STABILITY ANALYSIS OF THE SPACECRAFT ATTITUDE WITH CANONICAL VARIABLES

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ABSTRACT

The objective of this paper is to analyze the stability of the rotational motion of a symmetrical spacecraft (with two principal moments of inertia equal), in a circular orbit. The equilibrium points and regions of stability are established when components of the gravity gradient torque acting on the spacecraft are included in the equations of rotational motion. Andoyer's variables are used to describe the rotational motion of the satellite in order to facilitate the application of stability methods for Hamiltonian systems. The Andoyer's canonical variables are represented by generalized moments (L_1, L_2, L_3) and by generalized coordinates (l_1, l_2, l_3). The angular variables l_1, l_2, l_3 are angles related to the satellite system Oxyz (with axes parallel to the spacecraft's principal axes of inertia) and equatorial system OXYZ (with axes parallel to the axis of the Earth's equatorial system). The metrics variables L_1, L_2, L_3 are defined as: L_2 is the magnitude of the rotation angular momentum $\vec{L_2}$, L_1 and L_3 are, respectively, the projection of $\vec{L_2}$ on the z-axis's principal axis and projection $\vec{L_2}$ on the Z-equatorial axis.

The nonlinear stability of the equilibrium points of the rotational motion is analyzed here by the Kovalev-Savchenko theorem, which ensures that the motion is Liapunov stable if the following conditions are satisfied:

i. The eigenvalues of the reduced linear system are pure imaginary $\pm i\omega_1^o$ e $\pm i\omega_2^o$;
ii. $k_1\omega_1^o + k_2\omega_2^o \neq 0$ is valid for all k_1 and k_2 integer satisfying the inequality $|k_1| + |k_2| \leq 4$
iii. The Arnold determinant $D^o = -(\delta_{11}^o {\omega_2^o}^2 - 2\delta_{12}^o\omega_1^o\omega_2^o + \delta_{22}^o {\omega_1^o}^2) \neq 0$, where $\delta_{v,u}^o$ are the coefficients of the normal 4th order Hamiltonian.

Then it is necessary to reduce the Hamiltonian in its normal form up to the fourth order by means of canonical transformations around the equilibrium points.

The equilibrium points are found from the equations of motion described by the Andoyer variables. With the application of the Kovalev-Savchenko theorem, it is possible to verify if they remain stable under the influence of the terms of higher order of the normal Hamiltonian.

In this paper, numerical simulation were made for two hypothetical groups of artificial satellites, which ones have orbital data and physics characteristics similar to real satellites. In comparison with previous works, the results show a greater number of equilibrium points and an optimization in the algorithm to determine the normal form and stability analysis.

Several stable equilibrium points were determined and regions around these points have been established by variations in the orbital inclination and in the spacecraft principal moment of inertia. There were found 60 equilibrium points for the small size satellite (with some data similar to First Brazilian Data Collecting Satellite) with 10 Liapunov stable points. For the medium size satellite (with some data similar to the American satellite PEGAUS) it was found 60 equilibrium points, but with 2 Liapunov stable points. In both cases the fail was in the first condition of the Kovalev-Savchenko theorem.

The results for the stable regions show that in the linear stability there is a separation between the stable and unstable region when the spacecraft principal moments of inertia are equals. It is also possible to observe that the rotational motion is linearly unstable for the small satellite in a low orbital inclination. For considered equilibrium points, the second condition is valid for all values of k_1 and k_2 for any orbital inclination for the medium satellite but for the small satellite it is necessary an orbital inclination bigger than 0.1651 rad. In the nonlinear analysis it was possible to verify that the linear stability doesn't guaranty the non-linear stability and the stable regions are bigger for the small satellite. For the medium satellite there are two values for the orbital inclination in which the Arnold determinant is equal to zero, it means that the rotational motion is nonlinear unstable. For the small satellite there is nonlinear stability for orbital inclination bigger than 0.1651 rad.

Then the present analysis can directly contribute in the maintenance of the spacecraft's attitude. Once the regions of stability are known for the rotational motion, a smaller number of maneuvers to maintain the desired attitude can be accomplished. In this case, a fuel economy can be generated to the satellite with propulsion systems control, increasing the spacecraft 'lifetime.