OPERA: A tool for lifetime prediction based on orbit determination from TLE data

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Abstract: *Objects in Low-Earth Orbits (LEO) and Highly Elliptical Orbits (HEO) are subject to decay and re-entry into the atmosphere due mainly to the drag force. While being this process the best solution to avoid the proliferation of debris in space and to ensure the future sustainability of space activities, it implies a certain amount of risk as many of these reentries are done in an uncontrolled manner. In order to have a better insight on the objects reentering the Earth's atmosphere, in short and middle term, and on the risk posed by these re-entries, CNES has developed a tool named OPERA (Outil de PrEvision des dates des Rentrées Atmosphèriques non contrôlées).*

This paper will concentrate on the prediction of the in-orbit lifetime of a space object, based on publicly available TLE (Two Line Elements sets) data, as it has been implemented in OPERA. To this purpose, several operations are needed prior to the computation of the residual in-orbit lifetime of such object, by the propagation of its orbital position up to re-entry. These operations include the pre-processing of the TLE data by filtering their outliers and detecting maneuvers, as only non-maneuverable objects are considered for the analysis, the preliminary estimation of objects area-to-mass ratio and the final estimation of the orbits based on a weighted least-squares algorithm taking the TLE states as input pseudo-observations. Additionally, the accuracy (estimation error) of the results obtained for known past reentries will be presented.

Keywords: *lifetime prediction, orbit determination, equinoctial parameters, Two Line Element (TLE)*

1. Introduction

Objects in Low-Earth Orbits (LEO) and Highly Elliptical Orbits (HEO) are subject to decay and re-entry into the atmosphere mainly due to the drag force. While being this process the best solution to avoid the proliferation of debris in space and ensure the future sustainability of space activities, most of these re-entries are done in an uncontrolled manner.

This paper will concentrate on the prediction of the in-orbit lifetime of a space object based on publicly available TLE data.

Unfortunately the lifetime of an object in space is remarkably difficult to predict. This is mainly due to the dependence of the atmospheric drag on a number of uncertain elements such as the density profile and its dependence on the solar activity, the atmospheric conditions, the area-to-mass ratio of the object (which is very difficult to evaluate), its unknown attitude, etc.

In this paper we will present a method for the prediction of the residual in-orbit lifetime, of a non-maneuverable space object, based on publicly available Two-Line Elements (TLEs) from the American USSTRATCOM's Joint Space Operations Center (JSpOC). TLEs constitute an excellent source of information to access routinely orbital data for thousands of objects. However, it is a known issue that the reduced and unpredictable accuracy of the TLEs leads to imprecise and unreliable results, if used as such, hence the need of processing them in order to get more precise results.

The CNES, as the French Space Agency, is interested in the monitoring of the man-made space environment in order to firstly predict any potential damage that may be caused to the national territory by any space object or to a foreign territory by a French space object, and secondly to dispose of the necessary information to a-posteriori analyze any damage linked with a reentry event. OPERA (Outil de PrEvision des dates des Rentrées Atmosphèriques non contrôlées) constitutes the pillar of this monitoring process. OPERA, which is a Java-based tool is executed routinely at CNES to predict the orbital lifetime of a whole catalogue of objects. It uses CNES's STELA (Semi-analytic Tool for End of Life Analysis) as a library for efficient long-term orbit propagation (based on semi-analytical theory) and re-entry date prediction for a given orbit.

For this, several steps are performed:

1. Filtering of outlier data among the input TLE datasets.
2. Detection of maneuvering objects (for which the prediction is not performed). Both in-plane and out-of-plane maneuvers are detected.
3. Preliminary estimation of objects area-to-mass ratio, for drag and solar radiation pressure (to be a-priori values for successive final estimation process), based on an innovative method which uses the contribution of conservative and non-conservative forces to the time derivatives of the orbital parameters.
4. Final estimation of the objects' orbit (initial state vector and area-to-mass ratio for the drag and area-to-mass ratio for the solar radiation pressure) based on a weighted least-squares algorithm having the TLE states as input pseudo-observations. This orbit determination is implemented in equinoctial parameters (set used within STELA) to better control the significance given to each parameter (as the argument of latitude – last parameter in Eq. 1 - for instance) via their a-priori covariance and/or weight matrixes.
5. Prediction of the orbital lifetime for each object by propagation of the estimated orbit up to re-entry (when the object reaches a user-defined re-entry altitude).

This paper describes the algorithms implemented for outliers detection and removal, maneuver detection, preliminary area-to-mass ratio estimation and final orbit determination based on the TLE datasets for both LEO and HEO objects. Additionally, the accuracy (estimation error) of the results obtained for known past re-entries will be analyzed.

2. Software reuse

One of the key internal features is the long term propagation of the object, functionality which is covered in OPERA by the use of STELA. This library has been developed by CNES and IMCCE and is widely used in the analysis of the compliance to the French Space Act. Given the importance of the propagator used in the orbit determination process of OPERA – since some of the implementation choices are driven by it - this section presents a brief description of the CNES tool STELA. More detailed information can be found in [2] and [3].
STELA allows an efficient long-term propagation of LEO, GEO and HEO orbits using a semi-analytical extrapolation model. It considers the central body perturbation, the

perturbations due to the Earth potential irregularities (up to 7x7), the gravitational forces of the Moon and Sun, the perturbations due to the atmospheric drag and the solar radiation pressure.

