A.M.Lyapynov methodology in problems of modelling and dynamics of stabilization and orientation systems

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This work is devoted to the specific problems of avia-, aerospace systems, with reference to problems of the mathematical modelling, analysis and synthesis for the systems of stabilization and orientation with gyroscopic controlling elements (with using two-degrees gyroscopes). The research is developing the reduction principle, solving the decomposition problem for such systems by elaborated approach, that is based on A.M.Lyapunov methodology.

Nonlinearity, high dimensionality, multi-connectivity are causing the difficulties in obtaining of exact solution by analytical and analytic-computer methods in designing and control. It leads to the necessity of the simplifying of original model, with the revealing of main freedom degrees of the system, with the subsequent transition to the decomposed systems, to reduced submodels with the idealized physical properties. These problems are important both for general fundamental theory and for engineering applications; it is connected with the working-out of systematical methods for constructing of the correct reduced models and for determining of the acceptability domains in engineering practice.

In regard to the peculiarities of the stabilization and orientation systems with the gyroscopic controlling elements, it leads to the complex singularly perturbed problems, with the different singularities types, with critical cases, with the nonlinear singular generating systems as reduced systems.

The principal questions are discussed in research: the methodology of the reduction-decomposition for stability problems in first; the substantiation of reduced models in dynamics and control problems in second; the determination of conditions for the qualitative equivalence and correctness of reduced models.

Here above formulated problems are solved by strong method, following to the ideology of stability theory. General approach, based by A.M.Lyapunov, applied by N.G.Chetayev to mechanics problems, is extended here. The understanding of these problems via approach of singularly perturbed systems gives the perspective results both for theory and for applications. This method gives the efficient algorithm for systems of gyrostabilization and orientation. The constructiveness of this approach is illustrated on concrete examples of gyrostabilization systems; the cases of one-axis gyrostabilization, bi-axis gyrostabilization, … are considered for small satellites and for big stabilized objects (space stations).

Two principal tenets (stability postulate and singularity postulate) are accepted here as fundamental axioms. From this point, with reference to a specific problems and the peculiarities of gyrostabilization systems, the original objects are treated as the systems of the singularly perturbed class. Advanced approach is worked out, that is generated by the examples of concrete physical substance, those lead to the nonlinear, multi-dimensional dynamical problems (A.M.Lyapunov, N.G Chetayev, A.H.Nayfeh, R.O'Malley, L.K.Kuzmina, A.A.Voronov,…). For instance, in classical theory of the gyrosystems there is the well-known precessional system (PS) that is approximate reduced model (D.R.Merkin, K.Magnus,…). But it should be considered only as the result of mathematical decomposition

(D.Siljak) for original system. This shortened model does not take into consideration the high-frequency components of motion, and one is only formalized construction. The problem of precessional theory acceptability is not solved as yet. It is mechanics problem, that from mathematical view point is one of singularly perturbed class (L.K.Kuzmina). Also in dynamics of stabilization and orientation systems, that are used for spacecrafts control, the different reduced models are actually introduced (A.Yu.Ishlinskiy, B.V.Raushenbakh,…) as simplified models for original systems, with taken assumption about big (or small) parameters. From mechanical view point these models are ones with less number of freedom degrees (in general – non-integer one). But the strong validity of such reduced mathematical models is not examined in applied works. For this there are not the effective methods, the efficient simple algorithms. It is also mechanics problems of singularly perturbed class. Therefore main problems are: the elaboration of regular steps in modelling of complex systems, with possibility of substantiation for this mathematical decomposition; the constructing of correct reduced models (as comparison systems) by strong mathematical manners; the developing of regular methods of acceptability criteria (with the precise wording of the concept of "acceptability").

In this paper these problems are solved for singular systems in critical cases, that are inherent for gyrostabilization systems. Employed method permits to divide the variables and the motions in such system into different-frequency components, to get the conditions for admissible decomposition both for physical one and for mathematical. For considered gyrostabilization systems the regular scheme of decomposition is worked out, new interesting results are obtained. In reference to the problems of multi-channel, multi-axis stabilization systems with gyroscopic controlling elements, this approach allowed:

– to elaborate the systematical procedures of decomposition for original models, with the dividing original state variables on low-, middle-, high-frequency ones;
– to get an asymptotic models as minimal approximate models (on N.N.Moiseev);
– to determine the acceptability conditions for decomposed models;
– to validate approximate theory (PS theory);
– to obtain the decomposition conditions for original nonlinear models, with the possibility of splitting on separate channels for control and stabilization, in analysis and synthesis problems, including stability problems.

As illustration there is considered the family of the stabilization and orientation systems with gyroscopic controlling elements (the systems for small stabilized satellites, for big stabilized objects, …).

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