STABILITY ANALYSIS OF THE SPACECRAFT ATTITUDE WITH CANONICAL VARIABLES

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Abstract: *The objective of this paper is to analyze the stability of the rotational motion of a symmetrical spacecraft, in a circular orbit. The equilibrium points and regions of stability are established when components of the gravity gradient torque acting on the spacecraft are included in the equations of rotational motion, which are described by the Andoyer's variables. The nonlinear stability of the equilibrium points of the rotational motion is analyzed here by the Kovalev-Savchenko theorem, which ensures that the motion is Liapunov stable. In this theorem it is necessary to reduce the Hamiltonian in its normal form up to the fourth order by means of canonical transformations around the equilibrium points. With the application of the Kovalev-Savchenko theorem, it is possible to verify if they remain stable under the influence of the terms of higher order of the normal Hamiltonian. In this paper, numerical simulation are made for two hypothetical groups of artificial satellites. Several stable equilibrium points were determined and regions around these points have been established by variations in the orbital inclination and in the spacecraft principal moment of inertia. The present analysis can directly contribute in the maintenance of the spacecraft's attitude.*

Keywords: *Rotational motion, nonlinear stability, canonical variables, gravity gradient torque, normal form of Hamiltonian.*

1. Introduction

This work aims at analyzing the stability of the rotational motion of artificial satellites in circular orbit with the influence of gravity gradient torque, using the Andoyer's canonical variables. This stability analysis is very important in maintaining the attitude to ensure the success of a space mission.

In this paper, Kovalev-Savchenko theorem (*KST*) [1] is used for the study of the stability and it ensures that the motion is Liapunov stable if the following conditions are satisfied:

i. The eigenvalues of the reduced linear system are pure imaginary $\pm i\omega_1$ e $\pm i\omega_2$;

ii. $$k_1\omega_1 + k_2\omega_2 \neq 0 \quad (1)$$

is valid for all k_1 and k_2 integer satisfying the inequality $|k_1| + |k_2| \leq 4$;

iii. The Arnold determinant

$$D = -(\delta_{11}\omega_2{}^2 - 2\delta_{12}\omega_1\omega_2 + \delta_{22}\omega_1{}^2) \neq 0 \quad (2)$$

where $\delta_{\upsilon\upsilon}$ are the coefficients of the normal 4th order Hamiltonian.

The *KST* [1] was used for the stability analyses by Cabette et al [2,3] and Formiga [5]. The study of the stability of the rotational motion of artificial satellites in an elliptic orbit was developed in [2,3] and used the procedure presented by [4] to determine a normal form of the Hamiltonian up to the 4th order. The results of the analysis pointed out only few stables equilibrium points and the stability algorithm was very slow.

In [5] is described a numerical-analytical method for normalization of Hamiltonian systems with 2 and 3 degrees of freedom. The most important role of this work is the results obtained analytically for the generating function of 3rd order, necessary for determining the coefficients of the normal Hamiltonian of 4th order.

The objective of this paper is to optimize the stability analysis developed in [2,3] for the satellite in a circular orbit by applying the analytical expressions obtained in [5] for the coefficients of the normal 4th order Hamiltonian. The equilibrium points are established when terms associated with gravity gradient torque acting on the satellite are included in the equations of rotational motion. The stables regions around the equilibrium points are obtained by the variations in the orbital inclination and in the principal moment of inertia of the satellite.

In order to simplify the application of methods of stability in this is study, the Andoyer's variables [6] are used to describe the rotational motion of the satellite. These variables are represented by generalized moments (L_1, L_2, L_3) and by generalized coordinates (ℓ_1, ℓ_2, ℓ_3) that are outlined in Fig. 1. The angular variables ℓ_1, ℓ_2, ℓ_3 are angles related to the satellite system Oxyz (with axes parallel of the spacecraft's principal axes of inertia) and equatorial system OXYZ (with axes parallel to the axis of the Earth's equatorial system). Variables metrics L_1, L_2, L_3 are defined as: L_2 is the magnitude of the angular momentum of rotation $\overrightarrow{L_2}$; L_1 is the projection of $\overrightarrow{L_2}$ on the z-axis of principal axis system of inertia ($L_1 = L_2 \cos J_2$, where J_2 is the angle between the z-satellite axis and $\overrightarrow{L_2}$) and L_3 is the projection of $\overrightarrow{L_2}$ on the Z-equatorial axis ($L_3 = L_2 \cos I_2$, where I_2 is the angle between Z-equatorial axis and $\overrightarrow{L_2}$).

In this paper, numerical simulations are made for two hypothetical groups of artificial satellites, which have orbital data and physical characteristics similar to real satellites. The stables regions around the equilibrium points are obtained by the variations of the orbital inclination and variations in the principal moments of inertia of the satellite.

