022
Pointing Deviation (degrees)	0.265	0.262	0.308	0.264	0.265	0.303

Simulations for the SDC2 were made for 16 days. The results for the deviation between the computed values and actual values for right ascension, declination and spin velocity and the pointing deviation are shown in Tab.3 for the case that considers all torques actuating together. In the Fig. 6, Fig. 7, Fig. 8 and Fig. 9 are shown the results for the deviation between the computed values and actual values of the right ascension, declination and spin velocity and for the pointing deviation when it is considered each torque individually and all torques actuating together. The discontinuities in these figures occur due to the attitude control corrections applied by SCC. In the

Tab. 4 are shown the mean and standard deviation for each simulation. It is important to note that when the attitude control actuates, the computed values are assumed to be equal the real data because the control system is not included in the proposed theory.
For the test period of 16 days, the mean deviation error are also within the dispersion range of the attitude determination system of INPE's control center. The biggest influence in spin velocity is given by eddy currents torque, with the mean deviation error equal -0.009 rpm. The biggest influence in the right ascension and declination of the spin axis is due gravity gradient torque, with the mean deviation error equal 0.001^o and 0.002^o respectively.

Table 3 – Deviation between computed values and actual values when all torques actuating together for SCD2, with daily updated data

Day	Deviation in Right Ascension (degrees)	Deviation in Declination (degrees)	Deviation in Spin Velocity (rpm)	Pointing Deviation (degrees)
12/2/2002	0.000	0.000	0.000	0.000
13/2/2002	0.023	-0.018	-0.012	0.021
14/2/2002	0.023	0.004	0.000	0.011
15/2/2002	0.029	0.012	-0.007	0.018
16/2/2002	0.031	0.018	0.004	0.023
17/2/2002	0.028	0.024	-0.004	0.027
18/2/2002	0.040	0.036	-0.004	0.04
19/2/2002	0.061	0.060	-0.002	0.066
20/2/2002	0.087	0.086	-0.002	0.094
21/2/2002	0.119	0.088	-0.001	0.103
22/2/2002	0.126	0.095	-0.002	0.111
23/2/2002	0.108	0.095	-0.013	0.107
24/2/2002	0.000	0.000	0.000	0.000
25/2/2002	0.059	-0.087	0.051	0.092
26/2/2002	0.038	-0.084	-0.093	0.086
27/2/2002	0.033	-0.096	-0.028	0.097

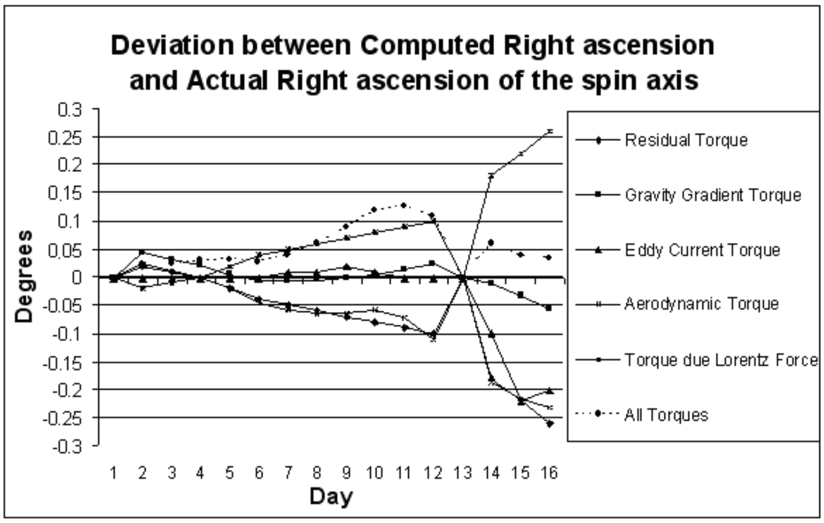

Figure 6 –Temporal variation for the deviation between the computed and actual right ascension of the spin axis for SCD2 and with the daily updated data

