FORMATION FLYING BY OUTPUT FEEDBACK CONTROLLERS

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Abstract: *In this paper formation flying along a circular orbit is considered. For relative orbit transfer, output feedback controllers are employed. Both continuous time controllers and pulse controllers are considered. In each case, a state feedback controller with desired total velocity change and settling time is designed by the linear quadratic regulator theory varying the weight parameter on the input. Then the identity observer is introduced and the observer gain is designed by thequadratic regulator theory for the dual system. The initial condition of the observer is determined as a small perturbation of the initial condition of the relative dynamics. Varying the weight parameter on the input of the dual system, the total velocity change for the relative orbit transfer and the settling time are calculated as functions of the parameter. The observer gain is designed which maintain the total velocity change and the settling time of the state feedback controller. An example is given to illustrate the design method. A state feedback and an observer gain for both continuous time and pulse inputs are designed and the controlled trajectories are given.*

Keywords: *Formation Flying, Hill-Clohessy-Wiltshire Equations, Output Feedback, Total Velocity Change, Settling Time.*

1. Introduction

The relative motion of a follower satellite with respect to the leader in a given circular orbit is described by nonlinear autonomous differential equations. The linearized equations around the null solution are known as Hill-Clohessy-Wiltshire (HCW) equations [1, 2]. The in-plane and out-of-plane motions are independent. The out-of-plane motion is always periodic, while the in-plane motion becomes periodic if the so-called CW condition is satisfied. Periodic solutions of the HCW equations are used by many authors as reference orbits of formation flying [2, 3, 4, 5, 6, 7]. Ichimura and Ichikawa [8] considered an open time formation problem by impulse control, where the number of impulses, the impulse times and the final position on the reference orbit are all free, and found optimal three (single) impulse strategies for the in-plane (out-of-plane) motion. The HCW system has the property of null controllability with vanishing energy and the L_2 norm of the input steering the initial state to the origin can be made arbitrarily small by taking a large control interval [10]. The consequence of this property is that the L_2 norm of the stabilizing feedback control designed by the algebraic Riccati equation of the linear quadratic regulator (LQR) theory decreases to zero as the weight on the state decreases to zero. The L_1 norm of the stabilizing feedback control, which represents the fuel consumption, also decreases monotonically with the weight on the state [7]. Using this property, suboptimal feedback controls are designed, which have the L_1 norm close to the minimum ΔV of the optimal strategies [7]. Feedback controls were also designed in [8] using the discretized system and the LQR theory. The NCVE property is maintained in the discrete-time system [10]. Jifuku, Ichikawa and Bando [11] considered the open time formation problem of [8] using pulse control and derived optimal three (single) pulse strategies for the in-plane (out-of-plane) motion. The optimal impulse strategies of [8] are derived as the limits of optimal pulse strategies as the pulse width goes to zero. They also designed suboptimal feedback controls using the discrete-time LQR theory.

Feedback controls used for asymptotic formation acquisition in [7, 8, 11] are all state feedback controls, and to employ them, the state of the HCW system is needed. In this paper we relax this condition, and consider the asymptotic formation problem under partial observation. First we consider the case of continuous time control and observation, and assume that the system is observable. As is known, the observer gain can be designed by the dual algebraic Riccati equation when the system is observable. Hence observer gains with different weight parameters can be designed. The dual of the HCW equations is NCVE. We shall examine how the observergain affects the ΔV of the output feedback controls. For this purpose we fix a suboptimal feedback control designed as in [7], design the observer gain with weight parameter and compute the ΔV of the output feedback control as a function of the weight parameter. We compare the ΔV of the state feedback and the output feedback. Using these results, we propose suboptimal output feedback controls. Secondly, we consider the case of pulse control. In this case, we set the sampling time equal to the quater of the period of the circular orbit. To design feedback and observer gains, we use the discretized system and the LQR theory. The discrete-time system remains NCVE. The case of impulse control is given as the limiting case of pulse width equal to zero.

2. Equations of relative motion

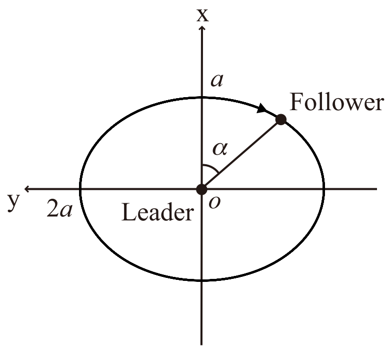
Consider a leader satellite in a circular orbit of radius R_0. The orbit rate and the period are given respectively by $n = (\mu_e/{R_0}^3)^{1/2}$ and $T = 2\pi/n$, where $\mu_e \equiv GM_e$ is the gravitational parameter of the Earth, G the universal gravitational constant and M_e the mass of the Earth. Introduce a coordinate system (x, y, z), fixed at the center of mass of the leader, where the x axis is along the radial direction, the y axis along the flight direction of the leader, and the z axis is out of the orbit plane and completing the right-handed reference frame. Then the equations of motion [1, 2] are given by

$$
\begin{aligned}
\ddot{x} &= 2n\dot{y} + n^2(R_0 + x) - \frac{\mu_e(R_0 + x)}{[(R_0 + x)^2 + y^2 + z^2]^{3/2}} + u_x, \\
\ddot{y} &= -2n\dot{x} + n^2 y - \frac{\mu_e y}{[(R_0 + x)^2 + y^2 + z^2]^{3/2}} + u_y, \\
\ddot{z} &= -\frac{\mu_e z}{[(R_0 + x)^2 + y^2 + z^2]^{3/2}} + u_z,
\end{aligned}
\tag{1}
$$

