Resonance Effects on Lifetime of Low Earth Orbit Satellites

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ABSTRACT

The subject of orbit lifetime estimation is particularly prominent these days in the context of an increasing population of Earth orbiting debris. A set of rules is commonly adopted (by France in particular in the frame of the French Space Operations Act) for satellites crossing the Low Earth Orbit Region. It states that if the impossibility to carry out a controlled reentry is duly proven, the satellite should be put on a disposal orbit at the end of its operational lifetime, either stable and outside of the LEO region, or such that an uncontrolled reentry occurs within 25 years. Of course, this implies having a good enough accuracy on the predicted trajectory for all that period of time.

Orbit lifetime computation for LEO satellites has been extensively studied in the literature. The main factor that makes lifetime prediction difficult is related to atmospheric drag. Recent studies conducted by CNES showed that lifetime uncertainty due to inaccurate prediction of solar activity can be satisfactorily handled by a "mean value" approach, so that LEO lifetime is not usually sensitive to initial conditions: changing the initial date or the local time of the ascending node for a Sun-synchronous orbit has limited consequences: lifetime varies by a few years only.

But there are a few cases for which initial conditions matter. This paper examines these particular situations (resonance cases) for which orbit lifetime more strongly depends on initial conditions such as RAAN, argument of perigee and date. They can for instance be induced by solar radiation pressure which can have, in particular situations, relatively strong impacts depending on the value of some initial angular orbital elements. The effects of other forces, mainly third body perturbation and some aspects of drag are also evaluated in the paper. Finally, the consequences in the context of the French Space Operations Act regarding the methods to be used to estimate lifetime with some confidence in these particular cases are discussed in the conclusion.

Acronyms

CNES	*Centre National d'Etudes Spatiales* - French Space Agency
LEO	Low Earth Orbit - Apogee altitude less than 2000 km (IADC definition)
RAAN	Right Ascension of the Ascending Node
SRP	Solar Radiation Pressure
STELA	Semi-analytic Tool for End of Life Analysis
sma	Semi-major axis
ecc	Eccentricity

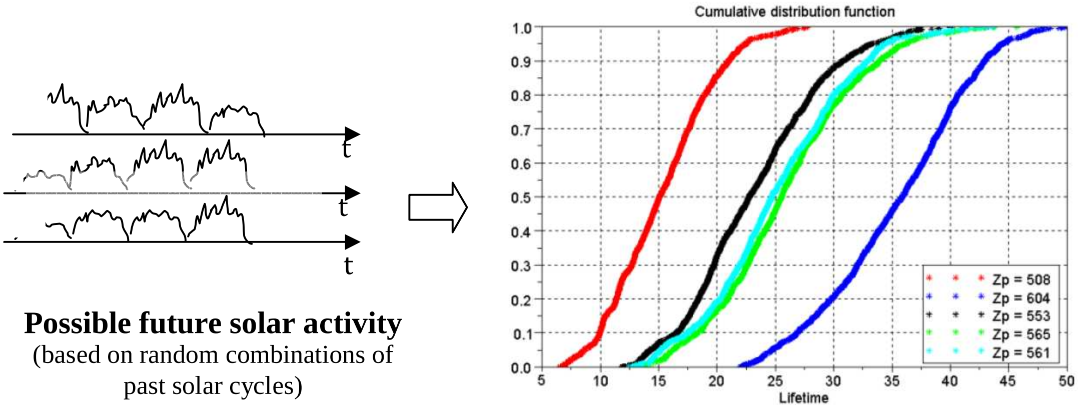
1. Introduction

In the frame of the French Space Operations Act, CNES is in charge of proposing and developing the technical methods that are recommended for a space mission in order that legal requirements are respected (See [1] for more details).
One of these requirements states that if the impossibility to carry out a controlled reentry is duly proven, the satellite should be put on a disposal orbit at the end of its operational lifetime, either stable and outside of the LEO region, or such that an uncontrolled reentry occurs within 25 years.
It is well known that lifetime estimation of LEO objects is difficult due to the uncertainty on atmospheric drag: prediction of attitude of the satellite, ballistic coefficient, density of the atmosphere, and most of all solar and geomagnetic activity can be very inaccurate. That's why a specific approach has been implemented to remove the uncertainty on solar and geomagnetic activity in lifetime prediction.

1.1 Handling of solar activity unpredictability

The problem of LEO lifetime unpredictability has been studied extensively in the literature (see [2] for instance). The consequences in the context of the French Space Operations Act have been analysed, which resulted in the definition of an efficient method based on an equivalent "mean solar activity" which guarantees (in most cases) that the actual lifetime will be less than the computed value with a probability of 50%. The required propagation model has also been defined and implemented in a tool named STELA ([1], [8]).

Let's take an example to illustrate the method.
Suppose that the initial orbit is a 800x800km circular orbit (perigee altitude = apogee altitude = 800km). Considering the recommended model for the dynamics (which includes the value of the constant solar flux) and propagating leads to the conclusion that the perigee should be lowered to 561 km for the lifetime to compliant with the 25 year rule. Then the "actual lifetime" distribution considering any possible future solar activity will look as shown in Figure 1 (light blue curve). The probability that the actual lifetime exceeds 25 years is less than 50%. And we can see that it is less that 35 years in about 95% of the cases. As a consequence the impact of solar activity unpredictability on lifetime is about +/- 10 years.



Possible future solar activity
(based on random combinations of past solar cycles)

Figure 1: Typical lifetime distribution (random solar activity)

It was also shown that (in most cases), Sun or Moon gravity perturbation or solar radiation pressure have limited effects on lifetime. Initial conditions, for instance the initial value of the ascending node local time for a Sun synchronous orbit or the initial date also have small impacts in comparison with the effect of drag uncertainty.

Thus, the constant solar flux approach simplifies lifetime estimation for LEOs by eliminating the major source of uncertainty. Some useful consequences are:
- only one simulation (i.e. propagation) case is needed to obtain an estimate of orbit lifetime, which makes the computation of disposal orbits easier, as well as the verification of these orbits.
- The results don't depend (or depend very little) on initial orbital elements and date, so that the conclusions remain valid even if the end of mission (and therefore the date of the disposal maneuver) happens to shift.