As the short-term periods have been analytically removed from the equations defining the considered perturbations, to only keep the middle and long-term effects, relatively big integration steps can be considered making possible the orbital propagation for long period of times in a quite efficient manner. It uses a numerical 6^{th} order Runge-Kutta method for the integration of the orbit.

3. Implemented algorithms

This section presents an overview of the overall architecture of OPERA. As it can be seen in Fig 1, there are four major steps: TLE filtering (by detection of outliers and maneuvered objects), estimation the dynamic parameters (area-to-mass ratio) used as initial guess of the orbit determination, orbit determination using an iterative least-squares estimation method and re-entry computation.

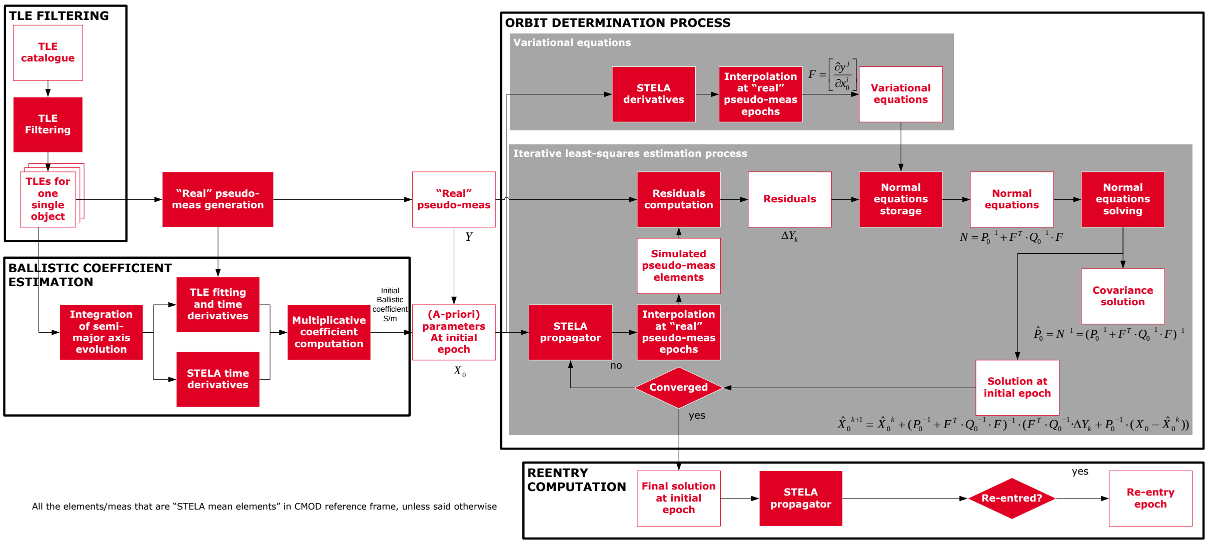

Figure 1 Architectural decomposition of OPERA

In the following sections more detail is given on each step of the process.

3.1. Filtering of outliers

The TLEs are the input data of this algorithm and they are also considered as the measurements of the orbit determination algorithm. Hence, it is of key importance to detect and filter any TLE outlier that may disturb the correct convergence of the estimation filter.

The outlier's detector consists on an iterative least-squares polynomial fitting of each orbital parameter. Given that no information is available about the uncertainty of the orbital elements of each TLE, the orbital elements are expressed in an equinoctial space (Eq. 1) as their temporal evolution is smoother and thus better suited for outlier detection and filtering in absence of any external information. Moreover, in order to avoid the sinusoidal evolution, the second and the third parameters are combined, and just the eccentricity evolution is considered for the filtering of the outliers. Hence, the LSM fitting to detect outliers is performed over the semi-major axis, the eccentricity and the inclination vector (ix, iy).

$$\begin{pmatrix} a \\ e \cdot cos(\omega + \Omega) \\ e \cdot sin(\omega + \Omega) \\ sin(^{i}/_{2}) \cdot cos(\Omega) \\ sin(^{i}/_{2}) \cdot sin(\Omega) \\ \omega + \Omega + M \end{pmatrix} \quad (1)$$

It is important to remark that the number of outliers on the space-track TLE sets is relatively low. Usually, most of these outliers appear during the re-entry phases as well as after injection of objects in orbit, where either the few amount of data or the high rate of change of orbital parameters (due mainly to drag), add a significant amount of uncertainty to the orbit determination problem.

3.2. Detections of maneuvers

OPERA has been defined to estimate uncontrolled short and middle term re-entry dates. Then, all the maneuvered objects (i.e. controlled) are filtered and they will not be considered for the process. It is important to filter the maneuvered objects, because any perturbing acceleration modifying the spacecraft orbital motion will be considered to be of natural origin in the differential correction algorithm.
The manoeuvers filtering is done based on TLE data directly, without any propagation. The mean nature of the orbital elements is suitable for manoeuvers detection as any abrupt discontinuity found on the mean orbital evolution will be considered as caused by a manoeuver. Then, it is only necessary to study the evolution of mean motion and inclination to detect in-plane and out-of plane manoeuvers respectively.

