UNCONTROLLED SPACECRAFT FORMATIONS ON TWO-DIMENSIONAL INVARIANT TORI

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ABSTRACT

The center manifold existing within the vicinity of the libration points provides a variety of natural periodic and quasi-periodic orbits. This includes, but is not limited to halo, lyapunov and quasi-halo orbits, which can be exploited for spacecraft formations [1]. The purpose of this study is to characterize the uncontrolled motion of spacecraft on a two-dimensional invariant torus and indicate properties of the motion that are beneficial for formation flying missions. Within the class of natural motions on two-dimensional tori the focus is set on quasi-halo orbits enabling relative spacecraft configurations with large distances among satellites. A two-dimensional invariant torus can be described as a set of orbits that start on a surface and stay on that surface during the dynamical evolution. A numerical method is used to obtain quasi-periodic orbits that envelope around a halo orbit in the Sun-Earth circular restricted three-body problem [2]. This method reduces the calculation for two-dimensional tori to a search for periodic orbits that return to a closed curve on a section plane. The curve is represented by truncated Fourier coefficients in position and velocity space. An example of several orbits evolving on a two-dimensional torus around a L1 northern halo orbit in the synodic frame is shown in Figure 1 (grey lines). The initial states for a spacecraft formation are defined at the intersection of a cutting plane with the torus. Initially, the set of position vectors describe a planar curve, and the evolution of this curve is indicated by red points at four moments in time. The motion in the yz-plane as seen from Earth is almost circular and constant with time, see Figure 1 (right). As the spacecraft proceed in time, they move longitudinally along

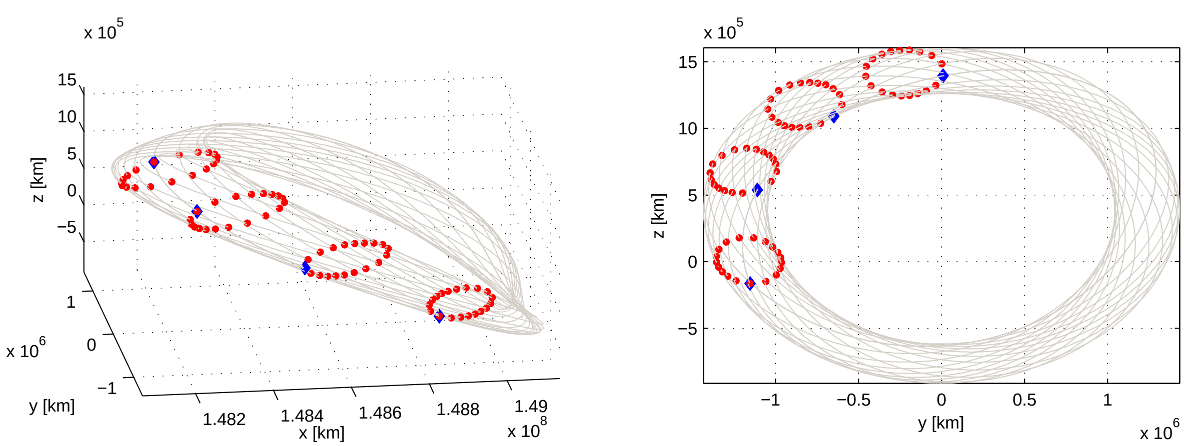

Figure 1. Formation trajectories in the synodic reference frame (grey lines). Four snapshots of the formation in time (red dots).

the underlying halo orbit, and describe latitudinally a winding motion. These components are significant aspects of the natural motion and effect the evolution of the curve in time. The curve's shape contracts and expands, and the orientation of the plane in space changes as the trajectories are propagated forward in time. The changes depend on the selection of the initial cutting plane with the torus, and therefore on the distribution of the spacecraft on the torus' surface. The appropriate orientation of the cutting plane comes from the linear subspaces of the monodromy matrix. Their eigenvalues and the associated eigenvectors indicate the linear stability of the halo orbit and characterize the nearby motion. Specially, the two-dimensional subspace spanned by the two complex eigenvectors define the plane for the initial states of the formation that stay bounded in the linear system. After establishing an appropriate cutting plane for the formation, a coplanar reference frame is introduced that lies in the formation plane with its origin at the underlying halo orbit. The x-direction is arbitrary chosen and defines a zero angle direction. In Figure 1 the blue points indicate this defined zero direction for each snapshot. The evolution of the curve, and therefore of the formation, in this reference frame over one orbital revolution can be characterized by in-plane and out-of-plane components. The out-of-plane components are caused by non-linear effects that are not considered in a linear analysis of the monodromy matrix. The tranformation matrix between the synodic and the coplanar reference frame and their rotation angles around the axes give the orientation and shifting rate of the formation plane. With these angles the plane stability can be defined and the two frequencies associated with the torus can be determined. After gaining a fundamental understanding of the motion associated with invariant tori, several different aspects of formation-keeping, station-keeping and reconfiguration problems can be studied. A whole family of two-dimensional invariant tori is available at a constant energy level and manoeuvres can be introduced to transfer the spacecraft from one torus to another. From the perspective of transfers, quasi-periodic orbits offer more efficient options than the underlying periodic halo orbit, as they have higher dimensional stable and unstable manifolds. The structure of the manifold branches are studied in detail, they could be beneficial for initially placing the spacecraft into the formation.

1. References

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