RELATIVISTIC ACCELERATION OF PLANETARY ORBITERS

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ABSTRACT

The Geocentric and Barycentric Celestial Reference Systems (GCRS and BCRS) are two commonly used relativistic systems of space-time coordinates for spacecraft orbit propagation and determination. Despite their names, these systems do not just differ from each other by a geometric translation, but also by a generalized Lorentz transformation. When propagating the orbit of a planetary orbiter, it is often convenient for numerical as well as for modeling reasons to integrate the trajectory relative to the planet center. In that case, the relativistic reference system is the BCRS with a geometrically translated frame of reference for the space coordinates. In this paper, when it is said that a planetary orbiter trajectory is integrated in the BCRS, it actually means in the translated BCRS.

The time coordinates for the GCRS and BCRS systems are called Terrestrial Coordinate Time (TCT) and Barycentric Coordinate Time (TCB), respectively. Since along the world line of an observer on the surface of the Earth, TCT and TCB are, to a good approximation, affine functions of the proper time, it is convenient to scale the GCRS and BCRS space-time coordinates by a linear factor. The resulting scaled GCRS and BCRS systems time coordinates are called Terrestrial Time (TT) and Barycentric Dynamical Time (TDB), respectively.

For most Earth orbiters, orbit integration and determination is performed in the GCRS. In that system, the main corrections to the Newtonian formulation of the acceleration are a relativistic correction in the central gravitational acceleration from the Earth, the Coriolis accelerations due to the Lense-Thirring precession (from Earth rotation) and due to geodesic precession (mainly due to the Sun). These effects are usually nine or ten orders of magnitude smaller than the total acceleration and at least one order of magnitude smaller than non-gravitational accelerations (e.g. atmospheric drag for lower orbits and solar radiation pressure) whose accurate modeling is challenging. Hence if there is no experiment on-board that allows to reduce the uncertainty in the non-gravitational acceleration (e.g. accelerometers or drag compensation system) and if there is no requirement for orbit reconstruction accuracy approaching the centimeter level, it is usual to neglect these relativistic corrections to the acceleration altogether.

The GCRS is only suitable in the immediate vicinity of the Earth. Therefore orbit propagation and determination of interplanetary spacecraft is usually performed in the BCRS. This requires that space-time events at a ground station on Earth appearing in the modeling of the observables are

Lorentz-transformed from the geocentric to the barycentric system. It is in theory also possible for planetary orbiters to use a local relativistic planetocentric reference system with an associated planetocentric coordinate time when propagating the trajectory, however this would require a second Lorentz transformation from the planetocentric to the barycentric system in modeling the observables.

In the BCRS, the relativistic correction to the gravitational acceleration is given by the Einstein-Infeld-Hoffman (EIH) equation, possibly adding the Lense-Thirring acceleration of a nearby planet. In the vicinity of a planet, the ratio of the corrective relativistic terms to the total acceleration can easily be two orders of magnitude higher in the BCRS than in the planetocentric relativistic system. However, an appropriate Lorentz transformation of the total acceleration from one system to the other shows that both systems are equivalent to a very good accuracy.

ESOC's interplanetary orbit determination software uses the BCRS system to propagate trajectories and model observables. Until recently, however, only a simple relativistic corrective term due to the Sun was introduced in the acceleration. This term does not model the main relativistic correction when in the vicinity of a planet. Hence, it was decided to implement the full EIH equation. For the sake of validation, a comparison of the total acceleration of a spacecraft in the vicinity of the Earth has been performed between the barycentric and geocentric systems using Lorentz transformations on position, velocity and acceleration. In order to reach the required accuracy in the validation of the relativistic terms, it was necessary to improve the hitherto employed Lorentz transformation on velocity and acceleration by introducing additional corrective terms.

The BepiColombo spacecraft is a future Mercury orbiter that ESA is planning to launch in 2015 for a Mercury orbit insertion in 2022. Using accurate accelerometers to measure non-gravitational accelerations and multi-band radio tracking capabilities, it is expected that the radio-science experiment will allow to derive the lower degree and order coefficients of the Mercury gravity field and the tidal Love number. The corrective relativistic term in the gravitational acceleration will not be negligible in comparison to the expected non-gravitational acceleration and to the uncertainty in the acceleration due to the limited a priori knowledge on some Mercury physical parameters to be estimated.

This paper gives an introduction to the BCRS and GCRS reference systems, the Lorentz transformation, coordinate time and scaling of coordinates. It presents the formulations for the relativistic corrections to the acceleration in both the BCRS and the planetocentric system (GCRS in Earth case). It discusses the pros and cons of each system. It describes the steps that were taken at ESOC to compare the formulations in the two systems, improve the Lorentz transformation and validate the improved Lorentz transformation to the required accuracy. Finally it presents the effects of the relativistic corrections on the orbit propagation in the BCRS of ESA planetary orbiters Mars Express, Venus Express, BepiColombo as well as Rosetta at one of its Earth swing-by.