1.2 Some sensitivity remains

As already mentioned in the previous section, drag is the main factor that affects LEO lifetime; other factors such as solar radiation pressure or third body gravitation are not usually considered as having a strong impact.

In fact that's not always the case as proved in Figure 2 which shows how orbit lifetime depends on initial conditions (RAAN in this case) for various inclinations. All parameters other than RAAN and inclination are constant. We can see that lifetime may vary at least 12 years depending on the RAAN's initial value, and for particular values of inclination.

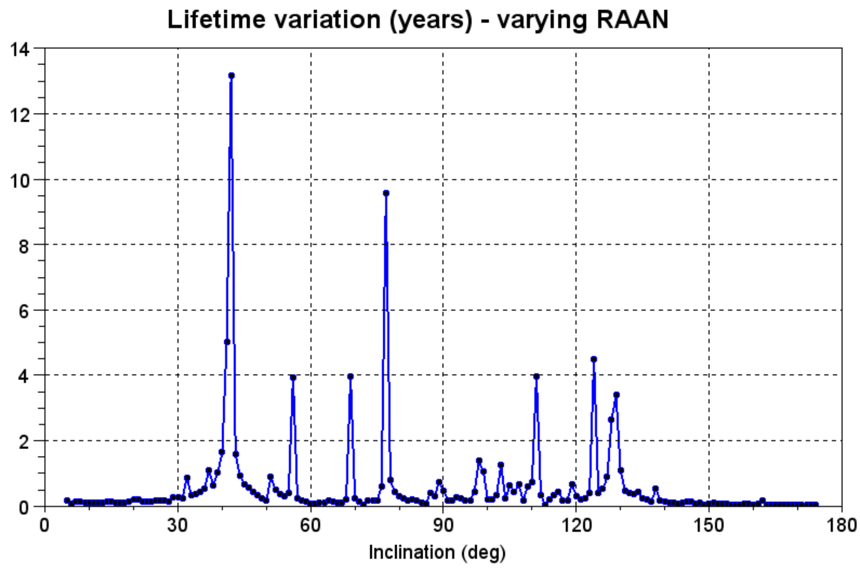


Figure 2: Lifetime variations as function of inclination (varying RAAN)

Lifetime was computed for each value of inclination (every degree) and each value of RAAN (every 30 degrees). The plot gives the maximum difference between 2 lifetime results corresponding to the same inclination.

Table 1: Simulation hypotheses

Semi-major axis	7240 km
Eccentricity	0.05
Altitudes Perigee/apogee	~500 km / 1200 km
Initial date	March 21st 1998
Area / Mass (drag or SRP)	15 m² / 1000 kg
Drag coefficient	2.2
Reflectivity coefficient	2
Solar activity	Constant : F10.7 = 140 sfu, Ap = 15
Atmospheric model	NRLMSIS-00
Propagation model	Standard STELA model for LEOs

The orbit considered is not that “exotic”, however lifetime varies more than could be expected: the uncertainty on lifetime can be as big as that coming from drag.

The same kind of sensitivity appears with the argument of perigee for which variations of more than 30 years (considering a fixed value for the initial RAAN) can be observed for some inclinations.

The objective in the following parts is to explain where the sensitivity comes from.
What matters most in this paper is not so much to be able to compute the exact effects of the perturbations for given initial conditions, as to know which orbits are concerned.

2. Analysis of the effects of the main influential perturbations

The perturbation forces that will be considered are: solar pressure, gravity of third body and atmospheric drag. Other perturbation causes will also be analyzed in the end of this section.

2.1 Solar radiation pressure (SRP)

2.1.1 Theoretical developments

The effect of SRP on the orbit is made more complicated due to eclipses (when the satellite is in the shadow of the Earth), so we’ll neglect them for now.

SRP effects on the Keplerian orbital elements when no eclipse occurs can be described by the following averaged equations:

$$\frac{da}{dt} = 0$$

$$\frac{de}{dt} = \frac{3}{2} \frac{\sqrt{1-e^2}}{n\,a} \gamma_Y$$

$$\frac{di}{dt} = -\frac{3}{2} \frac{e\ \cos(\omega)}{n\,a\,\sqrt{1-e^2}} \gamma_Z$$

$$\frac{d\omega}{dt} = \frac{3}{2}\left(-\frac{\sqrt{1-e^2}}{n\,a\,e} \gamma_X + \frac{e\ \sin(\omega)}{n\,a\,\sqrt{1-e^2}\ tg(i)} \gamma_Z \right) \tag{Eq 1}$$

$$\frac{d\Omega}{dt} = -\frac{3}{2} \frac{e\ \sin(\omega)}{n\,a\,\sqrt{1-e^2}\,\sin(i)} \gamma_Z$$

$$\frac{dM}{dt} = \frac{3}{2} \frac{1+e^2}{n\,a\,e} \gamma_X$$

With:
a: semi major axis
e : eccentricity
i : inclination
ω : argument of perigee
Ω : right ascension of ascending node
M: mean anomaly
n : mean motion = $\sqrt{\frac{\mu}{a^3}}$ (with μ : Earth gravitational constant)

These equations were obtained by averaging the Gauss equations over one orbit (average over the mean anomaly), assuming that no eclipse occurs and that the acceleration vector is constant. These equations apply in fact for any constant force, not only Solar radiation pressure.

γ_X , γ_Y , γ_Z are the components of the perturbing acceleration vector in the "orbit" frame. The "orbit" frame is defined by:
- X axis : directed towards the perigee,
- Z axis : perpendicular to the orbit plane and along the angular momentum vector (same sense) .

and we have: $\vec{\gamma} = -K\,\vec{u}$
where $\vec{u}$ is the unit vector from Earth center to Sun, and:

$$K = P_0\ C_r\ \frac{A}{M}\left(\frac{d_0}{d}\right)^2$$ with:

- P_0 : Solar radiation pressure at a distance d0 from the Sun
- C_r : Reflectivity coefficient (between 1 and 2; 1: absorbent surface, 2: reflecting surface)
- A : (mean) cross sectional area
- M : mass
- d: Earth-Sun distance
- d_0 : Earth-Sun reference distance (= 1AU)

These equations give an immediate feel as to the effect of SRP depending on where the Sun is relative to the orbit. Of course, these equations are not valid for a perfectly circular or equatorial orbit (e = 0 or i = 0). Specific orbital elements should be chosen in this case.