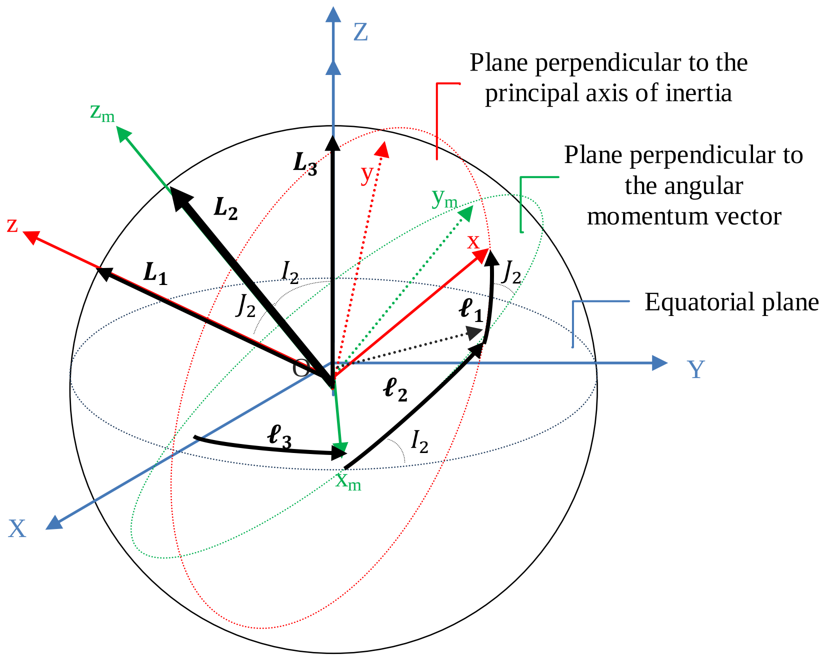


Figure 1 - Andoyer' canonical variables.

2. Equations of motion

Andoyer's variables $(L_1, L_2, L_3, \ell_1, \ell_2, \ell_3)$, defined above, are used to characterize the rotational motion of a satellite around its center of mass [6] and the Delaunay's variables describe the translational motion of the center of mass of the satellite around the Earth [8].

In this paper is assumed that the satellites are in a circular orbit, which differs from the study presented in [2]. This consideration was adopted to simplify the Hamiltonian of the problem, which is extensive [2] and to facilitate the stability analysis of the equilibrium points.

Thus, assuming that the satellites have well-defined circular orbit, the goal is to study the stability of the rotational motion of the satellite. Then the Hamiltonian of the problem is expressed in terms of the Andoyer and Delaunay variables (L, G, H, l, g, h) [8,9] as follows:

$$F(L_1, L_2, L_3, \ell_2, \ell_3, L, G, H, l, g, h) = F_o(L, L_1, L_2) + F_1(L_1, L_2, L_3, \ell_2, \ell_3, L, G, H, l, g, h)\ ; \quad (3)$$

where F_o is the unperturbed Hamiltonian and F_1 is the term of the Hamiltonian associated with the disturbance due to the gravity gradient torque, both are described respectively by [5]:

$$F_o = -\frac{\mu^2 M^3}{2L^2} + \frac{1}{2}\left(\left(\frac{1}{C} - \frac{1}{2A} - \frac{1}{2B}\right) {L_1}^2 + \frac{1}{2}\left(\frac{1}{A} + \frac{1}{B}\right) {L_2}^2\right) + \frac{1}{4}\left(\frac{1}{B} - \frac{1}{A}\right)\left({L_2}^2 - {L_1}^2\right)\cos 2\ell_1\ ; \quad (4)$$

$$F_1 = \frac{\mu^4 M^6}{L^6}\left[\frac{2C-A-B}{2}\hbar_1(\ell_m, L_n) + \frac{A-B}{4}\hbar_2(\ell_m, L_n)\right] \; ; \qquad (5)$$

where $m = 2,3$ and $n = 1,2,3$; A, B and C are the principal moments of inertia of the satellite on x-axis, y-axis and z-axis respectively ; $\hbar_1$ and $\hbar_2$ are functions of the variables (ℓ_m, L_n), where ℓ_2 and ℓ_3 appear in the arguments of cosines. The complete analytical expression for perturbed Hamiltonian F_1 is presented in [2] for eccentric orbit.

The equations of motion associated with the Hamiltonian F, Eq. 1, are given by:

$$\begin{cases} \frac{d\ell_i}{dt} = \frac{\partial F}{\partial L_i}, \\ \frac{dL_i}{dt} = -\frac{\partial F}{\partial \ell_i}; \end{cases} \qquad (i = 1,2,3). \qquad (6)$$

These equations are used to find the possible equilibrium points of the rotational motion when will be considered two of its principal moments of inertia equal, $B = A$ (symmetrical satellite). With this relationship, the variable ℓ_1 will not be present in the Hamiltonian, reducing the dynamic system to two degrees of freedom, a necessary condition for applying the stability theorem chosen for analysis of equilibrium points. The complete analytical expression for perturbed Hamiltonian F_1 for circular orbit and symmetrical satellite is presented in [9].

3. The Algorithm for Stability Analysis

To use the *KST* is necessary the normal Hamiltonian of the problem. It was discussed in [3,9] and the normal Hamiltonian $\mathcal{H}^o$ is an analytic function of generalized coordinates (q_v) and moments (p_v) to a fixed point P, expressed by [3,9]:

$$\mathcal{H}^o = \sum_{v=1}^{2} \frac{\omega_v}{2} \omega \mathcal{R}_v + \sum_{v,\upsilon=1}^{2} \frac{\delta_{v,\upsilon}}{4} \mathcal{R}_v \mathcal{R}_\upsilon + O_5 \; ; \qquad (7)$$

where O_5 represents higher order terms; ω_v is the imaginary part of eigenvalues associated with the matrix defined by the product of a 4th order matrix symplectic with the Hessian of the Hamiltonian expanded in Taylor series up to 2nd order around the equilibrium point; $\delta_{v,\upsilon}$ depend on the eigenvalues ω_v and the coefficients of the Hamiltonian expanded in Taylor series of 3rd and 4th order around the equilibrium point, which are presented analytically in Formiga [5]; and

$$\mathcal{R}_m = {q_m}^2 + {p_m}^2 \qquad \text{with} \qquad m = 1,2 \; . \qquad (8)$$

The *KST* says that a Hamiltonian reduced in its normal form up to 4th order, in the absence of the resonance condition of the eigenvalues associated and if the condition given by Eq. 2 is satisfied, it is guaranteed the existence of tori invariant in a neighborhood small enough of equilibrium position [10]. The associated process with the determination of the normal form of the Hamiltonian and the process of analyzing the stability of equilibrium points by the *KST* [6], are synthesized in a logical sequence of the algorithm and presented in Fig. 2.