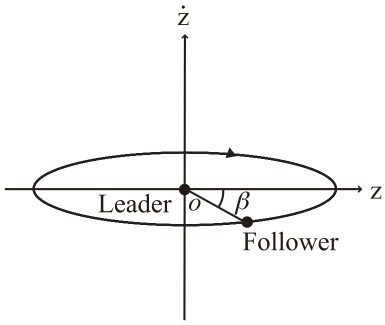
where (u_x, u_y, u_z) are the thrust accelerations. The linearized equations

$$
\begin{aligned}
\ddot{x} &= 3n^2 x + 2n\dot{y} + u_x, \\
\ddot{y} &= -2n\dot{x} + u_y, \\
\ddot{z} &= -n^2 z + u_z
\end{aligned}
\tag{2}
$$

are well-known as Hill-Clohessy-Wiltshire equations [1, 2].



(a) Elliptic orbit: in-plane.



(b) Elliptic orbit: out-of-plane.

Fig. 1. Periodic orbit.

For a given initial condition $\boldsymbol{x}(t_0) = [x_0\ y_0\ \dot{x}_0\ \dot{y}_0]^T$, the in-plane motion with $u_x = u_y = 0$ is parametrized by four constants a, c, d and α as

$$
\begin{aligned}
x(t) &= 2c + a\cos[n(t-t_0)+\alpha], \\
y(t) &= d - 3nc(t-t_0) - 2a\sin[n(t-t_0)+\alpha], \\
\dot{x}(t) &= -an\sin[n(t-t_0)+\alpha], \\
\dot{y}(t) &= -3nc - 2an\cos[n(t-t_0)+\alpha],
\end{aligned}
$$

where

$$
\begin{aligned}
&a = \left[(3x_0 + 2\frac{\dot{y}_0}{n})^2 + (\frac{\dot{x}_0}{n})^2\right]^{1/2}, \ c = 2x_0 + \frac{\dot{y}_0}{n}, \ d = y_0 - \frac{2\dot{x}_0}{n}, \\
&\cos\alpha = -\frac{1}{a}\left(3x_0 + \frac{2\dot{y}_0}{n}\right), \ \sin\alpha = -\frac{\dot{x}_0}{na}.
\end{aligned} \tag{3}
$$

Moreover,

$$
\left(\frac{x(t)-2c}{a}\right)^2 + \left(\frac{y(t)-d+3nc(t-t_0)}{2a}\right)^2 = 1.
$$

When $c = 0$, the trajectory is periodic with period $T = 2\pi/n$ and forms an ellipse with center $(0, d)$. The parameters a and d represent the size of the ellipse and the deviation of the center of the ellipse from the origin respectively, while the parameter α indicates the initial position on the ellipse as shown by Fig. 1 (a). The figure corresponds to the case $c = d = 0$. If relative orbits encircle the leader spacecraft, they are useful for rendezvous and formation flying. If $c \neq 0$, y contains the drift term $-3nc(t-t_0)$. The out-of-plane motion is given by $z(t) = b\cos(nt+\beta)$, where $b = [z_0^2 + (\dot{z}_0/n)^2]^{1/2}$, $\cos\beta = z_0/b$ and $\sin\beta = -\dot{z}_0/(bn)$. It is a sinusoidal motion as shown by Fig. 1 (b).

The state space form of Eq. (1) is given by

$$
\dot{\boldsymbol{x}} = A\boldsymbol{x} + B\boldsymbol{u} + Bg(\boldsymbol{x}), \tag{4}
$$

where

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 3n^2 & 0 & 0 & 2n & 0 & 0 \\ 0 & 0 & -2n & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -n^2 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

and

$$g(\boldsymbol{x}) = \begin{bmatrix} -3n^2x - (R_0 + x)(\mu/R^3 - n^2) & -y(\mu/R^3 - n^2) & -z(\mu z/R^3 - n^2) \end{bmatrix}^T.$$

The HCW system is described by (A, B) and the in-plane and out-of-plane motion are independent and are described respectively by (A_1, B_1) and (A_2, B_2), where

$$A_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 3n^2 & 0 & 0 & 2n \\ 0 & 0 & -2n & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 0 & 1 \\ -n^2 & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

3. Design of feedback and observer gains

Recall that the linear part of (4) is the HCW system, which is null controllable with vanishing energy (NCVE). This is a property that any state can be steered to the origin with an arbitrary small amount of control energy in the L_2 (square integral) sense, provided that the time duration is free. The system (A, B) is NCVE if and only if (A, B) is controllable and $Re\ \lambda \leq 0$ for every eigenvalue λ of A [9, 7, 10]. The second equivalent condition is that $X = 0$ is the unique nonnegative solution of the singular algebraic Riccati equation

$$A^T X + XA - XBR^{-1}B^T X = 0, \tag{5}$$

where R is any positive definite matrix. The HCW system (A, B) is NCVE, because it is controllable and the set of eigenvalues of A is $\{0, 0, \pm in, \pm in\}$. For the HCW system, the following is true [10].

