Periodic Orbits and Formation Flying near the Libration Points

Mai Bando *
Kyoto University, Gokasho, Uji, Kyoto 611-0011, Japan
and
Akira Ichikawa †
Nanzan University, 27 Seireicho, Seto 489-0863, Japan

Recently formation flying along a halo orbit near a libration point of the circular-restricted three-body problem (CR3BP) [1] has been studied [2]. As a basic dynamics, the linearized equations of motion along a halo orbit is used, and a control strategy to maintain a satellite near the orbit, which is based on the short-term relaive motion, is proposed. In this case the difficulty lies in the fact that the coefficients of the linearized system depend on the halo orbit and are periodic functions which are only numerically available. The paper [2] is motivated by earlier studies [3, 4] on formation flying in the vicinity of a libration point of the CR3BP with application to spacecraft imaging arrays. Reference orbits in [3, 4] are not natural orbits of the CR3BP but controlled orbits. In this case control implementation is straightforward, but good control strategies need to be designed.

In this paper we propose a simple method, which is based on the output regulation theory of periodic systems [5], to generate a relative orbit along a periodic orbit of the CR3BP. We design stabilizing feedback laws through the differential Riccati eqiation associated with the linearized system as in [6]. We compute ΔV neccessary to maintain the relative orbit as a function of the weight parameter in the Riccati equation and design a feedback law which is fuel efficient. We also employ the usual output regulation theory [7] to design reference orbits near the libration points. Taking the size and frequency of the reference orbit as parameters, we design reference orbits which require less ΔV for maintenance. Once a reference orbit is determined, we design a relative orbit and a control strategy for formation flying.

An example generated by the proposed method is given in the figures below. We consider the linearized equations at the L_2 point of the CR3BP and a halo orbit with period equal to the half of the two-body problem [8]. We generate by an exosystem a periodic orbit with size parameter A and frequency ω, where A is the non-dimensional amplitude in the along-track direction Y. By choosing A which minimizes the ΔV for maintenance, the reference orbit, which is closer to the periodic orbit, is determined. Feedback laws are designed by the algebraic Riccati equation associated with the linearized equations.

Table 1: L^1-norm for maintenance(ω=2).

A	L^1(in)[m/s]	L^1(out)[m/s]
Output regulation		
2.40	2.9774e+01	3.0475e+01
2.50	2.4502e+01	3.0481e+01
2.60	2.0529e+01	3.0486e+01
2.70	1.8227e+01	3.0491e+01
2.80	1.8901e+01	3.0495e+01
3.00	2.3606e+01	3.0501e+01
Frequency control		
2.85	1.9652e+01	3.0496e+01

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* Assistant Professor, Unit of Synergetic Studies for Space, m-bando@rish.kyoto-u.ac.jp. Member AIAA.
† Professor, Department of Systems Design and Engineering, aichika@nanzan-u.ac.jp. Member AIAA

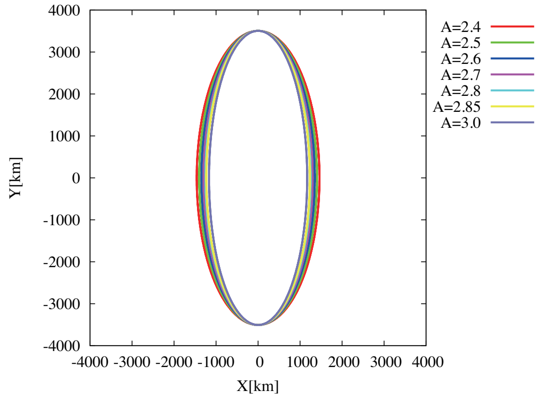

Fig. 1: Controlled trajectory (X-Y).

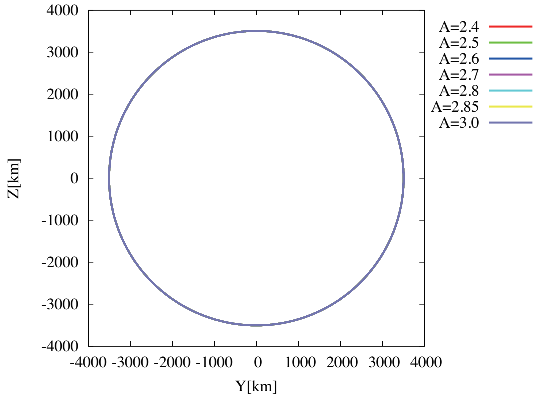

Fig. 2: Controlled trajectory (Y-Z).

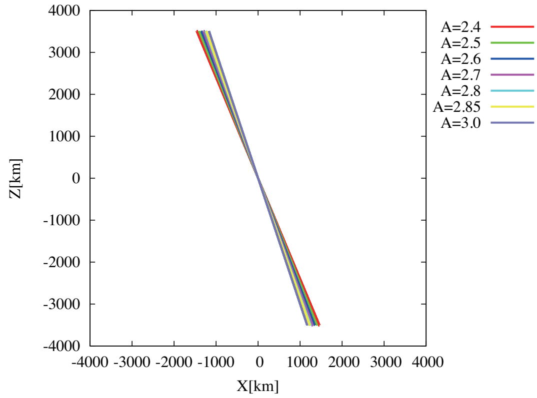

Fig. 3: Controlled trajectory (X-Z).

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