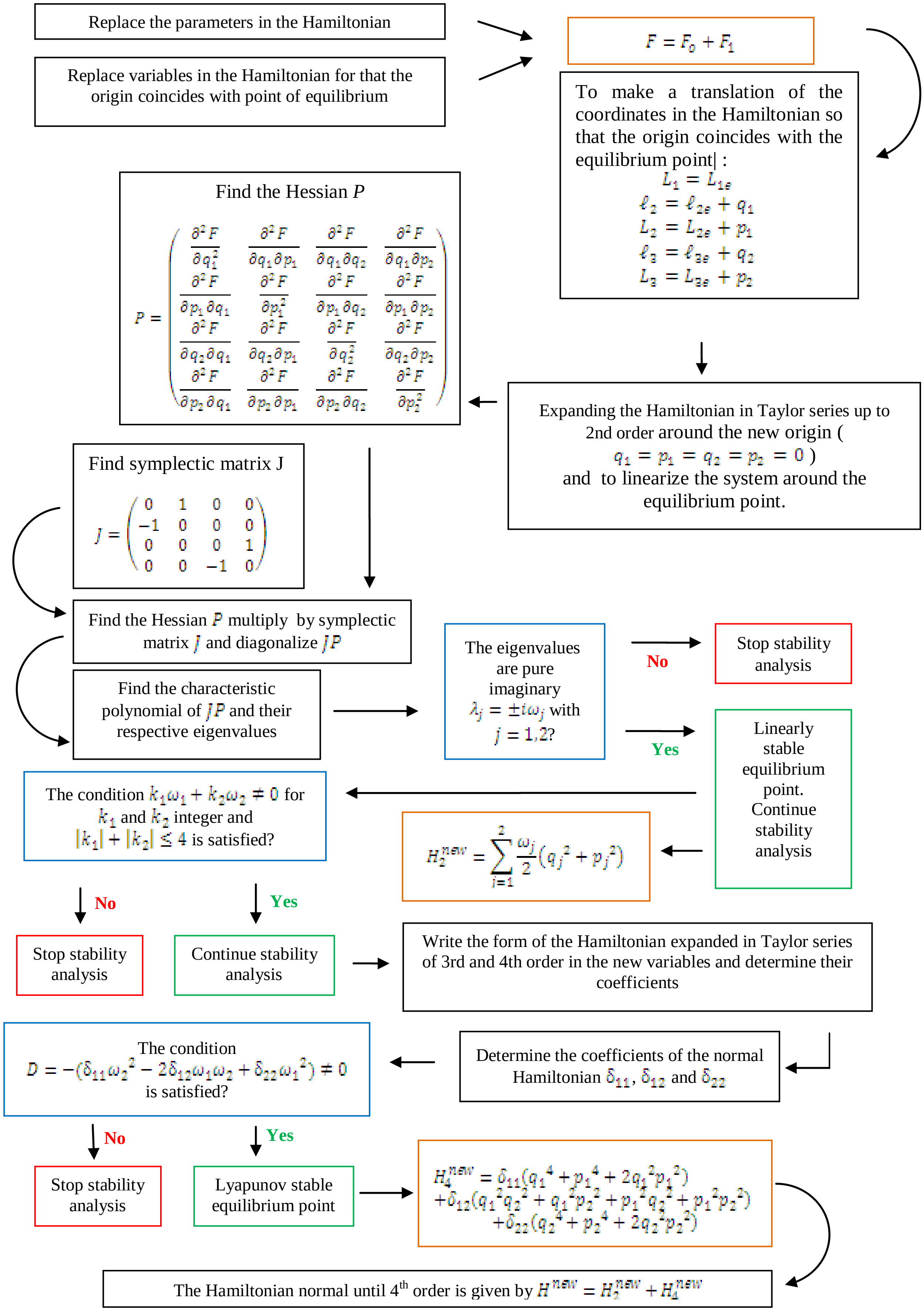


Figure 2 - Flowchart representative of the stability analysis of equilibrium points.

4. Numerical simulations

In order to do the numerical simulations, in this paper were considered two types of satellites: medium sized (*MS*), which has similar orbital characteristics with the American satellite PEGASUS [17] and small sized (*SS*), which has similar orbital characteristics of the First Brazilian Data Collection Satellite SCD-1 [2]. All the numerical simulations were developed using the software MATHEMATICA.

4.1. Results for the *MS* satellite

The initial data for the *MS* satellite are:

Orbital inclination: I = 0.5533rad;
Eccentricity: e = 0;
Orbital radius: r = 6959. 64km;
Principal moments of inertia on: x – axis, A = 0.39499 kg km^2;
y – axis, B = 0.39499 kg km^2;
z – axis, C = 0.10307 kg km^2;
Delaunay' variables: l = 0rad; g = 0rad; h = 0.5235rad;
L = 608336932. 2kg km^2/s;
G = 608336932. 2kg km^2/s;
H = 517570028. 7kg km^2/s.

