

# UNCERTAINTY PROPAGATION IN ANGLES ONLY TRACK INITIATION

Islam I. Hussein<sup>(1)</sup>, Mathew P. Wilkins<sup>(2)</sup>, and Paul Schumacher<sup>(3)</sup>

<sup>(1)(2)</sup>Applied Defense Solutions, 10440 Little Patuxent Parkway, Suite 600, Columbia, MD 21044,  
(410) 715-0005, [ihussein@applieddefense.com](mailto:ihussein@applieddefense.com), [mwilkins@applieddefense.com](mailto:mwilkins@applieddefense.com)

<sup>(3)</sup>Air Force Research Laboratory, 535 Lipoa Parkway, Suite 200, Kihei, HI 96753, (808) 891-  
7708, [Paul.Schumacher@maui.afmc.af.mil](mailto:Paul.Schumacher@maui.afmc.af.mil)

**Keywords:** Track Initiation, Angles Only Measurements, Uncertainty Propagation, Particle Methods

## ABSTRACT

Track initiation is a specialized method to make an educated guess for the initial orbit state that utilizes geometry, orbital dynamics, and iterative methods. The initial guess for the orbit state would then be fed into a higher fidelity orbit determination system. Angles-only observations have been a practical challenge for modern orbit determination for decades. Additional complications arise when one is uncertain if the observations are associated with each other and with the correct resident space object. In this case, we must not only try and determine an accurate orbit based upon line-of-sight information alone, but we must also try and determine through these methods if the observations are correctly associated and the resulting orbit is valid.

Previous work by the authors<sup>1</sup> sought to *a priori* reduce the number of association hypotheses by combining two techniques. The first is to apply physics-based boundaries on range hypotheses that may necessarily be overbroad to ensure that the actual orbit lies within the search space. The second technique is to iteratively solve for initial ranges plus physics-based boundaries to screen out bad solutions. This second step is the focus of the present paper, where we aim at developing information-based methods to eliminate the first step in this methodology.

For the second step, we use Lambert's Theorem to solve for the ranges. In this paper, we do so probabilistically (as opposed to deterministically as in Ref. [1]) using two separate approaches as follows. Both approaches require at least three observations in order for the Lambert's solution to be mathematically feasible. For the purpose of this paper, it is assumed that one has chosen three observations from the available data. In the first approach, a sufficiently large number of random particles will be generated from a given probabilistic model (assumed Gaussian or a sum of Gaussians) for the sensor error. These particles will then be mapped from the original measurement space to the range-angle space as prescribed by Lambert's Theorem. We will then graphically represent the distribution of the mapped particles in the range-angle space. This study will help in graphically understanding the evolution of uncertainty from the original measurement space to the full orbital element space.

In the second approach, we will follow an unscented transform approach. From the original measurement error model (assumed a sum of Gaussians, i.e., a Gaussian Mixture Model (GMM)) we will generate a set of sigma points. These points, like the particles in the first approach, will be transformed under the mappings described in Lambert's solution. Using the transformed

points we will reconstruct and graphically inspect the resulting GMM in the orbital element space.

This second method is particularly useful as it provides us with a way to solve the data association problem. This can be done as follows. Given a set of  $M$  measurements and taking three measurement combinations at a time, one can now compute the entropy<sup>2</sup> of the transformed probability distribution in the orbital element space. This can provide us with a description of the degree of self-consistency of the measurements and hence screen out bad associations. Additionally, using the notion of information divergence, one can then compute the degree of divergence<sup>2</sup> between any two sets of accepted three-measurement associations. This way, one can use entropy to determine whether any three (or more) measurements are associated or not (and to what degree) and the associated sets can be compared against each other using information divergence. Those associated sets that are deemed “close enough” to each other will then be patched together to form a single arc of a track. The two approaches will be tested in computer simulations.

[1] Schumacher, P. J., and Wilkins, M.P. “Scalable Track Initiation for Optical Space Surveillance.” Advanced Maui Optical and Space Surveillance Conference, Wailea, HI, Sept 11-14, 2012.

[2] Cover, T. M. and Thomas, J. A. *Elements of Information Theory*. Wiley-Interscience, Hoboken, N.J., 2006.