In-plane manoeuvers are performed in order to change the evolution of semi-major axis and/or eccentricity. So, it is possible to relate the cause of the manoeuver $\left(\frac{\Delta V}{V}\right)$ with its effects $\left(\frac{\Delta n}{n}\right)$ as Eq. 2 states. Then, the right side of the equation can be calculated with the data extracted from the TLE history, and it is considered that a maneuver may have taken place if this term – between two consecutive TLEs - is bigger than a predefined threshold.

$$\frac{\Delta V}{V} = \frac{\frac{-1}{3}\frac{\Delta n}{n}}{\frac{2}{1 - e \cdot cosE} - 1} \quad (2)$$

However, it is not possible to assure that there is a manoeuver just because this absolute criterion is fulfilled, so a second verification needs to be also accomplished to assure that a manoeuver has been detected. The second verification consists on relative criteria, using the data contained in a window centered at the date at which the maneuver has been detected. Moreover, considering that the effect of one manoeuver can be seen in consecutive TLEs, if the manoeuver is confirmed in consecutive TLEs, just one manoeuver will be considered.

The same logic is followed to detect out-of-plane maneuvers, but considering the inclination as stated in Eq. 3

$$\frac{\Delta V}{V} = \Delta i \tag{3}$$

Further details of sections 3.1 and 3.2 can be found in [1].

3.3. Preliminary estimation of objects area-to-mass ratio

In order to linearize the problem of orbit estimation, around an initial condition that assures the convergence of the orbit determination process, it is necessary to estimate such initial condition to be close enough from the unknown real orbit to guarantee the linearity of the estimation problem. Input TLEs contain information accurate enough to initialize the state vector, but there is no accurate information available to initialize the dynamic parameters, which are of key importance to describe the effect of perturbation on the motion of the satellite. Therefore, these dynamic parameters, mainly S_{drag}/m, and S_{SRP}/m, need to be estimated to initialize the orbit determination process.

As a first approximation to simplify the process, it is considered that the mean evolution of the semi-major axis with time is exclusively due to the drag perturbing acceleration. Hence, we'll assume that the estimated area-to-mass ratio will correspond to S_{drag}/m. If an HEO object is being analysed, this area-to-mass ratio value is also used to initialize the S_{SRP}/m (solar radiation pressure term), so we'll consider that $S_{SRP}/m = S_{drag}/m$. We'll also assume that the objects are randomly tumbling so it can be assimilated to a sphere whose cross sectional area is equal to the drag surface.

By the numerical integration of the second term of Eq. 4, it is possible to obtain a first guess of the area-to-mass ratio; hereafter called a-priori area-to-mass ratio.

$$\frac{1}{a(t_0)} - \frac{1}{a(t)} = -\frac{1}{\mu}\frac{S_{drag}}{m}\int_{t_0}^{t} \rho C_x V^3 dt \tag{4}$$

Being a(t) the semi-major axis at date t, μ the Earth's gravitational constant, ρ the atmospheric density, Cx a tabulated drag force coefficient from STELA (see [2]) and V the norm of the spacecraft velocity, as a non-rotating atmosphere is considered.

However, this method does not always give results accurate enough to initialize the process, mostly for HEO objects as the solar radiation pressure perturbing acceleration may be of the same order of magnitude or even bigger than the drag perturbing acceleration.

So as to refine the initial estimation of the area-to-mass ratio, for drag and solar radiation pressure, a new method has been developed based on the time derivatives of the orbital parameters provided by STELA propagator. These time derivatives can be split depending on their origin; hence we are able to know the contribution of the drag and solar radiation pressure forces in the semi-major axis and eccentricity evolution, as stated in Eq. 5 (please, note that the secular contribution of the conservative forces is null for the semi-major axis):

$$\left.\frac{da}{dt}\right|^{Prop} = \left.\frac{da}{dt}\right|^{drag} + \left.\frac{da}{dt}\right|^{SRP}$$
$$\left.\frac{de}{dt}\right|^{Prop} = \left.\frac{de}{dt}\right|^{drag} + \left.\frac{de}{dt}\right|^{SRP} + \left.\frac{de}{dt}\right|^{cons} \quad (5)$$

To compute the time derivatives of the TLEs, first a conversion of the TLE orbital parameters into equinoctial ones (Eq. 1) is performed and then the semi-major axis and the eccentricity vector are fitted with a configurable order polynomial (via least-squares). The time derivatives can afterwards be analytically computed using the coefficients of the polynomial.

Equation 6 gives the relation between the time derivatives provided by the propagator and those observed from the TLE catalogue. Since the time derivatives of the orbital parameters have been computed using the a-priori area-to-mass ratio, a multiplicative coefficient must be added to them so as to correctly fit the actual evolution observed from the TLEs. A polynomial fitting of the same order than the one for the TLEs is also applied to the time derivatives provided by the propagator.

$$\left.\frac{da}{dt}\right|^{TLE} = K_1 \left.\frac{da}{dt}\right|^{drag} + K_2 \left.\frac{da}{dt}\right|^{SRP}$$
$$\left.\frac{de}{dt}\right|^{TLE} - \left.\frac{de}{dt}\right|^{cons} = K_1 \left.\frac{de}{dt}\right|^{drag} + K_2 \left.\frac{de}{dt}\right|^{SRP} \quad (6)$$

Being $\left.\frac{\mathrm{da}}{\mathrm{dt}}\right|^{\mathrm{TLE}} / \left.\frac{\mathrm{de}}{\mathrm{dt}}\right|^{\mathrm{TLE}}$ the time derivative of the semi-major axis/eccentricity computed from the TLE catalogue, $\left.\frac{da}{dt}\right|^{drag} / \left.\frac{de}{dt}\right|^{drag}$ the time derivative of the semi-major axis/eccentricity due to the drag force provided by STELA, $\left.\frac{da}{dt}\right|^{SRP} / \left.\frac{de}{dt}\right|^{SRP}$ the time derivative of the semi-major axis/eccentricity due to the solar radiation pressure provided by STELA and $\left.\frac{de}{dt}\right|^{cons}$ the time derivative of the eccentricity due to the conservative forces provided by STELA.