Theorem 3..1. Let (A, B) be the HCW system and $Q \geq 0$. If $(\sqrt{Q}, A)$ is detectable, there exists a unique nonnegative solution X to the algebraic Riccati equation

$$A^T X + XA + Q - XBR^{-1}B^T X = 0 \tag{6}$$

such that $A - BR^{-1}B^T X$ is stable. If $(\sqrt{Q}, A)$ is observable, $X > 0$. As Q decreases to zero monotonically (equivalently, if R increases to infinity), X decreases monotonically to zero.

Let $\boldsymbol{x}_0$ be the initial condition of the follower and let $\boldsymbol{x}_{f0}$ be the initial condition of a virtual satellite in the final orbit. The dynamics of the follower and the virtual satellite are given respectively by

$$\begin{aligned} \dot{\boldsymbol{x}} &= A\boldsymbol{x} + B\boldsymbol{u}, \ \boldsymbol{x}(0) = \boldsymbol{x}_0, \\ \dot{\boldsymbol{x}}_f &= A\boldsymbol{x}_f, \ \boldsymbol{x}_f(0) = \boldsymbol{x}_{f0}. \end{aligned} \tag{7}$$

The feedback control

$$\boldsymbol{u} = -F\boldsymbol{\epsilon} = -R^{-1}B^T X\boldsymbol{\epsilon}, \ \boldsymbol{\epsilon} = \boldsymbol{x} - \boldsymbol{x}_f \tag{8}$$

bring the follower asymptotically to the final orbit. In fact

$$\dot{\boldsymbol{\epsilon}} = (A - BF)\boldsymbol{\epsilon}.$$

For a fixed Q, set $R = 10^r I$. Then the L_2-norm of the feedback (8) converges to zero as $r \to \infty$. Similarly, the L_1-norm of the feedback (8), which is equivalent to ΔV, decreases monotonically. Hence for a given upper bpund of L_1-norm, an admissible feedback can be designed by choosing r appropriately. If the state $\boldsymbol{x}$ is not available, the feedback controller (8) is not feasible. Suppose the observation of the system is given by

$$\boldsymbol{y} = C\boldsymbol{x}. \tag{9}$$

To estimate $\boldsymbol{x}$ an observer

$$\dot{\hat{\boldsymbol{x}}} = A\hat{\boldsymbol{x}} + B\boldsymbol{u} + H(\boldsymbol{y} - C\hat{\boldsymbol{x}}), \ \hat{\boldsymbol{x}}(0) = \hat{\boldsymbol{x}}_0 \tag{10}$$

is introduced, where $A - HC$ is stable. Then the estimation error $\boldsymbol{e} = \boldsymbol{x} - \hat{\boldsymbol{x}}$ decays to zero because

$$\dot{\boldsymbol{e}} = (A - HC)\boldsymbol{e}. \tag{11}$$

The feedback control (8) is replaced by

$$\boldsymbol{u} = -R^{-1}B^T X\hat{\boldsymbol{\epsilon}}, \ \hat{\boldsymbol{\epsilon}} = \hat{\boldsymbol{x}} - \boldsymbol{x}_f. \tag{12}$$

Then

$$\begin{aligned} \dot{\boldsymbol{\epsilon}} &= A\boldsymbol{x} - BF(\hat{\boldsymbol{x}} - \boldsymbol{x}_f) - A\boldsymbol{x}_f \\ &= (A - BF)\boldsymbol{\epsilon} + BF\boldsymbol{e}. \end{aligned}$$

H ence $\hat{\boldsymbol{x}} - \boldsymbol{x}_f$ converges asymptotically to zero. The observer gain H is designed through the dual Riccati equation

$$AY + YA^T + Q_0 - YC^T R_0^{-1} CY = 0 \tag{13}$$

and is given by

$$H = YC^T R_0^{-1}. \tag{14}$$

To examine the L_1-norm of the controller (12), Q_0 is fixed and R_0 is parametrized as $R_0 = 10^{r_o} I$.

4. Pulse control and sampled observations

The pulse instants of the optimal policies for in-plane motion and out-of-plane motion in [11] are different, it is convenient to consider two motions separately. First consider the in-plane motion. For simplicity, the equality $nt_0 = \alpha_0$ with $0 \leq \alpha_0 \leq \pi$ is assumed. Then the position of the follower at $t = 0$ is $\boldsymbol{x}(0) = [a_0 \; 0 \; 0 \; -2a_0 n]^T$. And the initial position of the follower at t_0 is $\boldsymbol{x}(t_0) = \exp(A_1 t_0)\boldsymbol{x}(0)$.

