EXTREMAL ANALYTICAL CONTROL AND GUIDANCE SOLUTIONS FOR POWERED DESCENT AND PRECISION LANDING

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ABSTRACT

The minimum- fuel optimal control and guidance problems for a planetary powered descent and landing at a specified landing site are formulated. Spacecraft is considered as a point mass with variable mass moving in a uniform and drag-free gravity field with gravitational and thrust accelerations from a given initial manifold in the state space to a given landing point. Although the theory of optimal trajectories is complete in the case of motion in the uniform gravity field, the number of studies on analytical trajectory and control solutions for optimal motion, and the corresponding guidance design solutions with their applications to specific maneuvers is very limited. In this work, the analytical solutions for three-dimensional, extremal and optimal powered descent and landing trajectories are presented. The state vector consists of the position and velocity vector components and the mass. The control vector includes the thrust magnitude, thrust direction cosines and an auxiliary parameter used to count for the thrust magnitude constraint. It is shown that the optimality conditions and the analysis of canonical equations reveal five different optimal control regimes and corresponding behavior of the switching function and the cost function for each control regime. These regimes determine the control sequence and consequently, the 14-th- order canonical system of equations are integrated completely analytically in terms of time, thereby providing 14 new arbitrary integration constants. It is shown that the integrals represent highly nonlinear relationships between the integration constants and the state and co-state variables, and other parameters. These constants are found by utilizing the initial and final conditions and the transversality conditions. The studies presented in this paper describe the first attempt to demonstrate that the integration constants play an important role in the design of an envelope of the descent trajectories and in the real-time targeting and guidance design for precision landing. Qualitative analysis can be conducted without any specific numerical results to construct the envelope of descent and landing trajectories, thereby accounting for the uncertainties, such as atmospheric conditions, wind turbulences and etc. In particular, the proposed solutions can be used to determine a manifold of the initial conditions from which the lander can be guided to prescribed landing site or to its vicinity determined by a terminal manifold in the state space. One of the new utilities of the analytical solutions is that the landing site can be re-designated on-board as many times as needed by redefining the integration constants, thereby solving the real-time re-targeting problem to allow for a hazard detection and avoidance and to provide a safe pin-point landing. Similar to

the Apollo guidance concepts, the platform and guidance frames are introduced to determine the metrics of landing accuracy. As a new development of these concepts, the consecutive planes of the platform and guidance frames will contain a sequence of the trajectories formed by re-targeting procedures. The distance between a newly re-designated landing site, which coincides with the origin of the guidance frame and the nominal landing site, which is contained in the newly formed platform frame determines the accuracy of landing.

A series of simulations have been conducted to demonstrate the utility of the proposed trajectory control, targeting and guidance solutions to achieve safe and precision landing on Mars. The algorithms used in the simulations do not have iterative procedure or approximations except for the numerical computation of some constants. These algorithms were designed to incorporate the solution of the minimum landing error problem by appropriate selection of the integration constants, maneuver time and the control parameters. The simulation results show that the analytical solutions obtained in this work can be implemented to generate feasible and extremal or optimal powered descent and landing trajectories with minimum landing error.
Feasibility of the trajectories is understood in the sense of connecting the initial and final conditions with some nonzero landing errors in the final position and velocity vectors, and satisfying the mass, control and time constraints. The results were comparable to those of the studies that use convex optimization and other numerical-analytical methods. Due to their explicitness, the proposed solutions can be used to determine manifolds of the initial and final conditions for atmospheric entry and powered descent, to generate the trajectory envelopes, to provide re-targeting design aimed to achieve the pin- point landing and to incorporate attitude guidance. Integration of the trajectory control, targeting and guidance solutions with attitude guidance is subject of further studies.