There was found 60 equilibrium points, but only 2 were stables by the application of *KST*. The others 58 equilibrium points failed in the first condition of the *KST*. It could be observed that, when the first condition was not satisfied, the eigenvalues associated with the matrix JP were real or were not pure imaginary. This equilibrium point is not linearly stable. Table 1 shows two equilibrium points found in the simulations for *MS* satellite, one Lyapunov stable and other unstable. For this stable equilibrium point, the following values were found to the inclinations angles I_2 = 2.20566rad, J_2 = 0.86245rad, the spin velocity ω = 0.003735rad/s and the rotation period T =1680.4s. These values characterize the non-existence of singularities on the Andoyer' variables in this point, it means that the angles I_2 and J_2 are not null or close to zero. When these angles I_2 and J_2 are null or close to zero, the Andoyer' variables ℓ_1, ℓ_2 and ℓ_3 are undeterminate (by Fig. 1, it is possible to observe that is difficult to determine the intersection between the involved planes in the definitions of these variables). This analysis was performed for all equilibrium points found in the simulations.

Table 1. MS Satellite: A=B = 3.9499*10^{-1} kgkm^2, C = 1.0307*10^{-1} kgkm^2

Equilibium points	Lyapunov stable	Unstable
$L_1(kg.km^2/s)$	2.50725*10^{-4}	- 1.34202*10^{-11}
$L_2(kg.km^2/s)$	3.85388*10^{-4}	- 4.19645*10^{-10}
$L_3(kg.km^2/s)$	- 2.28562*10^{-4}	- 3.53991*10^{-10}
$l_2(rad)$	- 1.6*10^{-7}	0.0934286
$l_3(rad)$	0.5235	0.41354

4.1.1 Stables region around the equilibrium points

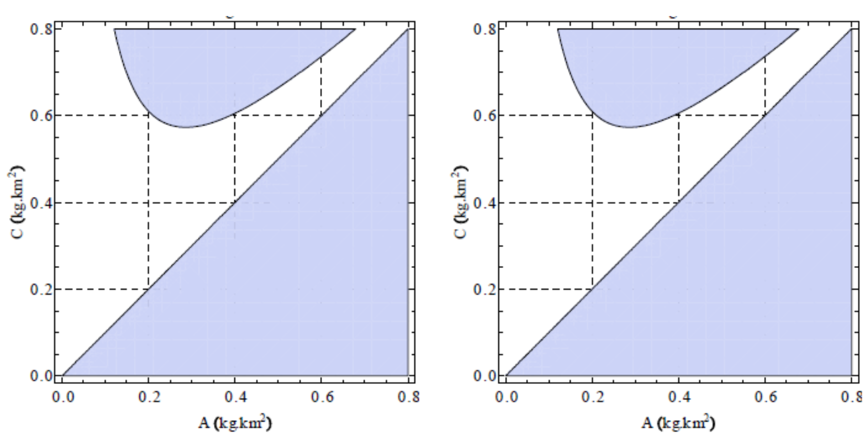
The behavior around the Liapunov stable equilibrium points were obtained by the variations in the orbital inclination (*I*) and in the principal moments of inertia of the satellite on x – axis (*A*) and on z - axis (*C*). For this procedure was used the algorithm developed by Silva [11], in which the Hamiltonian *F* is a function of (*A*, *C*, *I*) with the others parameters given by the others parameters of the equilibrium point. By this algorithm the analytical eigenvalues λ_i, *i* = 1,2, of the characteristic equation associated with the linear system of the rotational motion are given by:

$$\lambda_1 = \pm\sqrt{a(A,C,I)} \qquad (9)$$

$$\lambda_2 = \pm\sqrt{b(A,C,I)} \qquad (10)$$

with $a(A,C,I)$ and $b(A,C,I)$ are presented in Silva [11]. By this algorithm, with the variation of *A*, *C* and *I* , it is verified if the values of the *a(A, C, I)* and *b(A, C, I)* are smaller than zero. In this situation the eigenvalues are pure imaginary. The variations were arbitrarily chosen between 0 and 0.8 kgkm^2 for the principal moment of inertia and between 0 and 1rad for the orbital inclination.

Figure 3 show the result for the linear stability (eigenvalues are pure imaginary) when the orbital inclination is fixed (I_0 = 0.5533rad). There are two regions, one for the eigenvalue λ_1 and other for λ_2. In the Region 1 the blue color points out $a(A;C;I_0) < 0$ and in the Region 2 the blue color points out $b(A;C;I_0) < 0$, it means, by Eq. 9 and Eq.10, that the eigenvalues are pure imaginary. The results also show that in the linear stability there is a separation between the stable and unstable region when the spacecraft principal moments of inertia are equals.



Region 1- $a(A,C,I_0)$ *Region 2*- $b(A;C;I_0)$

Figure 3 – Stable regions around the equilibrium point for MS satellite, with I_0 =0.5533rad.

Figure 4 shows the analysis of the second condition for the *KST*, in the case that principal moment of inertia on the x –axis and the orbital inclination are fixed (A_0 = 0.39499kg km^2 and I_0 = 0.5533rad). It is possible to observe that the second condition ($k_1\omega_1^o + k_2\omega_2^o \neq 0$, k_1 and k_2 integer satisfying the inequality $|k_1| + |k_2| \leq 4$) is satisfied for C< 0.39499 kgkm^2 and C > 0.6028kgkm^2. The results conformed with the results show in Fig. 3, because the fixed values A_0 and I_0 in the Regions 1 and 2 of the Fig. 3, it can be observed that the eigenvalues are real or non-pure imaginary when

