COLLISION ASSESSMENT USING POLYNOMIAL CHAOS EXPANSIONS

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Keywords: *collision assessment, uncertainty propagation, polynomial chaos, space situational awareness*

ABSTRACT

This paper will present the application of polynomial chaos expansions (PCEs) for estimating the probability of collision between two spacecraft. Common methods of collision risk assessment use either Monte Carlo analyses or a Gaussian probability distribution function (pdf). When compared to Monte Carlo methods, non-intrusive PCE techniques greatly reduce the number of samples required to approximate the a posteriori pdf. Additionally, PCEs make no assumption that the propagated distribution will be Gaussian, i.e., the resulting expansion provides information on higher moments such as skewness. Accounting for these higher moments allows for a more accurate estimate of collision probability, thereby reducing false alarms and improving operation safety. PCE methods for propagating the absolute state orbit uncertainty have already been demonstrated[1]. Instead, this paper will describe the techniques available for approximating the pdf describing the relative distance between two satellites, which allows for the quantification of collision probability.

PCEs provide a means for approximating the solution of a stochastic ordinary differential equation that is square-measurable, possibly non-Gaussian, with respect to the input uncertainties. PCE-based techniques use a projection of the stochastic solution onto a basis of orthogonal polynomials in stochastic variables that is dense in the space of finite-variance random variables. Non-intrusive PCE methods treat a given differential equation solver as a black box and use resulting solutions to generate the PCE at a future point in time. Applications of non-intrusive polynomial chaos to astrodynamics, specifically the propagation of orbit state uncertainty, has already been demonstrated. For the tests considered to date, only 100-600 samples must be propagated to yield a representation of desired accuracy. The number of samples depends on the nonlinearity of the system, but previous tests include a highly nonlinear Molniya case.

Other methods exist for propagating orbit uncertainty, some of which have already been demonstrated for non-Gaussian conjunction assessment. The most common methods consist of Gaussian mixtures, state transition tensors (STT), or differential algebra (DA). The Gaussian mixture methods approximate the solution to the Fokker-Planck equation as an expansion of Gaussian distributions. Depending on the degree of nonlinearity present in the system, such systems may require the

[1] B. Jones, A. Doostan, G. Born, "Nonlinear Propagation of Orbit Uncertainty Using Non-Intrusive Polynomial Chaos", Accepted for publication in the Journal of Guidance, Control, and Dynamics, April 2012.

propagation of many Gaussians, thereby reducing their computational efficiency. Current demonstrations of the STT and DA methods rely on general perturbation methods for their implementation. General perturbation methods simplify the equations of motion to reduce computation time, but at the cost of reduced accuracy. Hence, the STT and DA methods may not provide solution precisions necessary for highly accurate collision risk assessment. The non-intrusive polynomial chaos methods allow for the use of existing special perturbation propagators, thereby allowing for higher propagation accuracy. In terms of computation cost, they are more costly than the STT and DA methods, but, depending on the scenario, may require fewer samples than Gaussian mixture methods.

This paper represents the first application of PCE methods to quantifying the risk of satellite collision. The cases considered for this paper will also extend to larger initial uncertainty volumes, thereby expanding on results already demonstrated for PCE-based uncertainty propagation. Such methods may allow for computationally efficient, and accurate, collision risk assessment. This is especially important for satellites flying in a relatively close formation, such as NASA's Magnetic-Multi-Scale (MMS) mission or any fractionated spacecraft.

We have already tested PCE-based methods of collision assessment for a MMS-like mission. In cases where the collision risk is small, but non-zero, a highly-accurate representation of the collision pdf is required. Generating an accurate estimate via Monte Carlo requires tens, if not hundreds, of thousands of orbit propagations. However, PCE methods generate an accurate estimate using a couple of hundred sample propagations. This paper will present these results, refinements in the techniques to improve accuracy, and expand to include other tests cases of interest to the community, e.g., close approach between two satellites in low-Earth orbit with large, non-Gaussian, orbit uncertainties.