NUMERICAL STABILITY STUDY OF MULTI-CIRCLE ELLIPTIC HALO ORBIT IN THE ELLIPTIC RESTRICTED THREE-BODY PROBLEM

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ABSTRACT

Circular restricted three-body problem (CRTBP) has drawn much attention in orbital dynamics especially in recent decades. New concepts and methods in space mission design, such as manifold theory[1], interplanetary superhighway[2], weak stability boundary theory and ballistic capture[3], have been developed under CRTBP and proved to be successful in practice. However, the motion of planets in the solar system can be better described by Kepler's elliptic orbits with eccentricity e ranging from 0.0086 to 0.2488[4]. Therefore, elliptic restricted three-body problem (ERTBP), where the two primaries revolve on elliptic orbits and the third body is infinitesimal, is a more realistic model for mission design. The equations of motion in ERTBP implicitly depends on time (or true anomaly f). This makes it a non-autonomous system which invalidates the methods developed in the CRTBP. The ERTBP has no Jacobi integral (or energy conservation), no constant zero velocity surface and no continuous halo orbit families as the CRTBP does. Broucke studied the stability of periodic orbits in ERTBP and categorized them[5], but his research was restricted in planar situation and only earth-moon mass and equal mass was considered. Sarris extended the study to spatial ERTBP but only focused on the periodic orbit with period 2π [6]. Campagnola calculated elliptic halo orbit revolving two circles and studied its stability in Earth-Moon system during the BepiColombo Mission design[7], but the orbits with other parameters behave differently. The problem is far from thorough understanding. As a complement to the whole ERTBP exploration, we construct ***multi-circle elliptic halo orbits*** with various parameters and numerically study their stability.

Since the equations of motion in ERTBP is periodic with period 2π, all the periodic orbits in ERTBP must have period 2π or multiples of 2π [5]. However, in most situations, the minimal period of circular halo orbit is too small and falls into the range about from 0.5π to π, so it is necessary to revolve M circles in order to reach a period $N \cdot 2\pi$. In this way, we obtain a periodic orbit in the ERTBP revolving M circles while primaries revolving N circles. We refer to this kind of orbit as ***multi-circle elliptic halo orbit***, with emphasis on its multi-circle property. Firstly we generate circular halo orbits families. Then we choose the circular halo orbit with desired period $T_C = N/M \cdot 2\pi$, and numerically continue it to specific eccentricity e to obtain a multi-circle elliptic halo orbit with parameters (M, N). During continuation, simple differential correction method fails to converge because of revolutions, so we rewrite it into an optimization problem and more advanced ***optimal method*** is exploited to solve it. After this, we calculate the monodromy matrix of the period orbit, and analyze the bifurcation of its eigenvalues with different parameters. The results are compared with those in CRTBP and planar ERTBP in order to find consistencies. Big mass ratio situation like Earth-Moon system and big eccentricity situation like Sun-Mercury and Sun-Mars system have attracted special attention during this study.

At the end of the study, compared with circular halo orbit in CRTBP, we propose possible applications of these orbits in future mission design. Observation missions, such as SOHO and JWST, have been revolving libration points for a long time, and they require orbit maintenance maneuver frequently which costs most of the fuel. Continuous communication with Mars can be built if we place a satellite on Martian libration points, but the eccentricity of Mars is relatively large and will inevitable enlarge the fuel cost. If we design such a mission flying along a multi-circle elliptic halo orbit, it will certainly save lots of fuel. The introduce of e in ERTBP is significant and far-reaching, because even a small variation of eccentricity e can cause circular halo orbits to diverge quickly. The multi-circle elliptic halo orbit is directly developed in ERTBP, which means that more nature dynamics is used, so that the quick divergence caused by e is successfully suppressed. Thus we can expect it to require orbit correction maneuver for longer intervals. In practice, because there exists other perturbations and mission duration is limited, a periodic orbit with too long a period is pointless, which on the other hand supports our choice of (M, N) in this paper.

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