Because pulse times t_j for optimal strategies in [11] are of the form $t_j = jT/2$, pulse inputs $\boldsymbol{u}_j$ on $[jT/2 - \frac{\tau}{2}, jT/2 + \frac{\tau}{2}]$ are considered, where τ is the pulse width. Set $\boldsymbol{x}_{j+1} = \boldsymbol{x}(jT/2 + \frac{\tau}{2})$. Then

$$\boldsymbol{x}_{j+1} = A_{1d}\boldsymbol{x}_j + B_{1d}\boldsymbol{u}_j, \tag{15}$$

where $A_{1d} = \exp(A_1 T/2)$ and $B_{1d} = \int_0^\tau \exp(A_1 t)dt B_1$ are given by

$$A_{1d} = \begin{bmatrix} 7 & 0 & 0 & \frac{4}{n} \\ -6\pi & 1 & -\frac{4}{n} & -\frac{3\pi}{n} \\ 0 & 0 & -1 & 0 \\ -12n & 0 & 0 & -7 \end{bmatrix}, \quad B_{1d} = \begin{bmatrix} \frac{1}{n^2}(1-\cos n\tau) & \frac{2}{n^2}(n\tau - \sin n\tau) \\ -\frac{2}{n^2}(n\tau - \sin n\tau) & \frac{4}{n^2}(1-\cos n\tau) - \frac{3}{2}\tau^2 \\ \frac{1}{n}\sin n\tau & \frac{2}{n}(1-\cos n\tau) \\ -\frac{2}{n}(1-\cos n\tau) & \frac{4}{n}\sin n\tau - 3\tau \end{bmatrix}.$$

The dynamics (7) of the virtual satellite is discretized as

$$\boldsymbol{x}_{f,j+1} = A_{1d}\boldsymbol{x}_{f,j},$$

where $\boldsymbol{x}_{f,0} = \boldsymbol{x}_f(\frac{\tau}{2}) = \exp(A_1\tau/2)\boldsymbol{x}_f(0)$ and $\boldsymbol{x}_f(0)$ is the initial condition of the virtual satellite on the final orbit. Then the error vector $\boldsymbol{\epsilon}_j = \boldsymbol{x}_j - \boldsymbol{x}_{f,j}$ satisfies the equation:

$$\boldsymbol{\epsilon}_{j+1} = A_{1d}\boldsymbol{\epsilon}_j + B_{1d}\boldsymbol{u}_j, \tag{16}$$

where $\boldsymbol{\epsilon}_0 = \boldsymbol{x}_0 - \boldsymbol{x}_{f,0}$.

Recall that a linear discrete time system (A, B) is NCVE if any initial state of the system can be steered to the origin by a control with arbitrary small l^2 norm [7, 10]. Necessary and sufficient conditions for this are that (A, B) is controllable and $|\lambda| \leq 1$ for any eigenvalue λ of A. Because (A_{1d}, B_{1d}) is controllable and the eigenvalues of A_{1d} are $(1, 1, -1, -1)$, Eq. (15) and hence Eq. (16) are NCVE. As in the previous subsection, feedback controls are designed via the algebraic Riccati equation (ARE) of a linear quadratic regulator, i.e.,

$$X = {A_{1d}}^T X A_{1d} + Q - {A_{1d}}^T X B_{1d}(R + {B_{1d}}^T X B_{1d})^{-1}{B_{1d}}^T X A_{1d}, \tag{17}$$

where $Q \geq 0$, $R > 0$ and $(\sqrt{Q}, A_{1d})$ is assumed to be observable. Then there exists a unique positive definite stabilizing solution of Eq. (17). The feedback control

$$\boldsymbol{u}_j^* = -(R + {B_{1d}}^T X B_{1d})^{-1}{B_{1d}}^T X A_{1d}\boldsymbol{\epsilon}_j. \tag{18}$$

is stabilizing, and $\boldsymbol{\epsilon}_j \to 0$ as $j \to \infty$. Hence the follower tracks the virtual satellite asymptotically, and the asymptotic transfer to the final orbit is assured. Note that if the system is NCVE, $X = 0$ is the unique nonnegative solution of the ARE with $Q = 0$ and that $X \to 0$ as $Q \to 0$ (or R increases to infinity) [10]. Moreover, the l^2 norm of the feedback control Eq. (18) goes to zero. Recall that the total velocity change of the feedback control is identified with the l^1 norm. As shown in [11], it also decreases as $Q \to 0$ (or as $R \to \infty$).

When the state is not available, the sampled observation

$$\boldsymbol{y}_j = C_1 \boldsymbol{x}_j \tag{19}$$

is assumed and the observer

$$\hat{\boldsymbol{x}}_{j+1} = A_{1d}\hat{\boldsymbol{x}}_j + B_{1d}\boldsymbol{u}_j + H_d(\boldsymbol{y}_j - C_1\hat{\boldsymbol{x}}_j) \tag{20}$$

is introduced. The feedback control (18) is replaced by

$$\boldsymbol{u}_j^* = -(R + {B_{1d}}^T X B_{1d})^{-1} {B_{1d}}^T X A_{1d} \hat{\boldsymbol{\epsilon}}_j, \tag{21}$$

where $\hat{\boldsymbol{\epsilon}}_j = \hat{\boldsymbol{x}}_j - \boldsymbol{x}_{f,j}$. The observer gain H_d is designed by the dual Riccati equation

$$Y = A_{1d} Y X A_{1d}^T + Q_0 - A_{1d} Y C_{1d}^T (R_0 + C_{1d} Y C_{1d}^T)^{-1} C_{1d} Y A_{1d}^T, \tag{22}$$

As in the case of consinuous control, Q_0 is fixed and R_0 is varied as $R_0 = 10^{r_0} I$.