$$A=0.39499 \text{ kgkm}^2 < C < 0.6028\text{kgkm}^2.$$

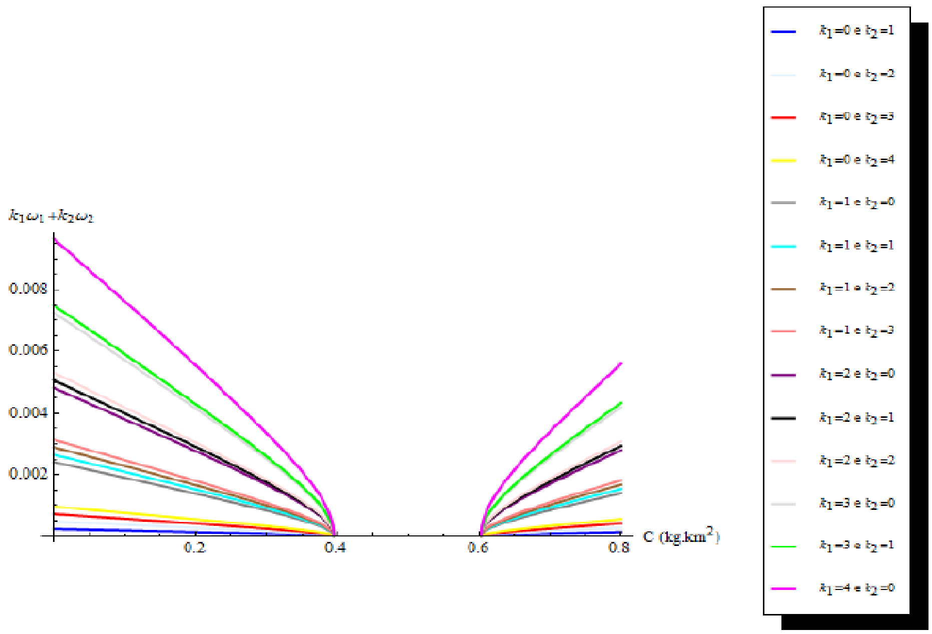


Figure 4 – Second condition for k_1 ≥0 e k_2 ≥ 0, considering A_0 =0.39499 kg km^2 and I_0 =0.5533rad.

Figure 5, 6 and 7 show the analysis for the third condition associated with the Arnold determinant D^o, given by Eq. 2. By Fig. 5, when the I_0 and A_0 are fixed, it is possible to see the unstable region for values of the principal moment of inertia on the z-axis (C), A_0<C<0.6028kgkm^2, it means that the rotational motion is nonlinear unstable ($D^o \neq 0$).

Figure 6 shows the values for the Arnold determinant when C_0 = 0.10307 kg km^2 and I_0=0.5533rad. In the region with A < 0.10307 kgkm^2 = C there is linear instability and the Arnold determinant doesn't assume any values because the eigenvalues are real or non-pure imaginary. When $A = C$ there isn't the non-linear instability, because $D^0 = 0$.

By Fig. 7, when A_0 and C_0 are fixed**,** there are two values for the orbital inclination in which the Arnold determinant is equal to zero: I = 0.159rad and I = 0.2353rad. These cases are associated with the tumbling of the satellite.

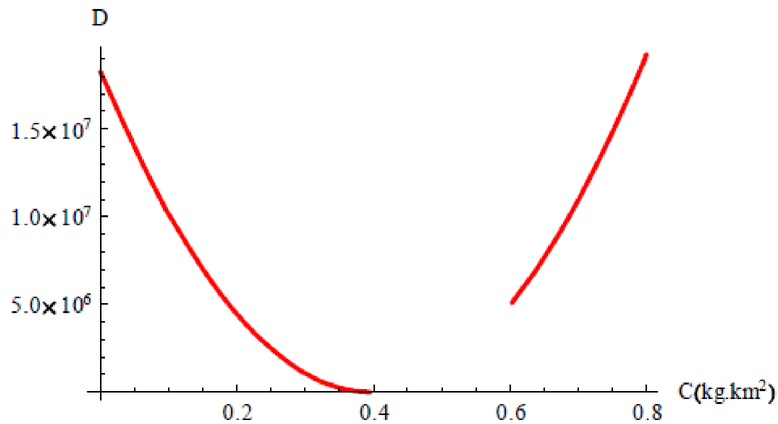


Figure 5 – Arnold determinant for A_0 = 0.39499kg km^2 and I_0 =0.5533rad.

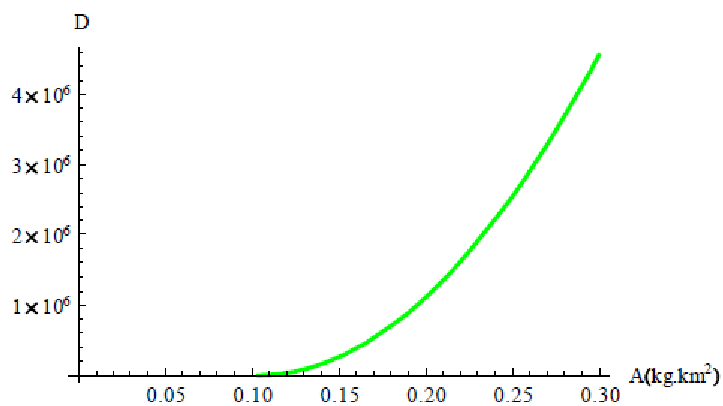


Figure 6 – Arnold determinant for C_0 = 0.10307 kg km^2 and I_0 =0.5533rad.