For each one of the TLEs in the analyzed catalogue, a pair of K_1 and K_2 is computed using the logic below:

- The mean of the time derivative of the semi-major axis cumulated from the epoch of the first TLE in the catalogue up to the current TLE is computed (based on the TLE fitting).
- The mean of the contributions to the semi-major axis (drag and SRP) cumulated from the initial epoch up to the current epoch is computed (based on the fitting of propagated values).
- The same goes for the eccentricity.
- The system of Eq. 6 can therefore be solved.

In order to avoid aberrant results due to numerical problems, whenever one of the contributions (drag or SPR) is a configurable number of times lower than the other, the smaller one is considered as negligible and is not considered in the resolution of the equations (and therefore, its multiplicative coefficient K is set to zero).

At last, the median of this set of values is computed to obtain the final multiplicative coefficient. Hence, the values of the dynamic parameters that will be used as initial guess for the orbit determination process - hereafter called refined area-to-mass ratio - are obtained as follows:

$$\left. S_{drag}/m \right|^{ref} = K_1 \left. S_{drag}/m \right|^{a\_priori}$$
$$\left. S_{SRP}/m \right|^{ref} = K_2 \left. S_{SRP}/m \right|^{a\_priori} \quad (7)$$

These parameters can also be used as initial conditions for a simple propagation – without the OD process- up to the re-entry date. This functionality can be useful in operational conditions to estimate mid-term reentries with a gain of time performance (with the drawback of a lower accuracy).

During the validation of the method (see section 4), several modifications have been included to enhance its performances, the most relevant being:

- Unexpected re-entry. It may happen that the propagation using the computed a-priori area-to-mass ratio makes the object to re-enter before the end of the TLE catalogue. In such cases, the dynamic of the orbit provided by the propagator is too far from the real one (from the TLEs), so the method is not able to get a precise multiplicative coefficient. To avoid such situations, the computation interval is dynamically reduced whenever the absolute value of the time derivatives of the semi-major axis from STELA propagator reaches a predefined threshold. In doing so, we focus on the interval where both dynamics are equivalent and hence the method provides much more accurate results.
- Frequency of TLE. It's well known that the frequency of the TLEs provided publicly by USSTRATCOM is not a constant (going from several TLEs in a single orbit to periods of several days without new data). Whenever two TLEs are too close to each other, the partial derivatives may present some sharp leaps – even if the evolution of the orbital parameters themselves is smooth – that cannot be correctly fitted with a low-order polynomial. Hence, an additional filter has been included so as to remove TLEs separated less than one orbit, and only the first one of them is kept (let's not forget that we're always dealing with mean elements).

3.4. Orbit determination

Once that accurate enough initial conditions have been calculated, it is possible to solve for the orbit determination problem using a standard least-squares batch filter. This is, the non-linear orbit determination problem will be linearized by a first order Taylor expansion, of the computed trajectory about the reference one, at each available point (i.e. measurements) in time.

The considered measurements are expressed in equinoctial elements (Eq 1). The choice of this set of parameters is driven by the fact that STELA propagator uses mean parameters with an integration step of the same order of magnitude that the orbital period, so the mean argument of latitude (fast moving parameter, here-after called L_{eq}) may not represent the actual orbital position of the space object at the date of release of the state vector. Hence, it's interesting to be able to activate/deactivate at will the estimation of the L_{eq}.

Given the source and the nature of the measurements (elements where the order of magnitude is completely different from one to another), a weighting matrix of the measurements is added to the process to achieve a better conditioning when inverting F. In order to give more flexibility to the operator, this weighting matrix can be defined in local (TNW frame) components or directly in equinoctial elements. For the first case, the input matrix is afterwards converted to equinoctial elements using the Jacobian of the transformation, as in Eq. 8

$$Q_0^{eq} = J_{cart \to eq} Q_0^{cart} J_{cart-eq}^T \tag{8}$$

The operator can also decide in that case if the full matrix is to be used, or only a diagonal one. The performed analysis have shown that the crossed terms in the converted equinoctial matrix have a non-negligible impact in the OD solution and their values are not always easy to predict from the weights given in the local reference frame. Given the need to fully control the weight of L_{eq}, the operator is also able to change the final value of the weight to be used for this parameter (σ_{Leq}^{new}). If the full matrix is considered, the crossed terms are modified as explained in Eq. 9. σ_i accounts for the standard deviation of any element except the Leq one, σ_{Leq} accounts for the standard deviation of the "fast moving" element (Leq) and ρ_{iLeq} accounts for the correlation between both elements.

$$\rho_{iLeq} \left(\sigma_i \sigma_{Leq} \right)^{new} = \rho_{iLeq} \sigma_i \sigma_{Leq} \frac{\sigma_{Leq}^{new}}{\sqrt{\sigma_{Leq}^2}} \tag{9}$$

The notion of the a-priori covariance (P_0) for the estimated parameters is also included in OPERA, so as to indicate the degree of trust we have on the knowledge of each parameter of the initial state-vector. Two possibilities are provided to define the a-priori covariance matrix:

- Equinoctial elements. The operator introduces the values directly in equinoctial elements.
- Computation from the TLE catalogue. Each TLE of the analyzed catalogue is propagated back-wards up to the epoch of the previous one and then a comparison of the propagated elements with the actual ones is performed. A diagonal covariance matrix is afterwards fulfilled with the average value of these differences.

