ANALYSIS OF A NEW NONLINEAR SOLUTION OF RELATIVE ORBITAL MOTION

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ABSTRACT

The objective of this research is to verify and explore a newly derived solution for relative translational motion between two space objects. This new solution, described in Ref. 1, is more accurate than existing solutions and was derived using multi-dimensional convolution modeling theory—specifically, Volterra kernels.[2] The solution, henceforth referred to as the Newman-Omran model, is similar to the Hill's-Clohessy-Wiltshire[3,4] (HCW) solution, in that it assumes two-body gravity and a circular chief orbit. However, the Newman-Omran model includes nonlinear terms, which the HCW and other linear models do not, and therefore its application is not limited to objects in close proximity. In fact, to the author's knowledge, the Newman-Omran model is the first successful derivation of a closed-form relative motion solution that captures nonlinear effects. It also yields additional dynamical system insight compared to other analytical relative motion solutions. The work presented here will entail full-scale verification and validation of the Newman-Omran model and investigation of the utility of the model for such practical mission applications as spacecraft navigation and orbit transfer.

Relative motion involves the motion of one or more space objects relative to a reference object or point in space, where all objects are under the influence of the same central gravitational body. The reference object is typically referred to as the "chief," while the other objects are referred to as "deputies." The relative motion between a chief and deputy is commonly characterized in the local-vertical, local-horizontal (LVLH) coordinate frame, as displayed in Fig 1, where x, y, and z denote the radial, along-track, and cross-track directions, respectively. The simplest model of relative motion in this frame can be generally described as a drifting ellipse in 3-D space whose projection in the x-y plane is a 2x1 ellipse (twice as long in the y direction as in x) and whose projection along the z axis is a constant amplitude oscillation. This motion is depicted in Fig 2.

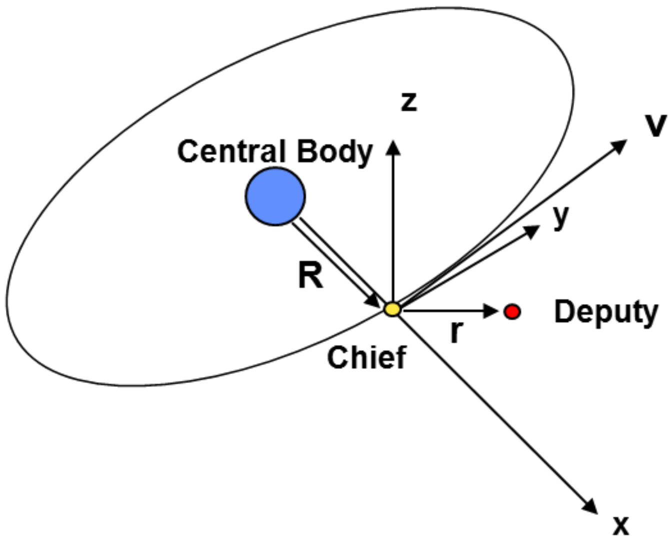

Figure 1. Characterization of an orbiting deputy and chief, with the LVLH frame defined.

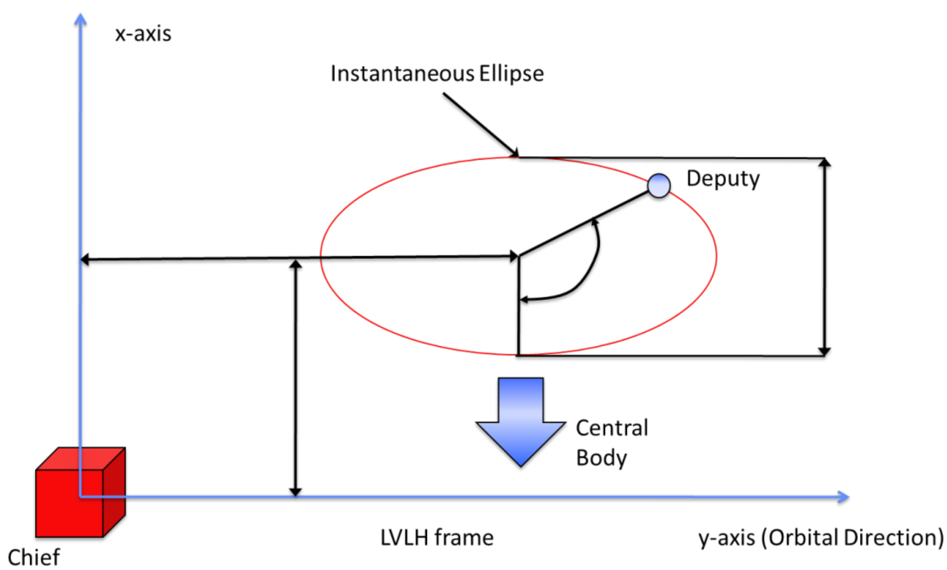

Figure 2. Typical motion of a deputy relative to a chief in the LVLH frame.

