REPRESENTATION OF PROBABILITY DENSITY FUNCTIONS FROM ORBIT DETERMINATION USING THE PARTICLE FILTER

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ABSTRACT

Statistical Orbit Determination (OD) is the estimation of the state of a satellite trajectory from noisy measurements. OD is required in almost all areas of space operations and science; from rendezvous and docking, to high-precision satellite geodesy. Classical methods of OD based on the least squares methods or the Extended Kalman filter (EKF), have been used for over 40 years. These methods are derived from an optimal estimation problem involving only the first and second moments of the state and measurement errors and thus can be found to minimize the mean square error, also known as the Minimum Variance Estimator (MVE). However, the MVE may not always be the best choice of an estimator for all problems because it only considers the second moments of the state. As long as the observation error and process noise can be accurately assumed to have a Gaussian distribution, these second moments are sufficient to infer all other statistics as well.

An example in which the non-Gaussian errors could arise is for observations based upon short arcs of tracking data when tracking orbit debris. An accurate prediction of the states over longer durations of time is imperative for orbit tracking and collision avoidance. Since even events with very low probabilities are of interest, we must accurately predict the tails of the distribution. Moreover, orbit dynamics are non-linear. Even if the a priori state vector has a Gaussian distribution, propagation with non-linear dynamics would evolve this distribution into a non-Gaussian one. These effects become more pronounced for orbits with higher eccentricity. Development of nonlinear filtering techniques capable of predicting a more complete representation of the state error distribution (a full Probability Density Function (PDF)), are necessary to address this type of problem.

The Particle Filter algorithm (PF) is used as a nonlinear filtering technique that is based on a sequential Monte Carlo approach representing the required PDF by a set of random samples or particles. The accuracy of the prediction and estimation of the states increases with an increase in the number of particles. The representation of the state PDF of an orbit with a large number of particles is not a practical representation for storing or distributing orbit state data. Common ephemerides, for example the NORAD Two-line elements, only represent the mean orbit state, providing no information on its probability distribution. The objective of this work, therefore, is to develop new methods for representing the full PDF of the orbit state, in a compact data record which could be distributed much in the same way as ephemerides are used today. We will approach this problem through investigating methods to compress the data with represented by the PF state to a much lower dimensional size and the number of particles used. This compact representation is particularly important for the orbit debris monitoring problem in which tens of thousands of objects must be tracked continuously. Conventional methods that perform orthogonal

transformations to obtain linearly uncorrelated variables, such as the principal component analysis, would lose information present in the higher moments if the distribution is not Gaussian. We have utilized a method known as the Independent Component Analysis (ICA) that has been developed that is capable of decorrelating non-Gaussian data by finding the local extrema of the kurtosis (fourth order moment) of a linear combination of the states and thus estimating the non-Gaussian independent components.

Given that we have an estimated a state vector $\mathbf{v_i}$ of dimension-N, with m particles $\mathbf{v_1}, \mathbf{v_2}, \ldots \mathbf{v_m}$, we form a linear combination of S-dimensional independent variables $\mathbf{s_1}, \mathbf{s_2}, \ldots \mathbf{s_m}$, where $S \leq N$, we then compute a mixing matrix $\mathrm{A}(N \times S)$, such that

$$\mathbf{v} = \mathbf{A}\mathbf{s} \tag{1}$$

where $\forall i, \mathbf{s_i}$ has unit variance. The data $\mathbf{v}$ is pre-whitened to obtain a new set of data $\mathbf{x} = \mathbf{M}\mathbf{v}$ that is uncorrelated and has a unit variance ($\mathbf{M}$ is the whitening matrix). We compute the orthonormal matrix $\mathbf{B}$ whose columns are the independent components computed using a fixed-point iteration scheme based on calculating the gradient ascent or descent of a defined objective function based on the kurtosis of $\mathbf{x}$, where $\mathbf{x} = \mathbf{M}\mathbf{v} = \mathbf{M}\mathbf{A}\mathbf{s} = \mathbf{B}\mathbf{s}$. Hence, our independent components $\mathbf{s}$ are given by

$$\mathbf{s} = \mathbf{B^T}\mathbf{x} \tag{2}$$

We will demonstrate the application of ICA to compress our state estimates for an orbit determination example obtained from using the PF. The orbit is a highly eccentric orbit with eccentricity of 0.9 and a 1 day period, similar to the phase 2 orbits of the Magnetospheric Multiscale(MMS) mission. The dimension of the estimated states are then reduced using the ICA by retaining the independent components with the highest collective eigenvalues ($\approx 97\%$). This initial stage of dimensional reduction is extremely useful for data allocation and transmission as well as navigation insight in the illumination of the directions where larger variances are concentrated. The data is easily reconstructed using the mixing matrix. The mean of the norm of the particles' distance from the original distribution to the reconstructed distributions is used as a measure of "goodness-of-fit" . It describes the average distance of the particles from the reconstructed distribution to the original distribution. Hence, the closer the distance is to zero, the closer the two distributions are similar to one another. The mean of the norm of the differences for position (X,Y,Z) is $[1.33, 1.46, 2.74] * 10^{-2} mm$. The figure below illustrates the initial data distribution(blue) and post distribution of the data after decorrelation, dimensional reduction and reconstruction(red).

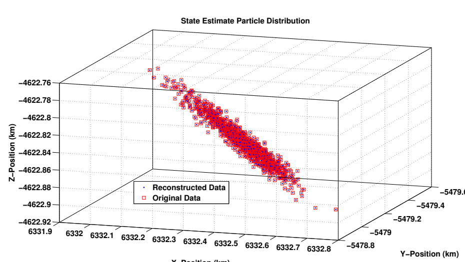

Figure 1. Original Data and Reconstructed Data State Estimate Distribution