Theoretical measurements, that are used to calculate the residuals w.r.t. the real measurements, are obtained with a STELA propagation starting from the initial state vector and the refined values of the dynamical parameters S_{drag}/m and S_{SRP}/m. If real and theoretical measurement's epochs do not match, an 8th order Lagrange interpolation is used to obtain the theoretical measurement at the same epoch as the real measurement and hence calculate the residual.

STELA also gives the transition matrix (partial derivatives) as output, which can be directly used for the orbit determination if the epochs fit with the epochs of the real measurements. Otherwise, a Lagrange interpolation of the partial derivatives is done. Finally, Eq. 10 expresses the orbit determination problem including all the assumption here mentioned.

$$\widehat{X_0^{k+1}} = \widehat{X_0^k} + [P_0^{-1} + F^T \cdot Q_0^{-1} \cdot F]^{-1} \cdot \left[F^T \cdot Q_0^{-1} \cdot \Delta Y_k + P_0^{-1}\left(X_0 - \widehat{X_0^k}\right)\right] \qquad (10)$$

This iterative process will end when one of the convergence criteria takes place. Several typical convergence criteria are applied (absolute and relative WRMS, maximum number of iterations,…)

3.5. Prediction of the orbital lifetime

Once the OD is finished, the initial state determined in the process is propagated with STELA until re-entry, in order to provide the re-entry date to the user. The object is considered as re-entered when it reaches a configurable altitude (typically 80 kms). The propagation is stopped after a configurable maximum duration (5 years), if the re-entry condition has not yet been fulfilled.

4. OPERA Results

This section shows some results obtained during the validation and operational use of the algorithms here-presented.

4.1. Refinement method

A study using the first part of OPERA, just up to the refinement method, has been performed. The goal is to analyze the differences, in terms of perigee and apogee height, between the used measurements (TLEs) and the orbit propagated from the refined conditions. It is important to know the accuracy offered by this propagation not only because it is the input of the orbit determination, but also because the re-entry date estimation can be done, if the user want to reduce the computational burden, without the orbit determination phase.

4.1.1. Objects population

A sample of objects from different populations has been selected and classified as:

- Launcher stages
    - Low LEO orbit (apogee altitude < 5000 km and perigee altitude < 500km)
    - Up LEO orbit (apogee altitude < 5000 km and perigee altitude > 500km)
    - GTO (apogee altitude > 5000 km)
- Objects that have reached their end of life (no longer maneuvered)
    - LEO (apogee altitude< 5000km)
    - GTO (apogee altitude > 5000km)

4.1.2. Analysis method

The analysis uses as input data the TLEs directly acquired from the www.space-track.com web site, which are selected for a period of time of 80 days taken 20 days after their launch (launcher stages) or their end of mission. These TLEs are used as input data and reference data; the ones that the solution is compared to. The outcome of the refinement method (S_{drag}/m and S_{SRP}/m) are used to initialise the final propagation and the results of this propagation are compared in terms of perigee and apogee height to the reference TLEs. In order to make possible this comparison, an 8th order Lagrange interpolation is performed to obtain the propagated measurements at the same epoch as the TLE and calculate the differences in apogee and perigee altitude.

4.1.3. Results of the refinement method

The most important conclusion taken from this analysis is that the accuracy of the resulting propagation (with respect to the TLEs) is inversely proportional to the eccentricity of the orbit, as it can be seen in Fig.2 to Fig.5 (the rest of results have not been shown here in order to maintain the required concision, but they follow the same scheme). As a result of this fact, the UpLEO group (e < 0.15) is the one that presents the better results as shown in Fig. 4 to Fig.5.
It is possible to conclude that the results obtained for eccentricities lower than 0.6 are satisfactory.

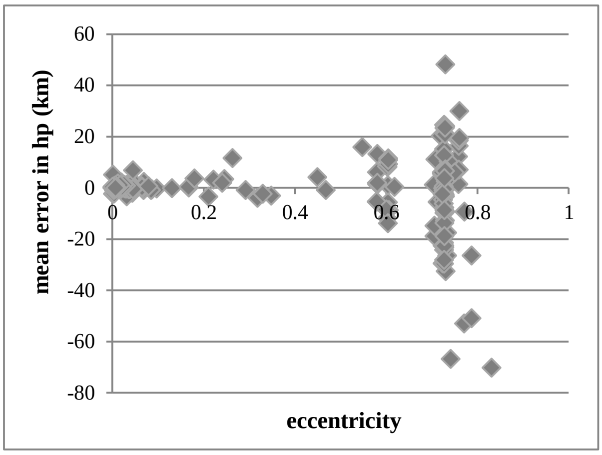

Figure 2. Mean error in perigee altitude with respect to eccentricity for LowLEO objects

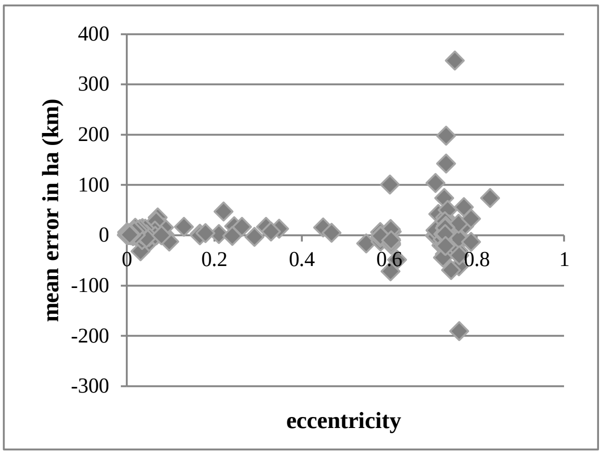

Figure 3. Mean error in apogee altitude with respect to eccentricity for LowLEO objects