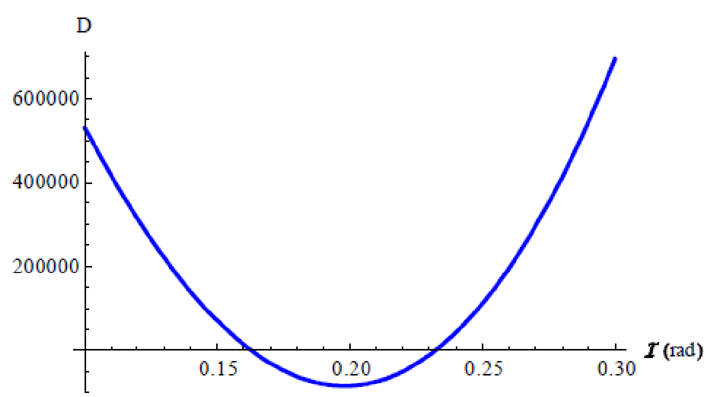


Figure 7 - Arnold determinant for C_0 = 0.1.0307 kg km^2 and A_0 = 3. 9499 *10^{-1}kg km^2.

4.2. Results for the PP satellite

The initial data for the SS satellite are:

Orbital inclination: I = 0.4364rad;
Eccentricity: e = 0;
Orbital radius: r = 7139. 61585km

Principal moments of inertia on: x – axis, $A = 9.855*10^{-6}$kg km^2;
y – axis, $B = 9.855*10^{-6}$kg km^2;
z – axis, $C = 13.000*10^{-6}$kg km^2;

Delaunay Variables: l = 0rad; g = 4.5420rad; h = 4.542rad;
$L = G$ = 5334653.709kg km^2/s;
H = 4834685.585kg km^2/s.

There were 50 equilibrium points and only 7 were stables, the others 43 equilibrium points had also failed in the first condition of the *KST* and were not linearly stable. Table 2 shows two equilibrium points found in the simulations, one Lyapunov stable and other unstable to *PP* satellite. The same way for the *MS* satellite, there aren't singularities on the Andover's variables in this point due to the values of the inclinations angles

$$I_2 = 1.03788\text{rad}, \quad J_2 = 1.55784\text{rad},$$

the spin velocity ω = 1.59939rad/s and the rotation period T =3.92848s.

Table 2. SS Satellite: A=B = 9.20*10^{-6} kgkm^2, C = 13*10^{-6} kgkm^2

Equilibrium points	Lyapunov stable	Unstable
$L_1(kgkm^2/s)$	$2.69365*10^{-7}$	$7.13725*10^{-11}$
$L_2(kgkm^2/s)$	$2.07921*10^{-5}$	$2.14416*10^{-10}$
$L_3(kgkm^2/s)$	$1.05633*10^{-5}$	$1.07843*10^{-10}$
$l_2(rad)$	0.066560	- 0.090962
$l_3(rad)$	0.4	0.07

4.2.1 Stable regions around the equilibrium points

The same procedure developed for MS satellite is now applied for the SS satellite, in order to observe the stability behavior around the equilibrium point. The variations of the principal moment of inertia on x-axis were arbitrarily chosen between 0 and $18*10^{-6}$kg km^2, variations of the principal momentum of inertia on z-axis(C) between 0 and $16*10^{-6}$kg km^2 and the variations in orbital inclination between 0 and 1rad.

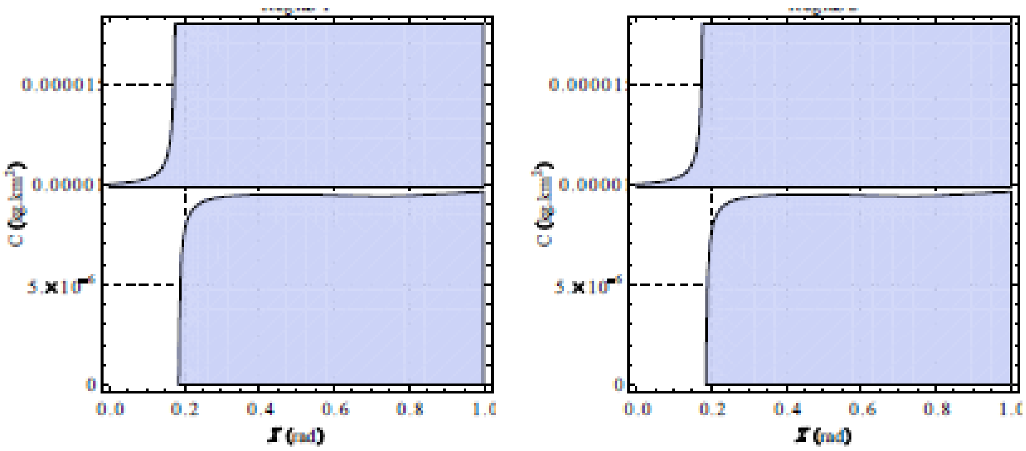
Figure 8 shows the linear stabilization when the principal moment of inertia on the x-axis is fixed ($A_0 = 9.855*10^{-6}$kg km^2). The blue color in the region 3 represents $a(A_0;C; I) < 0$ and the blue color in region 4 represents $b(A_0;C; I) < 0$, it means the eigenvalues are pure imaginary. By the results it is possible to note that there is non linear stabilization regions when C = A_0 or for low orbital inclination and there is linear stabilization regions for C< A_0

e C > A_0. In Fig. 9 the orbital inclination is fixed (I_0 = 0.4364rad). The blue color in region 5 represents $a(A;C;\ I_0) < 0$ and the blue color in region 6 represents $b(A;C;\ I_0) < 0$. Through the results it is possible to observe that there isn't linear stabilization for $A = C$, which was observed for the *MS* satellite.