Each relative motion model in the literature can be categorized by how it addresses three basic questions:

- What natural forces on each object are to be accounted for?
- Are the objects in close proximity, i.e., on closely neighboring orbits?
- Is the chief on a circular orbit?

Most natural forces included in a relative motion model have a nonlinear effect. If the objects are assumed to be in close proximity, a common practice is to linearize the motion about the chief orbit. Further, if the chief is assumed to be on a circular orbit, the relative motion dynamics are time-invariant. The HCW model, for example, involves both of the aforementioned assumptions and results in a *linear time-invariant* (LTI) solution. The Newman-Omran model assumes a circular chief orbit but not close proximity, and therefore results in a *nonlinear time-invariant* (NLTI) solution. Both models account only for two-body gravity force, therefore it is natural to compare the two. The Newman-Omran model is given as follows:

$$
\begin{aligned}
&\begin{bmatrix} x(t) & y(t) & z(t) \end{bmatrix}^T \\
&= \begin{bmatrix}
\left(4-3C_{n_o t}\right)x_o + \frac{\dot{x}_o}{n_o} S_{n_o t} + \left(1-C_{n_o t}\right)\left(\frac{2\dot{y}_o}{n_o} - \frac{3\left(2x_o^2-y_o^2-z_o^2\right)}{2R_o}\right) + \frac{6x_o y_o}{R_o}\left(n_o t - 2S_{n_o t}\right) \\
y_o - \frac{\dot{y}_o}{n_o}\left(3n_o t - 4S_{n_o t}\right) - 6x_o\left(n_o t - S_{n_o t}\right) - \frac{2\dot{y}_o}{n_o}\left(1-C_{n_o t}\right) + \frac{3\left(2x_o^2-y_o^2-z_o^2\right)}{R_o}\left(n_o t - 2S_{n_o t}\right) + \frac{3x_o y_o}{2R_o}\left(8 - 3n_o^2 t^2 - 8C_{n_o t}\right) \\
z_o C_{n_o t} + \frac{\dot{z}_o}{n_o} S_{n_o t} + \frac{3x_o z_o}{R_o}\left(1-C_{n_o t}\right)
\end{bmatrix}
\end{aligned}
$$

where R_o is the chief's orbit radius, n_o is the chief's mean motion, $S_{n_o t} = \sin\left(n_o t\right)$ and $C_{n_o t} = \cos\left(n_o t\right)$. The linear terms in this result are identical to the HCW solution. However, the Newman-Omran model contains nonlinear (bilinear and quadratic) terms as well. The comparison between the HCW and Newman-Omran models can be summarized as follows:

- The HCW model indicates no coupling between cross-track (z) motion and radial/in-track (x-y) motion. The Newman-Omran model indicates that the three motions are coupled, in that the $z(t)$ expression contains an x_o term and the $x(t)$ and $y(t)$ expressions each contain a z_o term.

- The HCW model indicates that the x motion is sinusoidal of frequency n_o, while the Newman-Omran model shows that this motion is a combination of sinusoidal motion with a secular drift term proportional to time.

- The HCW model indicates that the y motion is sinusoidal of frequency n_o with a drift term proportional to time; whereas the Newman-Omran model reveals a more complicated drift term proportional to time, as well as a quadratic drift term.

In this paper, the Newman-Omran solution will be verified against numerical two-body propagation for a variety of test cases, using the HCW model as a point of comparison. The characteristics of this solution will be explored in full detail, including a description of the geometry of the motion allowed by the solution and a determination of the conditions to be satisfied for no drift. In addition, the limits on the accuracy of this solution will be explored for increasingly large chief-deputy separation. Mission applications of the Newman-Omran model will also be investigated, including:

- **Relative navigation**: this involves determining a spacecraft's motion relative to a reference space object, given range and/or angle measurements between the objects. Of particular interest is determining the degree of observability afforded by the Newman-Omran model under angles-only navigation.[5]
- **Orbit transfer/maneuver planning**: whereas the HCW model is often considered accurate enough for planning a spacecraft's maneuvers for proximity operations relative to another object, a higher-order model could accommodate large enough separation distances to provide an orbit transfer algorithm between arbitrary points in space. The Newman-Omran model will be investigated in this respect and compared to a standard Lambert transfer.

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