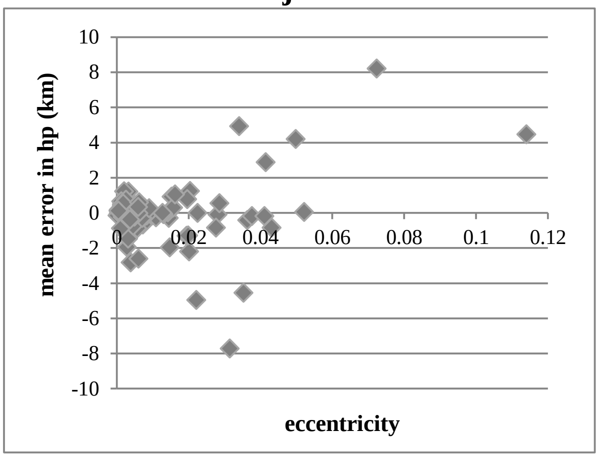

Figure 4. Mean error in perigee altitude with respect to eccentricity for UpLEO objects

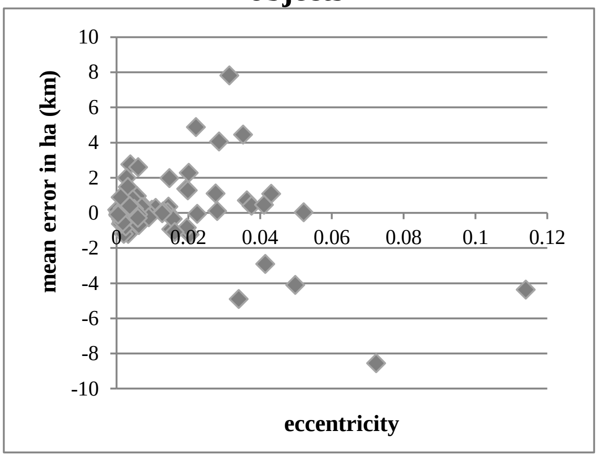

Figure 5. Mean error in apogee altitude with respect to eccentricity for UpLEO objects

The same analysis with respect to the inclination has been done and three inclination bands, where the most part of the aberrant results are gathered, have been discovered:

- Around 30º. This inclination corresponds to the Moon orbital plane, so it introduces short-term periods in the derivatives of these objects that are not properly fitted in the process (a 3rd order polynomial is used) and so do the errors are higher.

- Around 60º, close to the critical inclination value where the reliability of the performed STELA propagations is lower than for the rest of inclinations.

In order to have a better understanding of the origin of aberrant results, an analysis taking only into account the objects with the biggest errors has been done. It has been found that the errors are mainly caused by the differences between the derivatives obtained from STELA and the ones calculated from the TLEs. For the aberrant cases, these partial derivatives do not evolve in a similar way, so they cannot be adjusted by means of a single coefficient (Eq. 7). This behavior is more reflected in high eccentricity objects (e > 0.6) and low perigee altitude (hp < 5000km). We believe that this comes from the fact that, for such eccentricity domains, the differences between the model used to build the TLEs [4] and STELA are not compatible with the correct estimation of the dynamical parameters and the orbit of the analyzed objects.

4.2. Orbit determination method

Even if a previous reentry date can be obtained with the results extracted from the refinement method, the orbit determination is performed in order to offer a more accurate result.
One of the most important parameters to be configured in this process is the weighting matrix, which gives information to the algorithm about the confidence that the user has in the measurements considered for the orbit determination. As a matter of fact, further studies are being performed to characterize the optimal definition of this weighting matrix.
Table 1 shows a representation of different scenarios of weighting matrix configuration that have been considered for OPERA tests in order to evaluate the suitability of these scenarios regarding to the obtained results. All these scenarios have been run considering Leq weight as 1 radian.

Table 1. Scenarios configuration

Weight	Scenario1	Scenario2	Scenario3	Scenario4	Scenario5
Position_T (m)	100	1000	10000	1000	10000
Position_N (m)	100	100	100	1000	1000
Position_W (m)	100	100	100	1000	1000
Velocity_T (m/s)	0.1	1	10	1	10
Velocity_N (m/s)	0.1	0.1	0.1	1	1
Velocity_W (m/s)	0.1	0.1	0.1	1	1

These simulations have been performed for the whole population of LowLEO, which has been sub classified based on the eccentricity. The mean of the last iteration residuals of every component for every object is stored as result and the average and standard deviation of these means for every subpopulation are calculated. In order to represent information of all the residuals' components together, a normalization of these values with respect to their maximum reached along all the simulated scenarios is performed. Finally, the normalized contributions of all components are added to obtain the mean normalized error corresponding to each scenario as Eq.11 shows for scenario1.

$$mean\_normalized\_error = \sum_{i=1}^{6} \left| \frac{(mean\_res\_param(i))_{scen\_1}}{\max\_res\_param(i)} \right| \quad (11)$$

Fig.6 and Fig.7 illustrates these results for two subpopulations of LowLEO objects. The lower the mean normalized residual is, the closer the theoretical and real (TLE) measurements are, and so the better the result of the orbit determination is with respect to the considered measurements. Then, based on Fig.6 and Fig.7, the fourth scenario is the one that offers the best orbit determination performances for both subpopulations. This scenario considers a spherical weight in position (same weight for all components) and velocity, with a 1000 times ratio between position and velocity.
In order to obtain the lowest mean normalized residuals it is needed to use spherical weights (scenarios 1 and 4) for the low eccentricity subpopulation (Fig.6), whilst for the high eccentricity subpopulation (Fig.7) is more important to configure higher weights for N and W components (scenarios 4 and 5). This last point is directly related with the well-known loss of accuracy of TLEs for high eccentricities orbits, so the used weights must be configured accordingly.