The above equations can also be expressed as functions of orbital elements only (note that a quasi inertial and equatorial frame is implicitly defined).
Vector u 's components in the "orbit" frame can be written:

$$u = \begin{bmatrix} \cos(\delta_S)\,[\cos(\omega)\cos(\Omega-\lambda_S) - \cos(i)\sin(\omega)\sin(\Omega-\lambda)] + \sin(\delta_S)\sin(\omega)\sin(i) \\ \cos(\delta_S)\,[-\sin(\omega)\cos(\Omega-\lambda_S) - \cos(i)\cos(\omega)\sin(\Omega-\lambda_S)] + \sin(\delta_S)\cos(\omega)\sin(i) \\ \cos(\delta_S)\sin(i)\sin(\Omega-\lambda_S) + \sin(\delta_S)\cos(i) \end{bmatrix}$$

with: λ_S, δ_S : right ascension and declination of the Sun (in the equatorial frame).

And we have:

$$\cos(\delta_S)\cos(\lambda_S) = \cos(\alpha_S)$$
$$\cos(\delta_S)\sin(\lambda_S) = \cos(i_S)\cos(\alpha_S)$$
$$\sin(\delta_S)\sin(\lambda_S) = \sin(i_S)\cos(\alpha_S)$$

where α_S is the (true) argument of latitude of the Sun (angle in the Sun's orbit plane from the ascending node), and i_S is the Sun's orbit inclination (23 deg 26').

When Substituting in (Eq 1), one obtains an expression for $\frac{de}{dt}$:

$$\frac{de}{dt} = -\frac{3}{4}\frac{\sqrt{1-e^2}}{n\,a}\,K\sum_k A_k\,\sin(\Phi_k) \qquad \text{(Eq 2)}$$

where A_k and Φ_k are given in the following table:

Φ_k	A_k
$\Omega-\alpha_S+\omega$	$-\cos^2(i_S/2)\,(1+\cos(i))$
$\Omega-\alpha_S-\omega$	$\cos^2(i_S/2)\,(1-\cos(i))$
$\Omega+\alpha_S+\omega$	$-\sin^2(i_S/2)\,(1+\cos(i))$
$\Omega+\alpha_S-\omega$	$\sin^2(i_S/2)\,(1-\cos(i))$
$\alpha_S+\omega$	$\sin(i_S)\sin(i)$
$\alpha_S-\omega$	$\sin(i_S)\sin(i)$

Note that if the Sun can be considered "on average" in the equator (i_S =0), the expression simplifies in:

$$\frac{de}{dt} = \frac{3}{4}\frac{\sqrt{1-e^2}}{n\,a}\,K\left[\sin(\Omega-\alpha_S+\omega)\left(\cos(i)+1\right) + \sin(\Omega-\alpha_S-\omega)\left(\cos(i)-1\right)\right] \qquad \text{(Eq 3)}$$

Resonance effects

Resonance effects can occur if one of the angles Φ_k is nearly constant and $\sin(\Phi_k)$ is close to 1 or -1 for a maximum effect.

The method used to identify resonance cases is then the following:

- evaluate the amplitudes (A_k) and retain only the biggest ones,
- look for the values of the semi-major axis, inclination and eccentricity such that $\dot{\Phi}_k$ (time derivative) is 0.

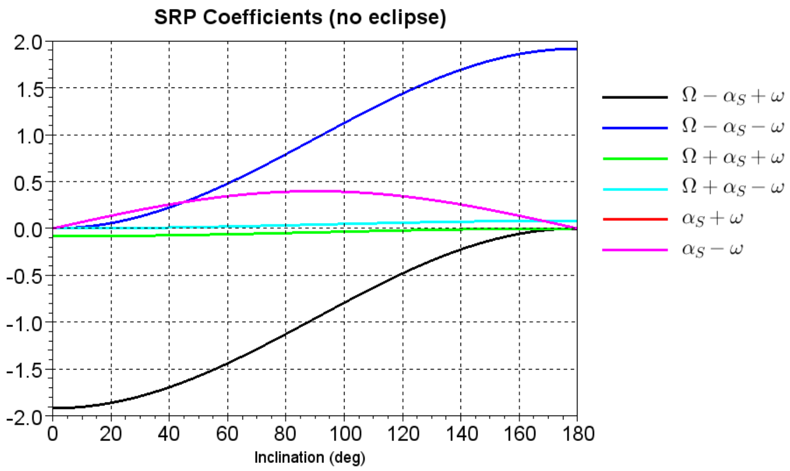


Figure 3: SRP coefficients (A_k)

We can see in Figure 3 that there are 4 most influential terms, the two also present in the approximate formula for $\frac{de}{dt}$ (Eq 3.) and 2 other ones.

The value of $\dot{\Phi}_k$ does not depend much on semi major axis as shown in Figure 4 for the 2 main terms. The dependence on eccentricity is also limited as $\dot{\Omega}$ or $\dot{\omega}$ are function of e^2 (if only J2 is taken into account), and e is less than 0.1 in the LEO domain.

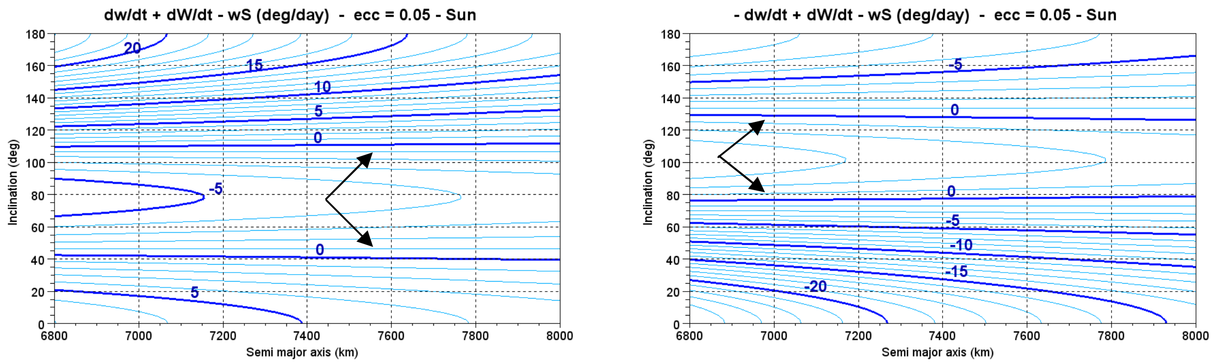


Figure 4: Time derivatives of resonance angles (J2)

The plots were obtained considering the J2 secular drifts only. More accurate calculations should consider other secular effects as well (third body perturbations…), but this would not change the results much.