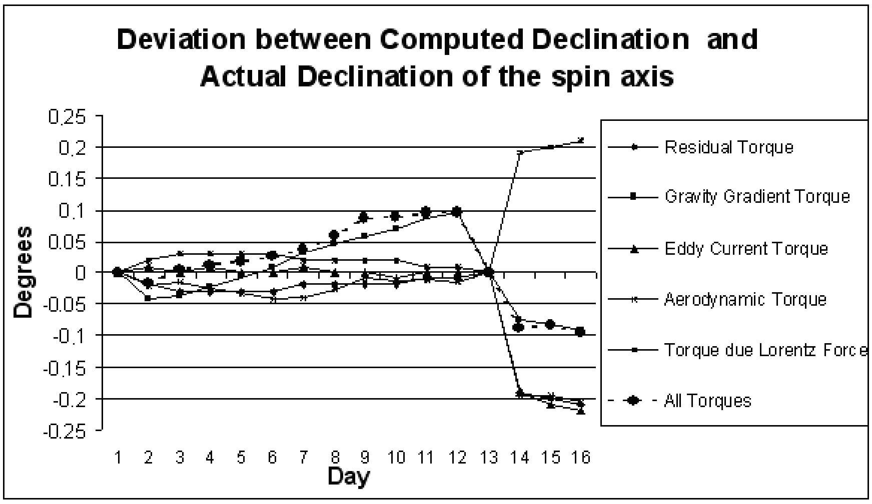

Figure 7 - Temporal variation for the deviation between the computed and actual declination of the spin axis for SCD2 and with the daily updated data.

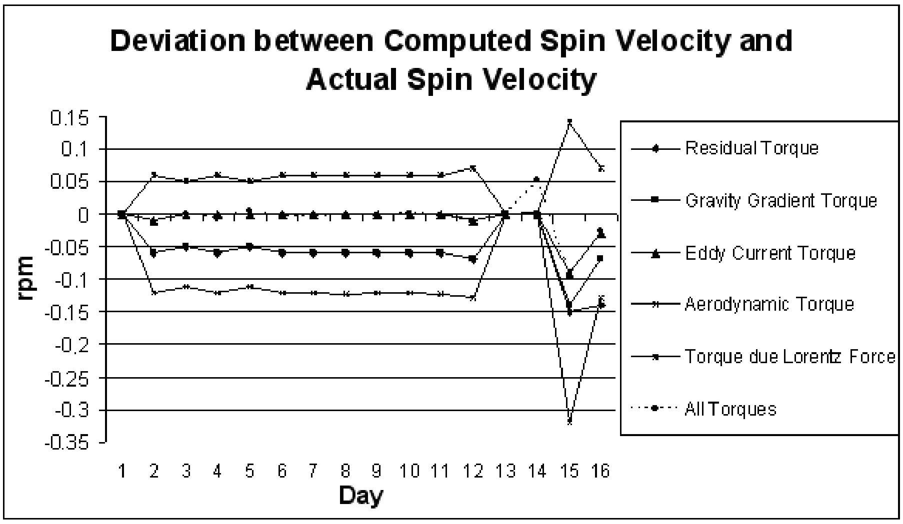

Figure 8 - Temporal variation for the deviation between the computed and actual spin velocity for SCD2 and with the daily updated data.

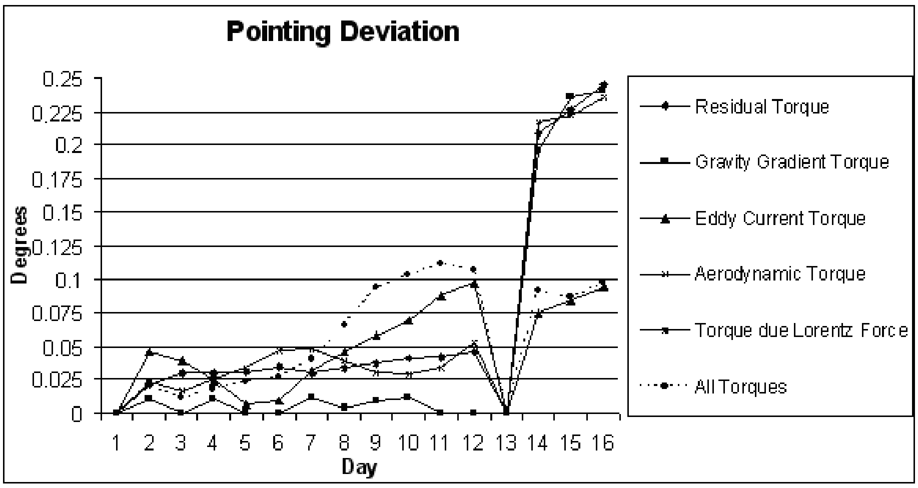

Figure 9 - Temporal variation for the pointing deviation for SCD2 and with the daily updated data.