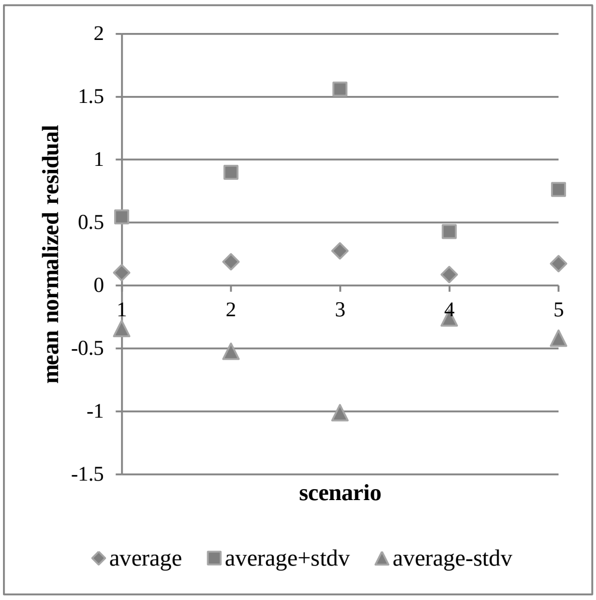

Figure 6. Mean normalized of the low eccentricity population (e<0.125) of LowLEO objects

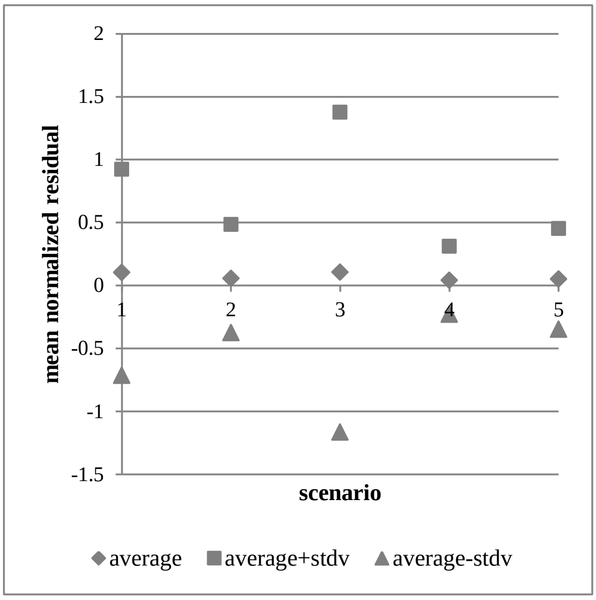

Figure 7. Mean normalized residuals of the high eccentricity population (e>0.6) of LowLEO objects

As mentioned in section 3.4, the orbit determination is performed in equinoctial parameters in order to allow the user to take or not take into account the “fast moving” variable (L_{eq}) via the measurement’s weight matrix.

Fig. 8 and Fig.9 show the evolution of the obtained error in the reentry date with respect to the Leq weight; both of them are run based on the scenario 5 configuration. The big effect of this parameter on the result of the GTO objects (Fig.9) confirms the importance of controlling this parameter, mainly for GTO.

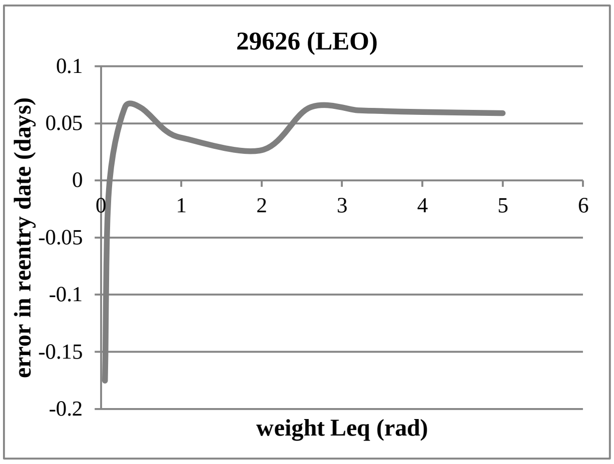

Figure 8. Error in reentry date with respect to the weight in Leq for a lowLEO object

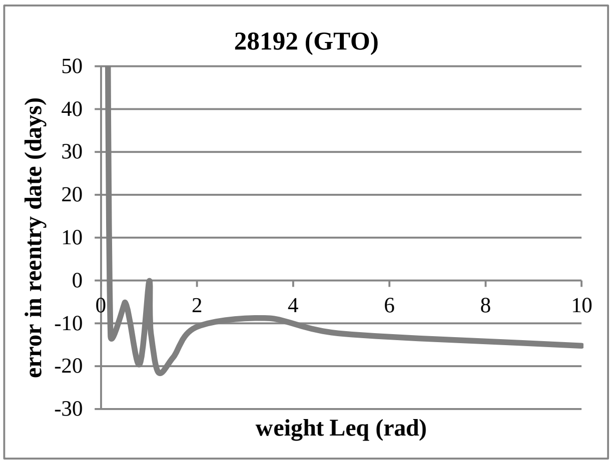

Figure 9. Error in reentry date with respect to the weight in Leq for a GTO object