The following table gives the (approximate) inclination values that corresponds to the 4 major terms, and the associated amplitudes (computed for sma = 7200km, ecc=0):

Φ_k	Inclination (deg)	A_k	Inclination (deg)	A_k
$\Omega - \alpha_S + \omega$	**41.6**	**-1.7**	**110.4**	**-0.6**
$\Omega - \alpha_S - \omega$	**77.1**	**0.7**	**128.6**	**1.6**
$\alpha_S + \omega$	68.1	0.4	111.9	0.4
$\alpha_S - \omega$	59.3	0.3	120.7	0.3

We see that the most "sentitive" inclinations are close to: 41.6, 77, 110.4, 128.6 deg.
All the inclinations found are in the ranges: 40 to 80 deg and 110 to 130 deg.

2.1.2 Illustration of impact on lifetime

We'll consider a worst case corresponding to $\dot{\Omega} - \dot{\theta} + \dot{\omega} \approx 0$: semi-major axis: 7240 km, eccentricity: 0.05, inclination: 41.6 deg. The initial values of RAAN and the argument of perigee are 0 and 90 deg respectively.
We propagate the initial state vector with STELA only considering the effects of Earth gravity (zonal terms), SRP and drag (constant solar activity). Ballistic and SRP coefficients are the same as given in Table 1. Depending on the value of RAAN, lifetime can vary from about 20 years to about 90 years (Figure 5 - red curve), which means that if the RAAN value is unknown, the result is (nearly) unpredictable. We note that at the initial date (21 March 1998 0h), $\sin(\Omega - \alpha_S + \omega)$ is close to 1 (=> maximum resonant effect).

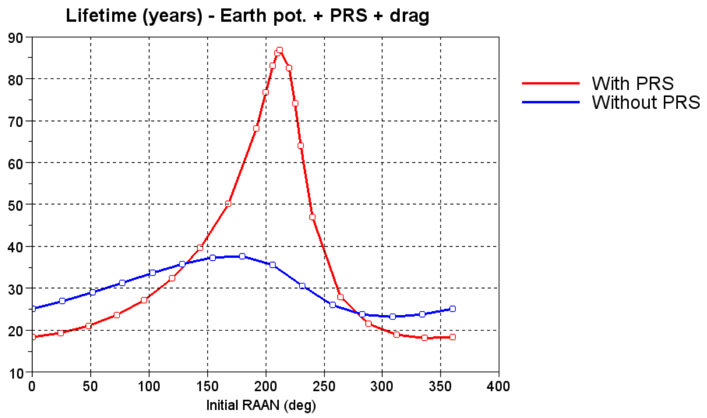


Figure 5: Lifetime with/without PRS as function of initial RAAN

In fact the observed evolution in not only due to PRS. There is also an effect of resonance with drag (see 2.3). But PRS explains most of what is happening.

The possibly long lifetimes originate in the evolution of the altitude of the perigee which increases at the beginning of the simulation (despite the presence of drag) as shown in Figure 6 for which the initial value considered for RAAN is 210 deg.

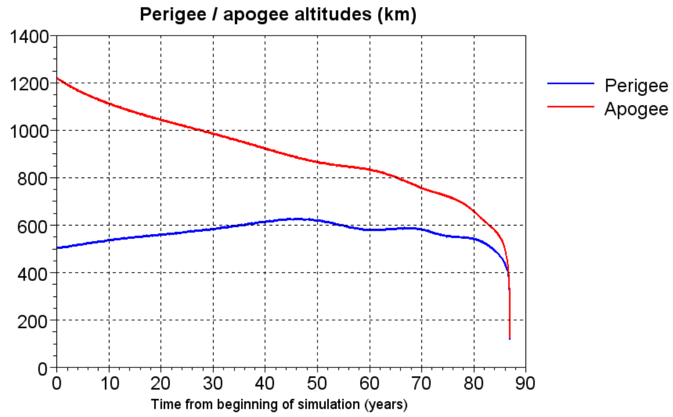


Figure 6: Perigee/apogee history

The time derivative of eccentricity due to SRP (Figure 7 – left plot) evaluated on the trajectory shows that eccentricity decreases nearly secularly at the beginning (as long as the resonance condition remains true). Also shown is the relatively small impact of eclipses (amplitude is slightly smaller with eclipses), which confirms, in this case at least, that they could be ignored in the theoretical developments. Another interesting point is the effect of SRP on the semi major axis (right plot). It would be unaffected without eclipses. In this case it increases, which makes the perigee altitude increase even more.

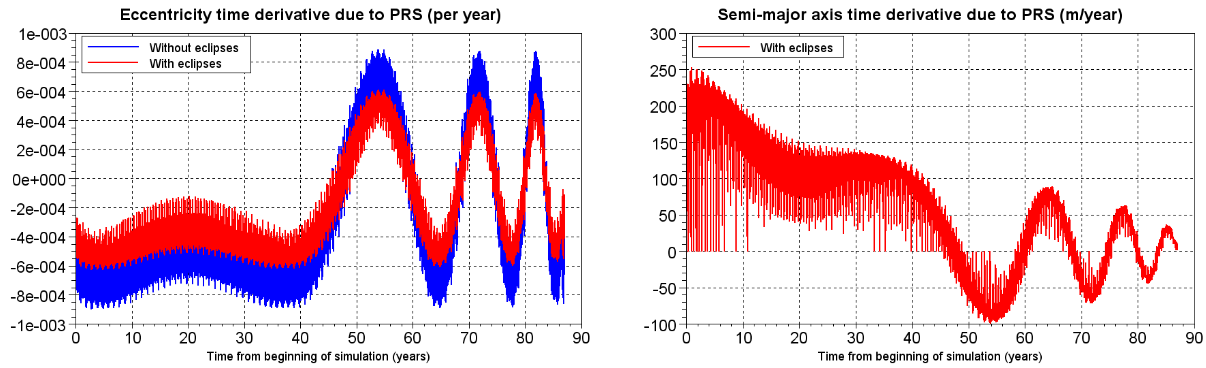


Figure 7: Semi major axis and excentricity

2.2 Third body perturbations

2.2.1 Theoretical developments

We proceed as for SRP.