Figure 10 shows the analysis of the second condition for the *KST*, for the case when the principal moment of inertia on x –axis and the orbital inclination are fixed:

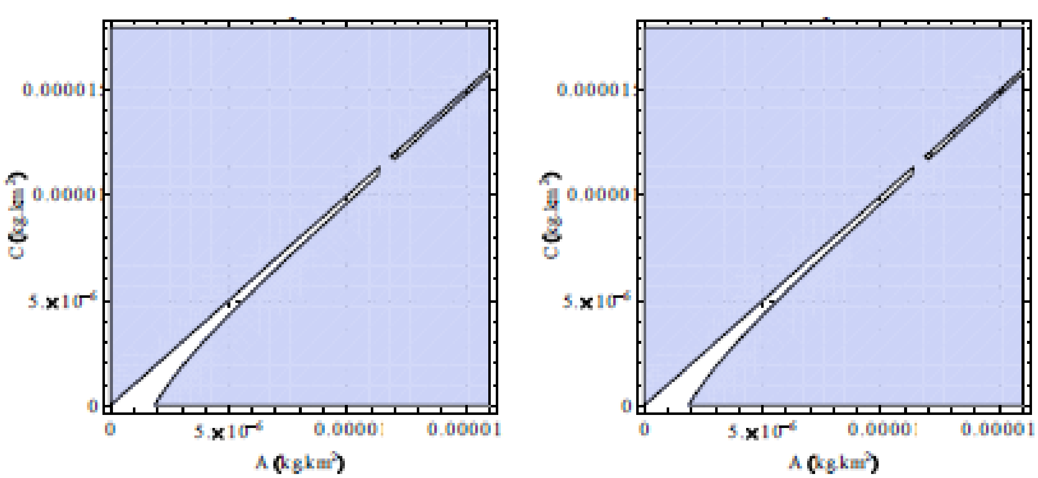
$$A_0 = 9.855{*}10^{-6}\text{kg km}^2 \text{ and } C_0 = 13\ {*}10^{-6}\text{kg km}^2.$$

Then the second condition ($k_1\omega_1^o + k_2\omega_2^o \neq 0$,) is satisfied for I > 0.1651rad, in according with the results presented in the Fig. 8.



Region 3- $a(A,C,\ I_0)$ *Region 4-* $b(A;C;\ I_0)$

Figure 8 – Linear stability region when A_0 = 9. 855* 10^{-6}kgkm^2.



Region 5- $a(A,C,\ I_0)$ *Region 6-* $b(A;C;\ I_0)$

Figure 9 – Linear stability region when I_0 = 0.4364rad.

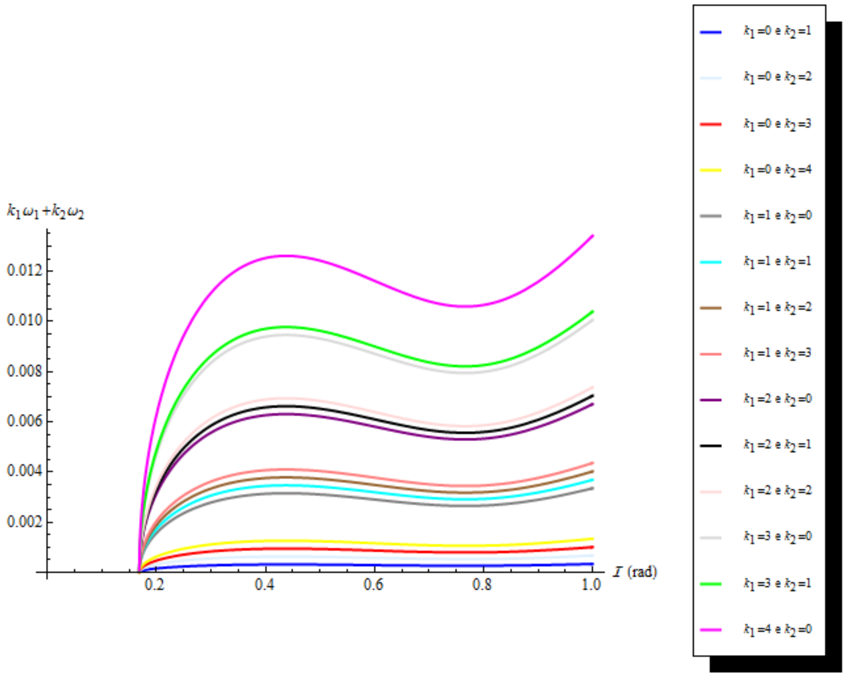


Figure 10 – Second condition for $k_1 \geq 0$ e $k_2 \geq 0$, considering A_0 = 9.855*10^{-6}kg km^2 and C_0 = 13*10^{-6}kgkm^2.

Figure 11, 12 and 13 show the analysis for the third condition associated with the Arnold determinant D^o for SS satellite. By Fig. 11, when the I_0 and A_0 are fixed, it is possible to see that the Arnold determinant is zero for values of the principal moment of inertia on z-axis (C), 9.5*10^{-6}kgkm^2 < C <A_0, than the third conditions are not satisfied and the rotational motion is nonlinear unstable .

Figure 12 shows the values for the Arnold determinant in terms of the principal moment of inertia on x-axis, when C_0 = 13*10^{-6}kg km^2 and I_0=0.4364rad. The third condition is not satisfied for A = C_0 and for C_0 < A < 13.268*10^{-6}kgkm^2.

By Fig. 13 , when A_0 and C_0 are fixed**,** the third condition is not satisfied for I< 0.1651rad, and there is nonlinear stabilization only for I > 0.1651rad.