4.3. OPERA applications in real world

At late October 2013, GOCE reached its end of life since it ran out of fuel. As long as it has been orbiting in low LEO regime, its re-entry into the atmosphere was close. As a consequence OPERA was used to estimate the predicted re-entry date, as well as to provide the dynamical parameters to be used as inputs in a short term re-entry estimation phase, using radar measurements. OPERA estimation was done using the TLEs obtained after the end of life, where no manoeuvers were done and the decay phase has started as Fig.10 shows.

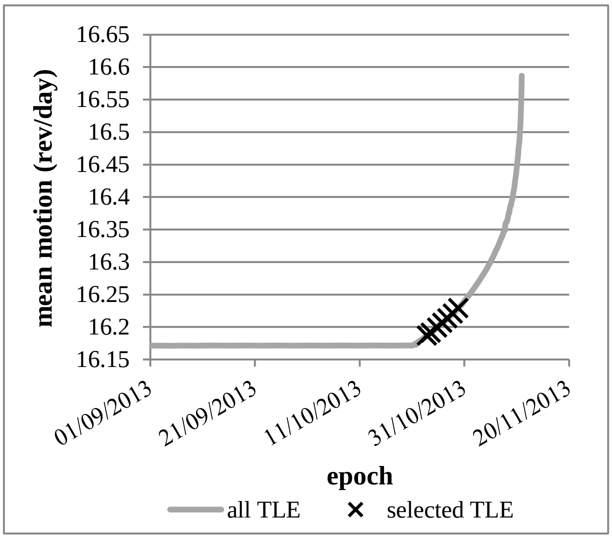

Figure 10. GOCE mean motion extracted from TLEs during its late life

The estimation has been done on 31/10/2013 considering the 7 TLEs highlighted in Fig. 10 and using the configuration that corresponds to the scenario 4 previously presented. The simulation gave as result the following reentry date: 2013-11-11 03:33:39.892. The difference between the real (2013-11-11 00:16 UTC +/-1 minute) and the estimated reentry date is less than 3 hours when it has been estimated 11 days before the actual re-entry took place. Given the main OPERA mission, to estimate the middle and long term uncontrolled reentries, this result is considered as highly satisfactory.

Moreover, while OPERA is being used in an operational environment to predict the re-entry of objects that are likely to reenter the Earth's atmosphere in a user configurable timespan, it allows CNES to anticipate the re-entry of objects by several weeks or even months. This anticipation allows to analyze and flag some re-entering objects as dangerous, in order to perform a precise re-entry prediction, using radar measurements, several days before the effective re-entry date.
Since most of the re-entries predicted by OPERA can be also accessed publicly (www.space-track.com), most of the time with a fewer anticipation. We have found interesting to present in this paper some of the cases that we could find at Space-Track website only a-posteriori. This is, for these objects JSpOC only was able to certify that the space object have re-entered the Earth's atmosphere oncethe event have taken place.
Some of these objects are listed in Tab. 2, where the estimated reentry date 30, 20 and 10 days before the actual reentry date is shown. As seen in the Tab. 2, the results, corresponding to the ones obtained after the refinement method, are well within the margins of a reasonable accuracy in spite of the object 23584, whose last available TLE was from 23/08/2013. Then, even if the reentry date is calculated closer to the actual reentry epoch, the last used TLE is older than the operational date and the catalogue becomes smaller (as the initial date is getting closer to the last available TLE).

Table 2. Predicted reentry dates

NORAD Id	Semi-major axis (km)	Eccent	Incl (º)	Actual reentry date	Estimated reentry date 30 days ahead	Estimated reentry date 20 days ahead	Estimated reentry date 10 days ahead
26805	9490	0.31	8.5	17/08/2013	21/07/2013	09/08/2013	21/08/2013
11844	26554	0.75	62.7	24/05/2013	22/05/2013	20/05/2013	21/05/2013
23584	26495	0.75	62.5	23/09/2013	22/08/2013	23/08/2013	23/08/2013
32997	26295	0.74	63.0	26/08/2013	17/08/2013	20/08/2013	23/08/2013

5. Conclusions

This paper has presented the basis of the algorithms of OPERA, as well as some results obtained during its validation and operational use.
Bearing in mind the mean nature of the considered measurements (TLEs) and the used propagator (STELA), it has been proved that it is of key importance performing the orbit determination in equinoctial parameters, where the determination of the parameter regarding the position within the orbit (L_{eq}) can be "activated / deactivated" via the measurement's weight matrix.
The validation campaigns performed against known past reentries show that the accuracy obtained in the results for high eccentric orbits ($e > 0.6$) could not fulfill the same level of accuracy as for lower eccentricities due to the differences between the SGP4 [4] and STELA models [2].
OPERA has been tested in an operational environment and trust-worthy results of real reentry campaigns have been obtained.
Even though the results obtained after the orbit determination are usually more accurate than those obtained after the refinement method, due to performance issues in operational context, OPERA could be used only with the refinement method as a first approximation. Then, for some particular re-entries, the operator is free to re-run the re-entry estimation date using the orbit determination phase.

6. References

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