Table 4 – Mean values for SCD2 simulations with the daily updated data.

Included Torques	Residual	Gravity Gradient	Eddy current	Aerod.	Due Lorentz force	All torques
Right Ascension Deviation (degrees)	-0.071	0,001	-0,029	-0.068	0.071	0.050
Declination Deviation (degrees)	-0,053	0,002	-0,037	-0.053	0.056	0.015
Spin Velocity Deviation (rpm)	-0,059	-0,054	-0,009	-0.118	0.054	-0.007
Pointing Deviation (degrees)	0,066	0,046	0.048	0.066	0.066	0.056

6.2. Second approach: without daily updated data

For SCD1 satellite the simulation was performed considering all torques actuating together. The results in term of the difference between computed and actual right ascension, declination and spin velocity and pointing deviation are shown in Tab. 5. The results show a good agreement between the computed values and the actual satellite behavior only for 1 day simulation . For more than 1 day the mean deviation error for the right ascension is higher than the precision required for INPE’s SCC (0.5º)

Table 5 - Deviation between computed values and actual values when all torques actuating together for SCD1, without daily updated data

Day	Right ascension	Declination	Spin velocity	Pointing deviation
17/08/93	0	0	0	0
18/08/93	-0.871	0.346	0.021	0.373
19/08/93	-1.511	0.702	0.035	0.741
Mean	-0.794	0.349	0.0219	0.371

The Table 6 presents the results obtained when the same simulation is applied to SCD2. The results show a good agreement between the computed values and the actual satellite behavior for the entire simulated time interval of 11 day.

Table 6 - Deviation between computed values and actual values when all torques actuating together for SCD2, without daily updated data.

Day	Right ascension (degrees)	Declination (degrees)	Spin velocity (rpm)	Pointing deviation (degrees)
12/2/2002	0	0	0	0
13/2/2002	-0.023	0.018	0.012	0.021
14/2/2002	-0.048	0.029	0.014	0.036
15/2/2002	-0.077	0.026	0.026	0.043
16/2/2002	-0.104	0.008	0.028	0.047
17/2/2002	-0.143	-0.027	0.227	0.069
18/2/2002	-0.196	-0.084	0.241	0.122
19/2/2002	-0.198	-0.116	0.253	0.146
20/2/2002	-0.211	-0.16	0.265	0.186
21/2/2002	-0.238	-0.216	0.275	0.241
22/2/2002	-0.212	-0.257	0.288	0.274
23/2/2002	-0.196	-0.311	0.309	0.323
Mean	-0.137	-0.091	0.110	0.126

7. Conclusions

In this paper a numerical approach was presented to the spin-stabilized satellite attitude propagation taking into account the residual torque, eddy current torque, aerodynamic torque, torque due Lorentz force and gravity gradient torque. The modeling scheme is entirely developed in terms of quaternion parameterization.

The theory was applied to the spin stabilized Brazilian's satellites SCD1 and SCD2. The results show that, for SCD1 and SCD2, the influence of the eddy current torque is bigger than the others ones, not only due to the orbit altitude, but also due to other specific satellites characteristics. The influence of the torque due Lorentz force is smaller than the others ones because of the dimension and the electrical charges of the SCD1 and SCD2.

Two approaches were presented. In the first one, where the attitude and orbital data are daily updated with real attitude data supplied INPE, the results shown a good agreement between the computed and actual data during the simulated time interval. The mean pointing deviation was of 0.303^{o} for SCD1 and of 0.056^{o} for SCD2, which are within the dispersion range of the attitude determination system used for these satellites.

In the second approach the attitude and orbital data are not updated. For SCD1, the obtained results showed a good agreement between the analytical solution and the actual satellite behavior only for one day simulation. For more than 1 day the mean deviation of the right ascension, declination and pointing deviation were higher than the precision required for SCC (0.5º). For the satellite SCD2, over the test period of the 11 days, the difference between computed values and actual data remained within an acceptable dispersion range over all simulated time interval.

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