# Formation Flying by Output Feedback Controllers

Mai Bando *
Kyoto University, Gokasho, Uji, Kyoto 611-0011, Japan
and
Akira Ichikawa †
Nanzan University, Seto, Aichi 489-0863, Japan

The relative motion of a follower satellite with respect to the leader in a given circular orbit is described by nonlinear autonomous differential equations. The linearized equations around the null solution are known as Hill-Clohessy-Wiltshire (HCW) equations [1, 2]. The in-plane and out-of-plane motions are independent. The out-of-plane motion is always periodic, while the in-plane motion becomes periodic if the so-called CW condition is satisfied. Periodic solutions of the HCW equations are used by many authors as reference orbits of formation flying [2–7]. Ichimura and Ichikawa [8] considered an open time formation problem by impulse control, where the number of impulses, the impulse times and the final position on the reference orbit are all free, and found optimal three (single) impulse strategies for the in-plane (out-of-plane) motion. The HCW system has the property of null controllability with vanishing energy and the $L_2$ norm of the input steering the initial state to the origin can be made arbitrarily small by taking a large control interval [9]. The consequence of this property is that the $L_2$ norm of the stabilizing feedback control designed by the algebraic Riccati equation of the linear quadratic regulator (LQR) theory decreases to zero as the weight on the state decreases to zero. The $L_1$ norm of the stabilizing feedback control, which represents the fuel consumption, also decreases monotonically with the weight on the state [7]. Using this property, suboptimal feedback controls are designed, which have the $L_1$ norm close to the minimum $\Delta V$ of the optimal strategies [7]. Feedback controls were also designed in [8] using the discretized system and the LQR theory. The NCVE property is maintained in the discrete-time system [9]. Jifuku, Ichikawa and Bando [10] considered the open time formation problem of [8] using pulse control and derived optimal three (single) pulse strategies for the in-plane (out-of-plane) motion. The optimal impulse strategies of [8] are derived as the limits of optimal pulse strategies as the pulse width goes to zero. They also designed suboptimal feedback controls using the discrete-time LQR theory.

Feedback controls used for asymptotic formation acquisition in [7, 8, 10] are all state feedback controls, and to employ them, the state of the HCW system is needed. In this paper we relax this condition, and consider the asymptotic formation problem under partial observation. First we consider the case of continuous time control and observation, and assume that the system is observable. As is known, the observer gain can be designed by the dual algebraic Riccati equation when the system is observable. Hence observer gains with different weight parameters can be designed. The dual of the HCW equations is NCVE. We shall examine how this property affects the $\Delta V$ of the output feedback controls. For this purpose we fix a suboptimal feedback control designed as in [7], design the observer gain with weight parameter and compute the $\Delta V$ of the output feedback control as a function of the weight parameter. We compare the $\Delta V$ of the state feedback and the output feedback. Using these results, we propose suboptimal output feedback controls. Secondly, we consider the case of pulse control. In this case, we set the sampling time equal to the half of the period of the circular orbit as in [8, 10]. To design feedback and observer gains, we use the discretized system and the LQR theory. The discrete-time system remains NCVE. The case of impulse control is discussed as the limiting case of pulse width equal to zero. Finally, we shall discuss the extension to the Tschauner-Hempel equations of the elliptic orbit [7].

## References

[1] Wie, B., "Space Vehicle Dynamics and Control", AIAA, 2008, pp. 299-303.

[2] Alfriend, K. T., Vadali, S. R., Gurfil, P., How, J. P., and Breger, L. S., "Spacecraft Formation Flying: Dynamics, Control and Navigation," Elsevier, Amsterdam, 2010, pp. 83-103.

*Mission Research Fellow, Research Institute for Sustainable Humanosphere, +81-774-38-3883, m-bando@rish.kyoto-u.ac.jp. Member AIAA

†Professor, Department of System Design and Engineering, aichika@nanzan-u.ac.jp. Member AIAA

[3] Kapila, V., Sparks, A. G., Buffington, J. M., and Yang, Q., "Spacecraft Formation Flying: Dynamics and Control," *Journal of Guidance, Control, and Dynamics*, Vol. 23, No. 3, May-June, 2000, pp. 561-564.

[4] Kang, W., Sparks, A., and Banda, S., "Coordinate Control of Multisatellite Systems," *Journal of Guidance, Control, and Dynamics*, Vol. 24, No. 2, 2001, pp. 360-368.

[5] Campbell, M. E., "Planning Algorithm for Multiple Satellite Clusters," *Journal of Guidance, Control, and Dynamics*, Vol. 26, No. 5, 2003, pp. 770-780.

[6] Vaddi, S. S., Alfriend, K. T., Vadali, S. R., and Sengupta, P., "Formation Establishment and Reconfiguration Using Impulsive Control," *Journal of Guidance, Control, and Dynamics*, Vol. 28, No. 2, 2005, pp. 262-268.

[7] Shibata, M., and Ichikawa, A., "Orbital Rendezvous and Flyaround Based on Null Controllability with Vanishing Energy," *Journal of Guidance, Control, and Dynamics*, Vol. 30, No. 4, 2007, pp.934-945.

[8] Ichimura. Y., and Ichikawa. A., "Optimal Impulsive Relative Orbit Transfer Along a Circular Orbit," *Journal of Guidance, Control, and Dynamics*, Vol. 31, No. 4, July-August, 2008, pp. 1014-1027.

[9] Ichikawa, A., "Null Controllability with Vanishing Energy for Discrete-Time Systems," *Systems & Control Letters* , Vol. 57, No. 1, 2008, pp. 34-38.

[10] Jifuku, R., Ichikawa, A., and Bando, M., "Satellite Formation by Pulse Control Along a Circular Orbit," *Journal of Guidance, Control, and Dynamics*, Vol. 34, No. 5, September-October, 2011, pp. 1329-1341.