Now consider the out-of-plane motion. Suppose the follower lies on the orbit (b_0, β_0) with $nt_0 = \beta_0$. Then at $t = 0$, $\boldsymbol{x}(0) = [b_0 \ 0]^T$. Because pulse times t_j for optimal strategies in this case are of the form $t_j = jT/4$, a natural choice of the pulse interval for feedback control is $T/4$. Now

$$A_{2d} \equiv \exp(A_2 T/4) = \begin{bmatrix} 0 & \dfrac{1}{n} \\ -n & 0 \end{bmatrix}, \quad B_{2d} = \begin{bmatrix} \dfrac{1}{n^2}(1 - \cos n\tau) \\ \dfrac{1}{n}\sin n\tau \end{bmatrix}$$

and (A_{2d}, B_{2d}) is controllable. The discrete time system (A_{2d}, B_{2d}) is NCVE, and feedback controllers and observer gains can be designed by the regulator theory as in the case of the in-plane motion.

5. Simulation results

In our simulation the height h_c of the circular orbit of the leader satellite is assumed to be 400 km. Then the period of this circular orbit is $T = 5565$ s (1.55h) and the orbit rate $n = 1.1291 \times 10^{-3}$ rad/s. The parameters a, c and d below are given in km's and the time in seconds. The parameters of initial and final orbits for the in-plane motion are assumed respectively $(a_0, c_0, d_0, \alpha_0) = (50, 0, 0, \pi)$, $(b_0, \beta_0) = (10, \pi)$ and $(a_f, c_f, d_f, \alpha_f) = (5, 0, 0, *)$, $(b_f, \beta_f) = (0, 0)$.

Table 1. Parameters of circular orbit

Constants	Values
R_e	6378 km
μ	398601 km^3/s^2
R_0	R_e+400 km
T	5565 s

5.1. Continuous control

To design feedback controls via the ARE (6), the matrices $Q = 10^{-7}I$ and $R = 10^{r}I$ are assumed. To introduce a stopping rule for simulation, let d_{min} denote the minimum distance of a point in the final orbit from its center and v_{min} the minimum velocity of the chaser in the final orbit. The follower is regarded in the final orbit when $|e_x|, |e_y| < 10^{-5} \times d_{min}$ and $|\dot{e_x}|, |\dot{e_y}| < 10^{-5} \times v_{min}$, where $e_x = x - x_f$ and $e_y = y - y_f$. The time required to get on the final orbit is called the settling time and is denoted by T_s. The total velocity change as a function of the parameter r for the in-plane motion is given in Fig. 2(a). It decreases monotonicaly to a constant, as r increases. Finally, Fig. 2(b) is the settling time, which is monotone increasing. The L_1-norm and the settling time can be regarded as performance indices. $T_s = 24644$ [s] for $r = 7$ and it is less than $5T = 27825$. The L_1-norm is 34.935[m/s]. This state feedback denoted by F will be used below.

In the case of partial observation, the observation matrix C is assumed to be

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}. \tag{23}$$

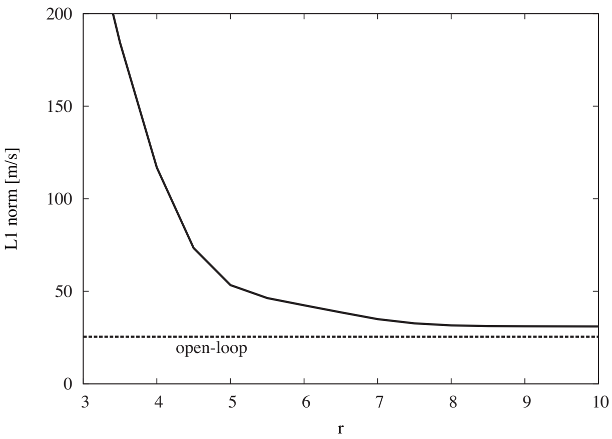
The performance of the output feedback (12) depends critically on the initial condition $\hat{\boldsymbol{x}}_0$. If $\hat{\boldsymbol{x}}_0 = \boldsymbol{x}_0$, the controller (12) coincides with the state feedback controller (8). To examine the role of the observer gain, the initial condition of observer is given as

$$\hat{\boldsymbol{x}}_0 = \boldsymbol{x}_0 + \begin{bmatrix} dx & dy & 0 & 0 & 0 & 0 \end{bmatrix}^T \tag{24}$$

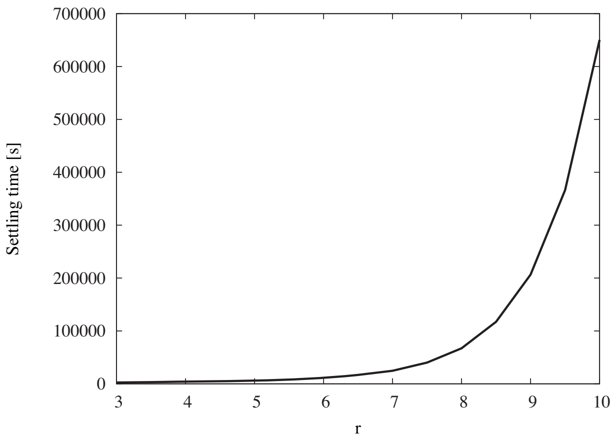
where dx, dy indicates the uncertainty of the initial position. The feedback gain F with settling time less that $5T$ is fixed. To design observer gains, $Q_0 = 10^{-7}I$ is fixed and $R_0 = 10^{r_0}I$ is considered. Fig. 3 shows L_1-norm and settling time as functions of r_0 and the relation between them, when $dx = dy = 2$ and the output feedback controller is used. Two functions are no longer monotone functions of r_0, but the range of r_0 exists such that the L_1-norm and settling time less than those of the state feedback controller. Figs. 4 and 5 are the controlled trajectories for $r = 6$ and $r = 7$ respectively.