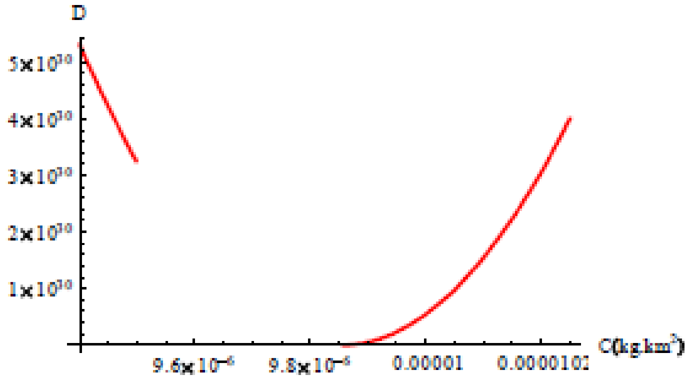


Figure 11 – Arnold determinant in terms of principal moment of inertia, for A_0 = 9.855*10^{-6}kgkm^2 and I = 0.4364rad.

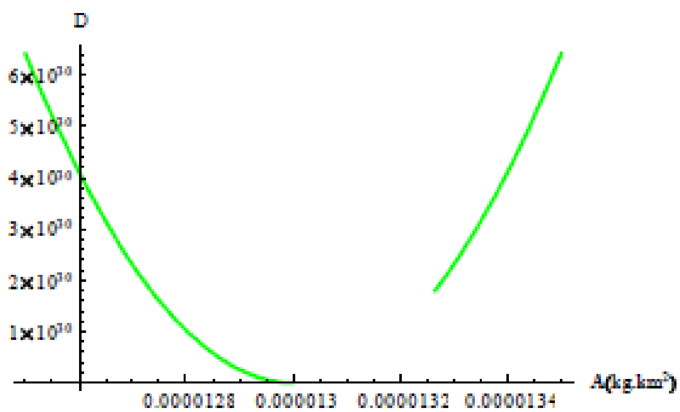


Figure 12 – Arnold determinant in terms of principal moment of inertia on x-axis, C_0 = 13 *10^{-6}kg km^2 and I_0=0.4364rad.

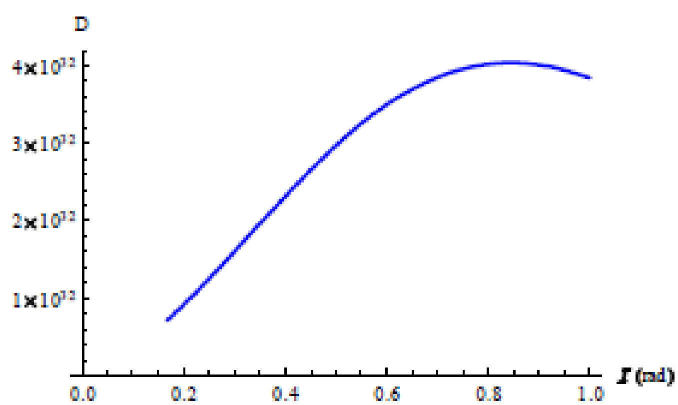


Figure 13 – Arnold determinant in terms of orbital inclination, C_0 = 13 *10^{-6}kg km^2 and A_0 = 9.855*10^{-6}kg km^2.

5. Conclusions

In this paper it was presented a semi-analytical stability of the rotational motion of artificial satellites, considering the influence of gravity gradient torque for symmetric satellite in a circular orbit. Applications were made for two types of satellites: medium (*MS*) and small (*SS*).

Initially the points of equilibrium were determined using the physical, orbital and attitude characteristics of each satellite. Then the algorithm for stability analysis was applied and it was obtained 2 stable equilibrium points for the *MS* satellite and 7 stable points for the SS satellite.

Several stable equilibrium points were determined and regions around these points have been established by variations in the orbital inclination and in the spacecraft principal moment of inertia. There were found 50 equilibrium points for the small size satellite (with some data similar to First Brazilian Data Collecting Satellite) with 10 Liapunov stable points. For the medium size satellite (with some data similar to the American satellite PEGASUS) there were found 60 equilibrium points, but only with 2 Liapunov stable points. In both cases the fail was in the first condition of the Kovalev-Savchenko theorem (*KST*).

For the *MS* satellite it was gotten only two equilibrium point because this satellite has similar characteristics to the satellite PEGASUS, which is tumbling [11]. For *SS* satellite were obtained many others equilibrium points, but most of them were discarded, because it lead to the Andoyer's variables a condition of uniqueness (it means that the angles I_2 and J_2 are null or close to zero, and the Andoyer' variables ℓ_1, ℓ_2 and ℓ_3 are indeterminate). In comparison with previous works, the results show a greater number of equilibrium points and an optimization in the algorithm to determine the normal form and stability analysis.

The results for the stable regions show that in the linear stability there is a separation between the stable and unstable region when the spacecraft principal moments of inertia are equals. It is also possible to observe that the rotational motion for the small satellite is linearly unstable in a low orbital inclination. For considered equilibrium points, the second condition is valid for all values of k_1 and k_2 for any orbital inclination for the medium satellite but for the small satellite it is necessary an orbital inclination bigger than 0.1651 rad. In the nonlinear analysis it was possible to verify that the linear stability doesn't guaranty the non-linear stability and the stable regions are bigger for the small satellite. For the medium satellite there are two values for the orbital inclination in which the Arnold determinant is equal to zero when principal moments of inertia on x-axis and z-axis are fixed it means that the rotational motion is nonlinear unstable. For the small satellite there is nonlinear stability for orbital inclination bigger than 0.1651 rad.

Then the present analysis can directly contribute in the maintenance of the spacecraft's attitude. Once the regions of stability are known for the rotational motion, a smaller number of maneuvers to maintain the desired attitude can be accomplished. In this case, a fuel economy can be generated to the satellite with propulsion systems control, increasing the spacecraft 'lifetime.

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