The classical third body perturbation equations given by Kaula [4] are used to derive the averaged effects on eccentricity (by averaging over the mean anomaly). The equations are developed to order 1 only (l=2 with Kaula's notation), although order 2 is recommended for the Moon. The time derivative of eccentricity is given by:

$$\frac{de}{dt} = -\frac{15}{4}\frac{e\sqrt{1-e^2}}{n}\frac{\mu_b}{d_b^3}\sum A_k \sin(\Phi_k) \qquad \text{(Eq 4)}$$

where μ_b is the gravitational constant of the body (Sun or Moon), d_b its distance to Earth center, and A_k and Φ_k are given in the following table:

Φ_k	A_k
$2\omega - 2\alpha_b$	$-3/8\,\sin^2(i_b)\,\sin^2(i)$
$2\omega + 2\alpha_b$	$-3/8\,\sin^2(i_b)\,\sin^2(i)$
$2\omega - 2\alpha_b + (\Omega - \Omega_b)$	$-1/4\,\sin(i_b)\,(1+\cos(i_b))\,\sin(i)\,(1+\cos(i))$
$-2\omega - 2\alpha_b + (\Omega - \Omega_b)$	$1/4\,\sin(i_b)\,(1+\cos(i_b))\,\sin(i)\,(\cos(i)-1)$
$2\omega - 2\alpha_b + 2(\Omega - \Omega_b)$	$-1/16\,(1+\cos(i_b))^2\,(1+\cos(i))^2$
$-2\omega - 2\alpha_b + 2(\Omega - \Omega_b)$	$1/16\,(1+\cos(i_b))^2\,(\cos(i)-1)^2$
2ω	$3/4\,(\sin^2(i_b)-1/2)\,\sin^2(i)$
$2\omega + (\Omega - \Omega_b)$	$1/4\,\sin(2\,i_b)\,\sin(i)\,(1+\cos(i))$
$-2\omega + (\Omega - \Omega_b)$	$-1/4\,\sin(2\,i_b)\,\sin(i)\,(\cos(i)-1)$
$2\omega + 2(\Omega - \Omega_b)$	$-1/8\,\sin(i_b)^2\,(1+\cos(i))^2$
$-2\omega + 2(\Omega - \Omega_b)$	$1/8\,\sin(i_b)^2\,(\cos(i)-1)^2$
$2\omega + 2\alpha_b + (\Omega - \Omega_b)$	$-1/4\,\sin(i_b)\,(\cos(i_b)-1)\,\sin(i)\,(1+\cos(i))$
$-2\omega + 2\alpha_b + (\Omega - \Omega_b)$	$1/4\,\sin(i_b)\,(\cos(i_b)-1)\,\sin(i)\,(\cos(i)-1)$
$2\omega + 2\alpha_b + 2(\Omega - \Omega_b)$	$-1/16\,(\cos(i_b)-1)^2\,(1+\cos(i))^2$
$-2\omega + 2\alpha_b + 2(\Omega - \Omega_b)$	$1/16\,(\cos(i_b)-1)^2\,(1-\cos(i))^2$

One may note that (Eq 4) is equivalent to the more compact form (taken from [6]):

$$\frac{de}{dt} = -\frac{15}{2}\,\frac{\mu_b}{d_b^3}\,\frac{e\sqrt{1-e^2}}{n}\,u_X\,u_Y$$

(u_X, u_Y, u_Z are the components of the Earth->body unit vector in the "orbit" frame, see 2.1.1)

Remark: Ideally, we would have used doubly averaged equations as those derived by Chao in [3]. But the application of the same formulas to LEOs would be invalid (in most cases) due to the rapid variation of the angular orbital elements compared to the rotation rate of the Sun or Moon.

If we plot the amplitudes of the major terms for the Sun, we obtain the plot in Figure 8.
We see that the 2 main contributions come from the same resonant terms as for SRP (except for the factor 2).
We also note that the amplitudes for the Moon slightly differ from those for the Sun as the Moon’s inclination oscillates with an amplitude of ±5 deg with a period of 18.6 years (the ascending node too with an amplitude of ±13 deg, but this has no effect on the amplitudes, only the phase angles). The factor $\frac{\mu_b}{d_b^3}$ has different values for the Sun and the Moon: the value for the Moon is 2.2 times as big as the one for the Sun, which should theoretically be considered when selecting the terms.

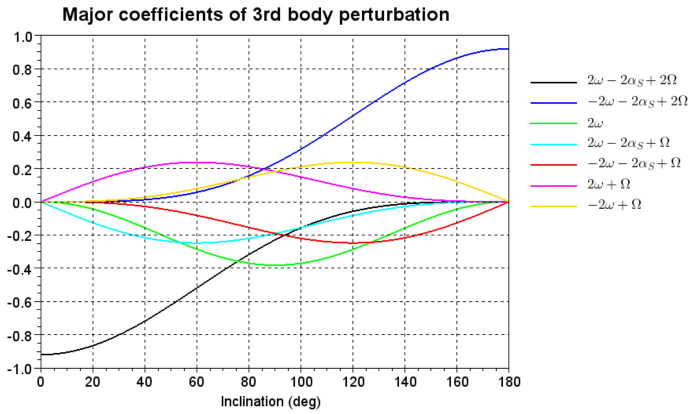


Figure 8: Major coefficients (A_k) for third body perturbation

An examination of the resonance cases (as for SRP) for the Sun and Moon leads to the following results:

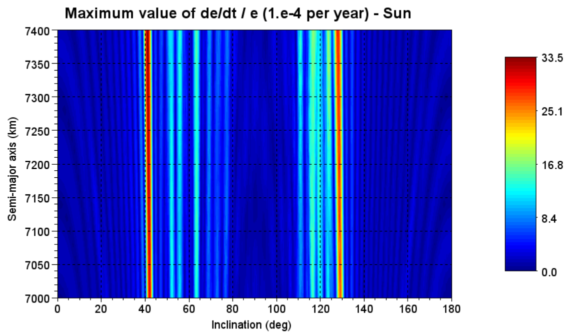
	Sun		Moon	
Φ_k	Inclination (deg)	A_k	Inclination (deg)	A_k
$2\omega - 2\alpha_b + 2(\Omega - \Omega_b)$	**41.6**	**-0.7**	147.5	~0
	110.4	-0.1		
$-2\omega - 2\alpha_b + 2(\Omega - \Omega_b)$	77.1	0.1		
	128.6	**0.6**		
2ω	63.4	-0.3	**63.4**	**-0.3**
	116.6	-0.3	**116.6**	**-0.3**
$2\omega - 2\alpha_b + (\Omega - \Omega_b)$	51.7	-0.25	158.3	0
	114.8	-0.1		
$-2\omega - 2\alpha_b + (\Omega - \Omega_b)$	73.3	-0.1		
	119.1	-0.25		
$2\omega + (\Omega - \Omega_b)$	56.1	0.25	*Same as for Sun*	
	111.0	0.1		
$-2\omega + (\Omega - \Omega_b)$	69.0	0.1	*Same as for Sun*	
	123.9	0.25		

Cells with a grey background correspond to cases where no solution was found in the LEO domain.
The strongest effects appear for the inclination values: 41.6, 128.6, 63.4 and 116.6 deg.

Remarks: We have considered for the calculations "mean" values for the Moon's RAAN and inclination: 0 deg and 23 deg 26' respectively (same as for the Sun). The inclination values are a bit approximate as they actually slightly depend on semi-major axis and eccentricity (in the table: computed with: sma=7200km, ecc=0, $\dot{\alpha}_b$ for the Moon = 2π/27.3 rad/day).

The results are now checked numerically.

The effects coming from the Sun and Moon are averaged over one year for various (random) values of RAAN and argument of perigee in order to retain the highest value of |de/dt|/e. Only J2 and third body effects are considered here. The initial date is 21 March 1998, eccentricity is constant (0.05). Semi-major axis is varied over a limited domain (7000-7400 km), inclination is varied with a step of 0.2 deg. One may note that the results are almost independent of eccentricity.



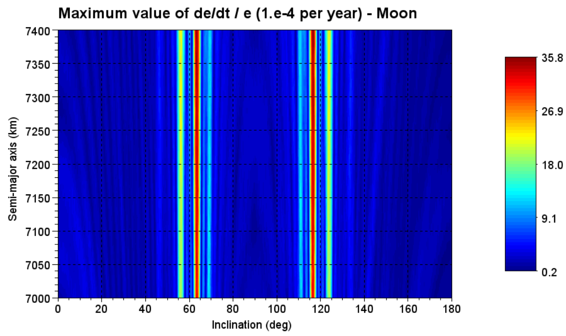


Figure 9: Resonance due to third body perturbation (Sun:left, Moon:right)

We clearly see the 4 main “critical” inclinations (red vertical bars): 2 for the Sun (41.6 and 128.6 deg) and 2 for the Moon (63.4 and 116.6 deg), plus other less influential ones, all in the expected range. The impact of the Moon can be as strong as that coming from the Sun as predicted by the equations (considering the 2.2 factor for the Moon). Also, there is no visible effect for Sun-synchronous orbits.

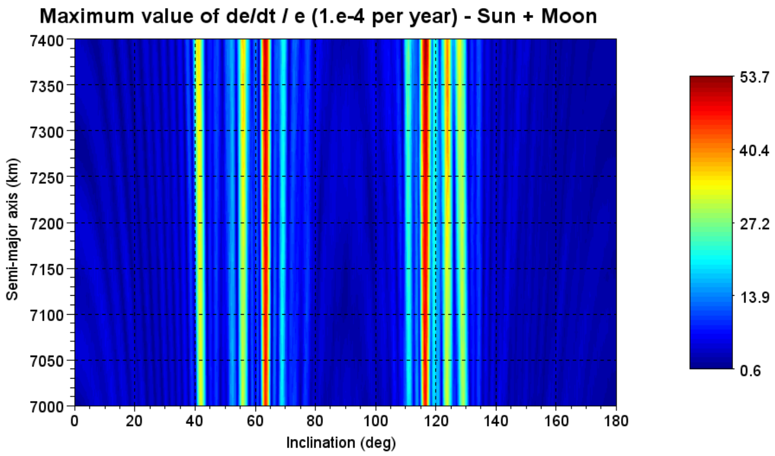


Figure 10: Resonance due to third body perturbation (Sun + Moon)

Figure 10 shows the cumulative effect (sum of absolute values), considering that the perturbations coming from the Sun and Moon can (potentially) add up.

We can see about 10 areas for inclination where de/dt can be constant. The potentially strongest effect appears at the critical inclinations (63.4 and 116.6 deg), and effectively the effect coming from the Sun and Moon can (almost) add up in this case.

As an example, if eccentricity = 0.05 and semi-major axis = 7200 km, the maximum computed effect on the perigee altitude is: 53.7 * 0.05e-4 * 7200 ~ 2 km/year.

2.1.2 Illustration of impact on lifetime

We'll consider the same orbit as in 2.1.2, except for the value of inclination which is 56 deg (so that $2\dot{\omega}+\dot{\Omega}$ is close to 0).
We propagate the initial orbital elements using STELA, considering the effects of Earth gravity (zonal terms only), third body gravity (Moon and/or Sun or none) and drag.
The initial value of RAAN is varied from 0 to 360 deg.

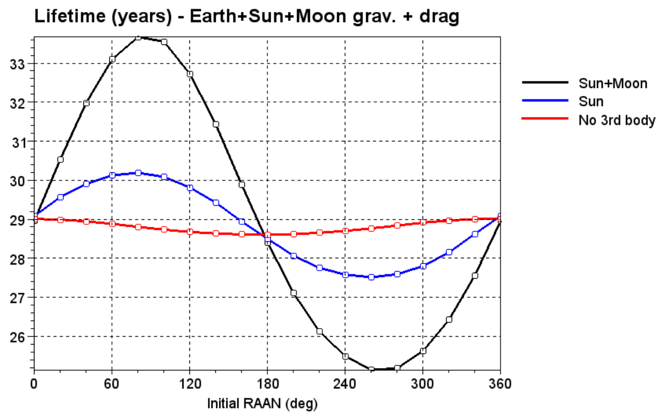


Figure 11: Resonance due to third body perturbation (lifetime)

We can see lifetime variations due the effects of Sun and Moon together, Sun alone or if no 3rd body is taken into account. We observe in particular that the contribution due to the moon is bigger than the one due to the Sun (as expected). Although there is some sensibility to RAAN (about 8 years between the minimum and maximum values), the effect of the third body perturbation is not extremely strong in this case.