5.2. Pulse control

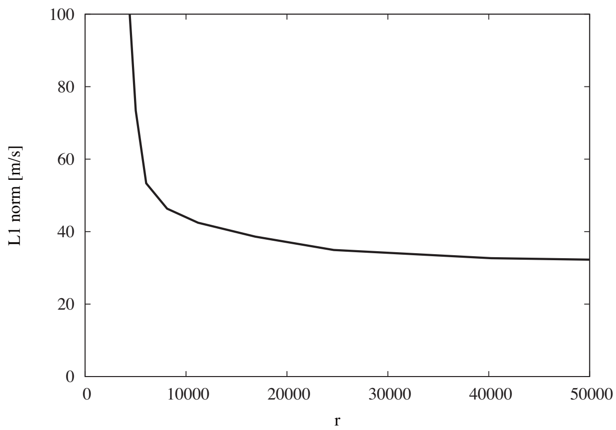
The pulse width is taken as $\tau = T/30 = 3.09$ minutes. The parameters a, c and d below are given in km's. The parameters of initial and final orbits are assumed respectively $(a_0, c_0, d_0, \alpha_0) =$



(a) L_1-norm vs. r.

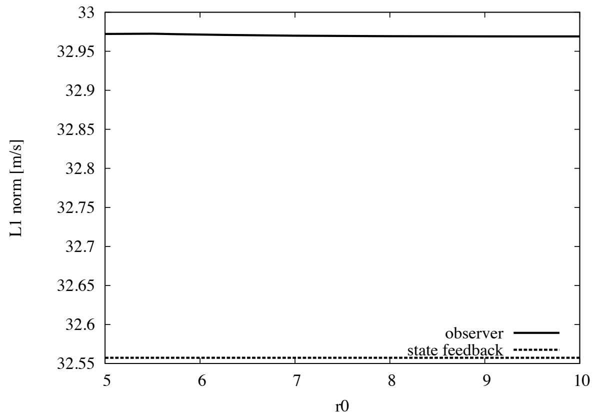


(b) T_f vs. r.

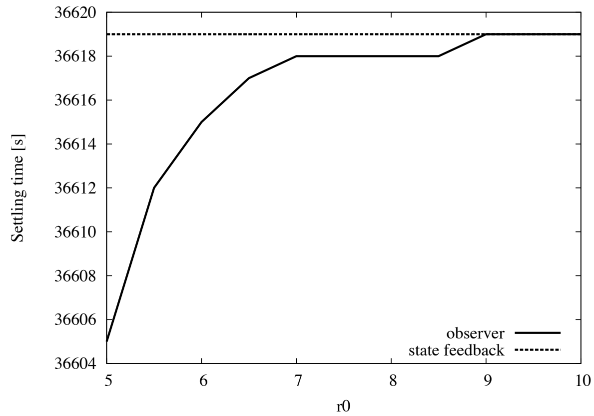


(c) L_1-norm vs. T_f.

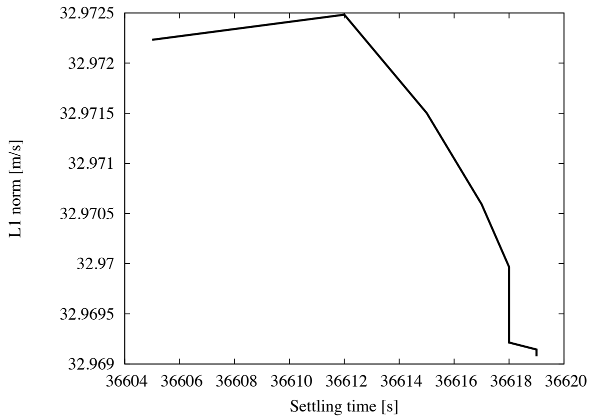
Fig. 2. L_1-norm and settling time: state feedback.



(a) L_1-norm vs. r.



(b) T_f vs. r.



(c) L_1-norm vs. T_f.

Fig. 3. **L_1-norm and settling time: output feedback(r=7).**

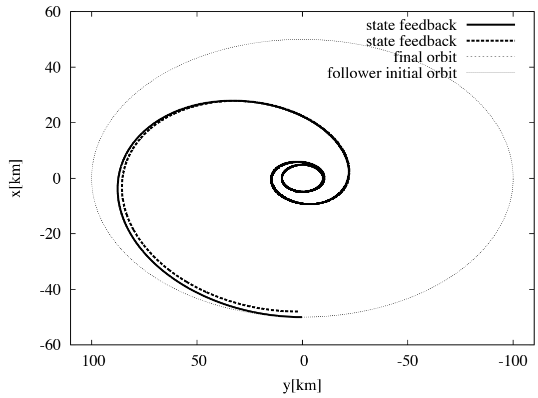


Fig. 4. Controlled trajectory $dx, dy = 2$ **(**$r = 6, r_0 = 5$**).**

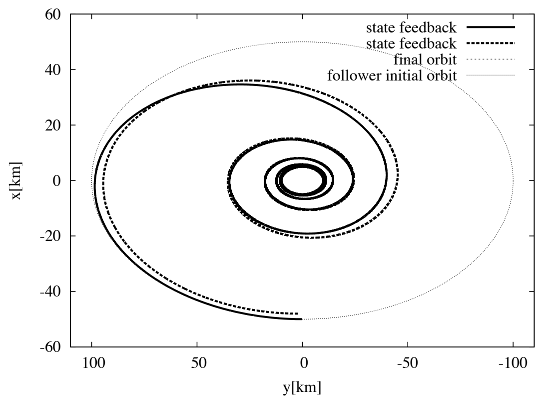
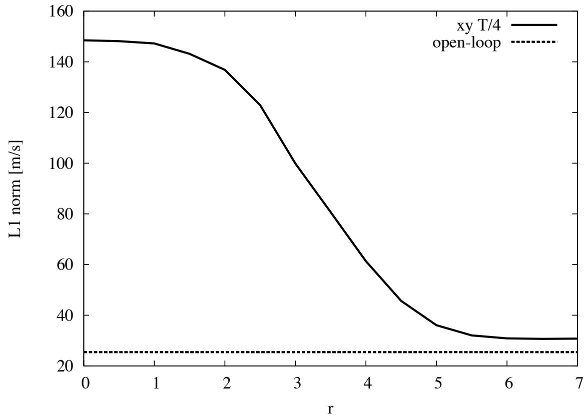
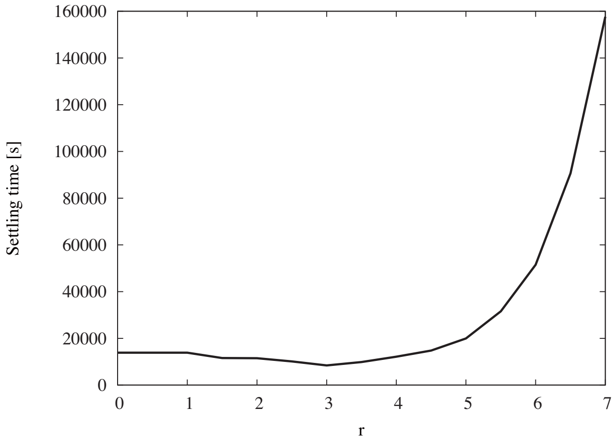


Fig. 5. Controlled trajectory $dx, dy = 2$**(**$r = 7, r_0 = 5$**).**

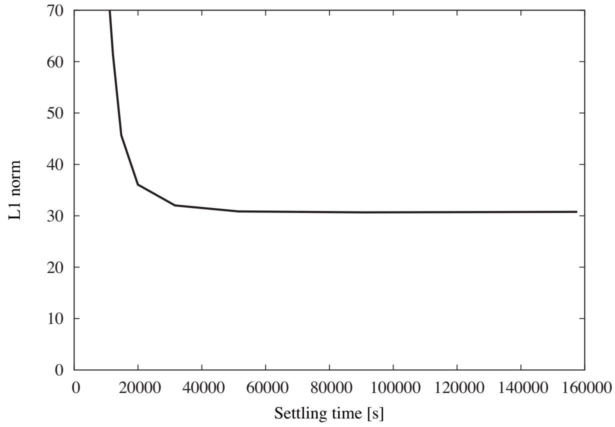
$(50, 0, 0, \pi/2)$, $(b_0, \beta_0) = (10, \pi/2)$ and $(a_f, c_f, d_f, \alpha_f) = (5, 0, 0, *)$, $(b_f, \beta_f) = (0, 0)$. Because $\alpha_0 = \pi/2$, it follows that $t_0 = T/4$. Thus the follower is initially (at time t_0) at $(x_0, y_0, z_0) = (0, -100, 0)$. To design a feedback pulse controller, Q is fixed small as $Q = 10^{-5}I$ and the parametrized $R = 10^r I$ of the ARE is considered. The total velocity change as a function of the parameter r for the in-plane motion is calculated and given in Fig. 6(a). Note that it decreases to the value of an optimal three pulse strategy as r increases. Thus the LQR theory provides a good design method of feedback controllers. The settling time is given in Fig. 6(b). As r increases, ΔV decreases but the settling time increases. Hence a compromise is necessary between these two performance indices. $T_s = 51519$ [s] for $r = 5$ and it is less than $10T = 61500$. The L_1-norm is 30.855[m/s]. This state feedback denoted by F_d will be used below.
To design observer gains, $Q_0 = 10^{-7}I$ is fixed and $R_0 = 10^{r_0}I$ is considered. Fig. 7 gives L_1-norm and settling time as functions of r_0 and the relation between them, when the output feedback controller is used. Two functions are no longer monotone functions of r_0, but the range of r_0 exists such that the L_1-norm and settling time less than those of the state feedback controller. Figs. 8 and 9 are the controlled trajectories for $r = 6$ and $r = 7$ respectively.



(a) L_1-norm vs. r.