2.3 Atmospheric drag

We'll show here an example of resonance with drag.

The atmospheric density is not uniformly distributed as the Sun heats up some parts of the atmosphere more than other ones. Consequently, the mean effect of drag (average over one orbit) will depend on the orientation of the orbit with respect to the Sun.

This is illustrated below.
The semi major axis, eccentricity and inclination are respectively 7240 km, 0.05 and 41.6 deg. Other hypotheses are the same as in Table 1.
Figure 12 shows the mean semi-major axis decrease rate over one orbit as a function of ascending node local time and argument of perigee. The date is March 21st.

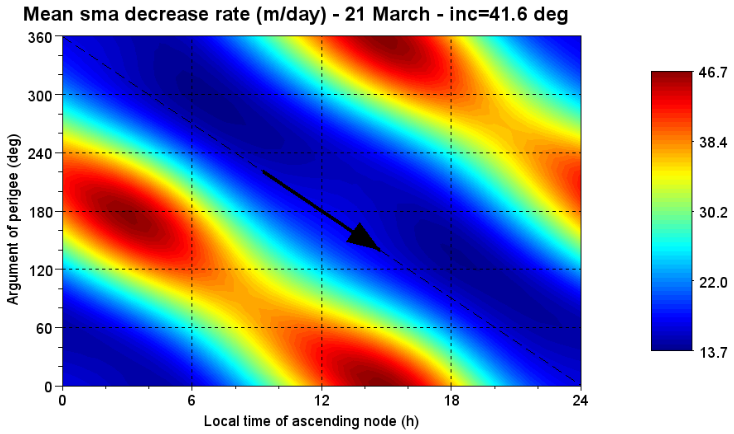


Figure 12: Semi major axis mean decrease rate

The greatest decrease rate is obtained for a local time of the ascending node of 15h (or 3h), due to the atmosphere response time delay.
The arrow shows how the argument of perigee and local time of ascending node would change in time (the perigee and the local time move at the same rate for this orbit as $\dot{\Omega} - \omega_S + \dot{\omega} \sim 0$).
In reality the average decrease rate would change in time (for fixed values of the argument of perigee and local time of ascending node) because of the varying declination of the Sun. But the overall aspect of the plot would look similar.

Figure 13 represents the mean effect of drag on the semi major axis over 2 years evaluated for all possible initial values for the ascending node local time and argument of the perigee. The model takes J2 to J6 zonal terms into account. The decrease rate of the semi major axis is evaluated on the "nominal" (osculating) orbit, that is to say unaffected by drag.
The values are slightly smaller than for the previous plot because of the changing declination of the Sun along the year.

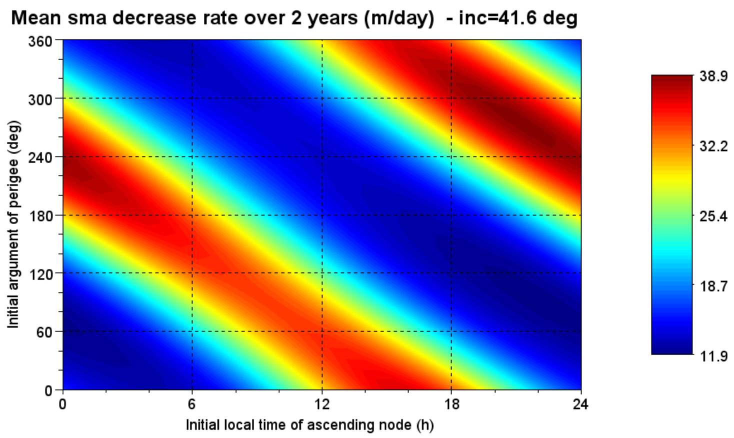


Figure 13: Semi major axis mean decrease rate over 2 years

Among all possible resonance cases (all possible pairs of integers such that $i(\dot{\Omega}-\omega_S)+j\dot{\omega}=0$, Only the cases |i| = |j| = 1 have been shown to have a significant impact. This leads to the table

Resonance angle	Inclination 1 (deg)	Inclination 2 (deg)
$\Omega-\alpha_S+\omega$	**41.6**	**110.4**
$\Omega-\alpha_S-\omega$	**77.1**	**128.6**

(evaluated for sma=7200km and ecc=0)

For the other cases, semi major decrease rate variations as function of local time of ascending node and argument of perigee are smaller that the ones obtained for a Sun-synchronous orbit.

We can expect that these variations on the mean semi-major axis decrease rate will have an impact on lifetime.
These impacts are evaluated by simulation with STELA where all parameters are fixed except the initial argument of perigee and right ascension of ascending node which are drawn uniformly in [0, 360 deg]. The hypotheses are the same as in 2.1.2 except that only Earth gravity and drag are considered (solar activity is constant).
Figure 14 shows lifetime results as function of the resonance angle ($\Omega-\omega_S+\omega$). Lifetime is maximum for a angle of about 250 degrees, which is very close to what could be foreseen from Figure 13. We also observe the range of the results: between 21 and 38 years.

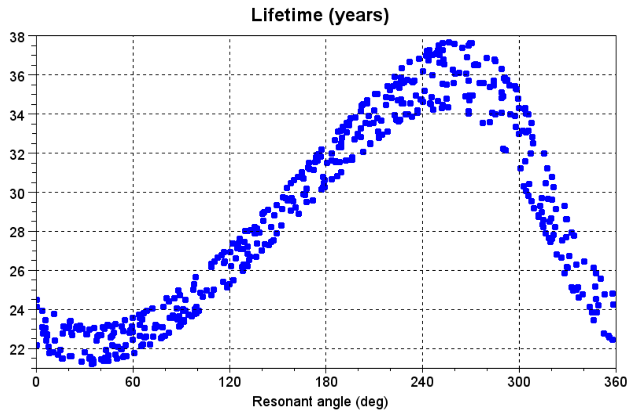


Figure 14: Lifetime as function of resonant angle

2.4 Earth gravity
There are two factors that may have some impact: zonal terms of the Earth gravity field at the critical inclination and tesseral terms.
We will only discuss them briefly.