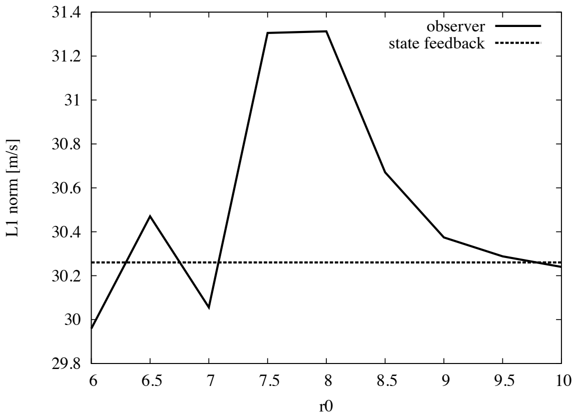


(b) T_f vs. r.

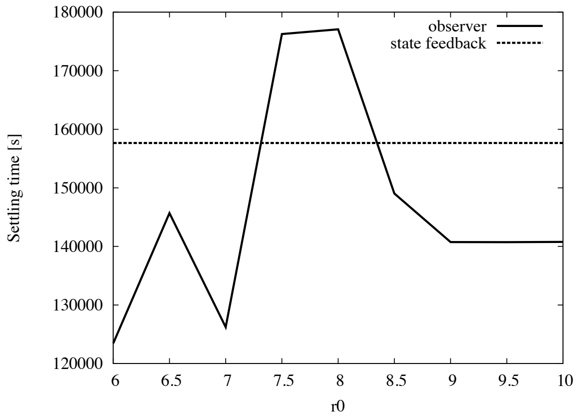


(c) L_1-norm vs. T_f.

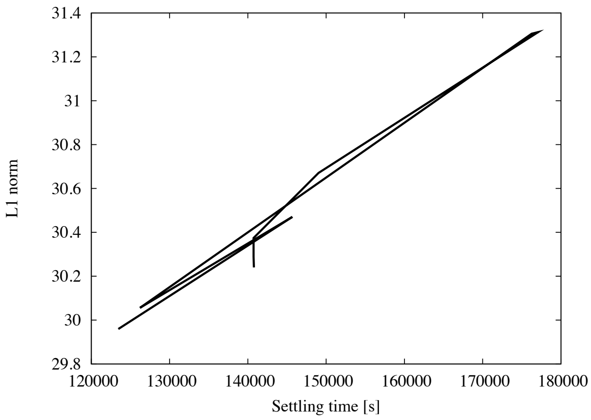
Fig. 6. L_1-norm and settling time: state feedback.



(a) L_1-norm vs. r_0.



(b) T_f vs. r_0.



(c) L_1-norm vs. T_f.

Fig. 7. L_1-norm and settling time: state output feedback (r=7).

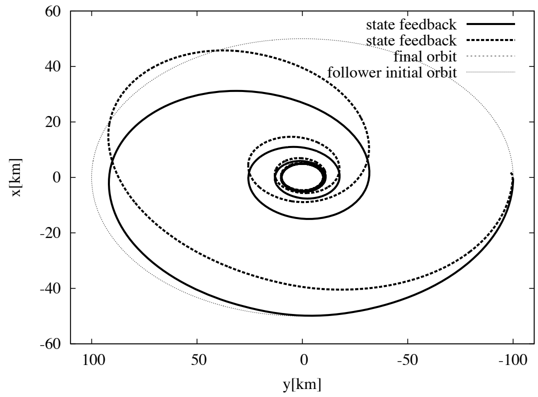


Fig. 8. Controlled trajectory $dx, dy = 2$ **(**$r = 5, r_0 = 5$**).**

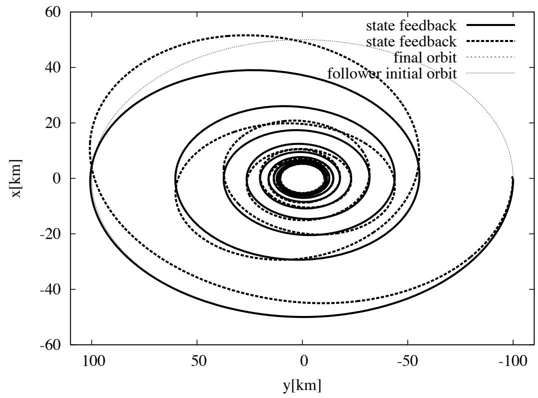


Fig. 9. Controlled trajectory $dx, dy = 2$**(**$r = 6, r_0 = 5$**).**

6. Conclusions

In this paper formation flying along a circular orbit by output feedback controllers is considered. Both continuous time controllers and pulse controllers are considered. In each case, a state feedback controller with desired total velocity change and settling time is designed by the linear quadratic regulator theory varying the weight parameter on the input. The total velocity change is a monotone decreasing function of the parameter, while the settling time is monotone increasing function. To realize output feedback, the identity observer is introduced and the observer gain is designed by thequadratic regulator theory for the dual system. The initial condition of the observer is determined as a small perturbation of order 10 % of the initial condition of the relative dynamics. Varying the weight parameter on the input of the dual system, the total velocity change for the relative orbit transfer and the settling time are calculated as functions of the parameter. They are no longer monotone functions of the parameter, the observer gain can be designed which keeps the increase of the total velocity change within 10%. A numerical example is given to illustrate the design method.

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