Zonal terms at the critical inclination
We'll only show an example. Orbital elements are the same as in 2.3 except for the inclination and the argument of perigee which vary. The orbit is propagated using STELA and Earth gravity only (zonal terms up to J15) is considered.
Figure 15 show the mean value of eccentricity over 30 years as a function of argument of latitude.

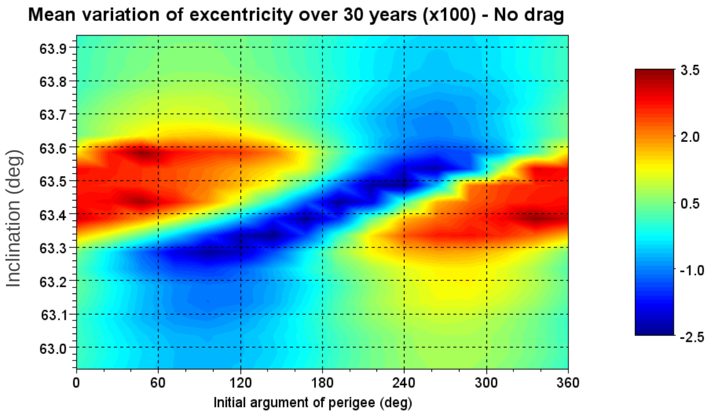


Figure 15: Mean eccentricity over 30 years (Earth gravity only)

We can see correlations with Figure 16 which shows lifetime in the same domain of inclination/argument of perigee. Small lifetime values can be seen in the regions where the mean variation of eccentricity is maximum (and positive). But other effects appear as the decrease rate of the semi major axis is affected by the initial value of the argument of perigee (almost constant) through altitude, since density is a function of the geodetic altitude.

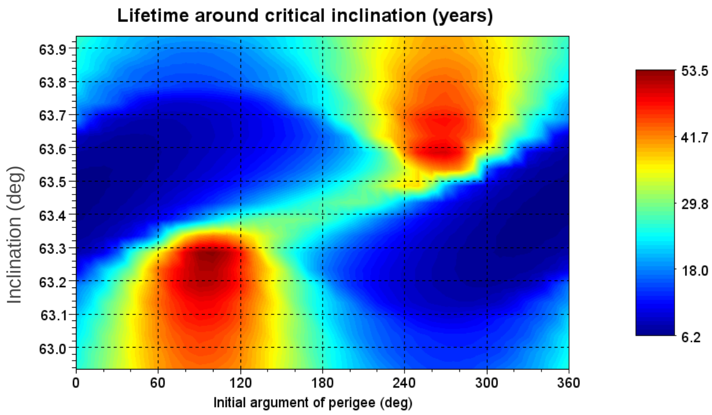


Figure 16: Lifetime around critical inclination (Earth gravity + drag)

All these causes make lifetime vary rapidly around the critical inclination. This means that lifetime prediction should consider accurate enough values of the orbital elements to be meaningful.

<u>Tesseral terms</u>
The effect of tesseral terms haw been roughly estimated to be very small: a few months on lifetime, that is to say negligible.

3. Conclusion

Resonance effects due to various perturbation sources (solar radiation pressure, third-body perturbation and drag in particular) have been shown to have significant effects on LEO lifetime in some particular cases.

The “sensitive” low Earth orbits are mainly characterized by their inclination as the dependence of resonance conditions to semi-major axis and eccentricity is weak. The fact that the angular orbital elements’ time derivatives don’t depend much on semi-major axis and eccentricity in the LEO domain also explains why the resonance conditions can last for an extended period of time despite the effect of drag.

The “sensitive” inclinations have been found in the range: 40-80 deg and 110-130 deg. This means that particular attention should be given to orbits with inclinations in these ranges. Fortunately, there are few operational LEO missions with such inclinations (but some exist).

Another fact is that some effects are only observed with eccentric orbits. That’s the case for the third-body perturbation for which there is no effect on eccentricity if the orbit is circular. This is different for SRP for instance.

If a disposal orbit falls into the “sensitive” domain, evaluating its lifetime may lead to imprecise estimates if not all the initial orbital elements are accurately known. For instance the RAAN value may be unknown at the time of the disposal maneuver as it may depend on the date of end of mission which may shift. More generally, it is clear from the developments in the paper that RAAN, argument of perigee and of course inclination are key parameters.

From a disposal-orbit-choice and lifetime-verification viewpoint there are practical consequences that have to be dealt with. The main question is whether the orbit may potentially be affected by resonance phenomena or not. This is of course of importance, first because it may lead to longer lifetimes, but also because the methods normally used to evaluate the lifetime and to verify it may be not well adapted to this case. There are in fact several possibilities:

- If no resonance can occur (because inclination is not in the “sensitive” domain”) then the usual methods apply (see 1.1). That should be the most frequent situation.

- If resonances can occur and the initial conditions can be chosen so as to control the lifetime (and it can be proved that the strategy envisaged will actually be useable), then the sensitivity disappears, and the usual methods also apply. The only difference with the previous case is that initial conditions (position of disposal maneuver …) matter more.

- If resonances can occur and the sensitive parameters cannot be controlled accurately (e.g. the RAAN value cannot be predicted accurately enough and it has an influence on lifetime), then one solution is to adopt the same strategy as for GTOs for which lifetime sensitivity to initial conditions is the common case rather than the exception. This implies running several simulations (with adequate inputs reflecting the uncertainties on parameters and their ranges of variation) and to process them statistically [7]. Of course orbit lifetime estimation is then more time-consuming.

In addition, if the answer to whether resonances can occur or not is not obvious, a parametric study, i.e. varying the initial conditions and model parameters to analyze how lifetime depends on them, may help to answer the question.

Thus, having a good a priori knowledge of the orbits for which resonance may occur is of major importance to reduces the size of the "sensitive" domain. There are some useful elements in this paper but additional work aiming to refine the domain (through a finer analysis of the effect of eclipses or of the combination of various effects for instance) remains to be done so that a truly applicable method can be implemented in the context of the French Space Operations Act.

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4. References

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