

# COORDINATE EFFECTS ON THE USE OF ORBIT ERROR UNCERTAINTY

**James Woodburn<sup>(1)</sup> and Sergei Tanygin<sup>(2)</sup>**

<sup>(1)</sup> *Analytical Graphics, Inc., 220 Valley Creek Blvd., Exton, PA 19340, USA, +16109818082, [woodburn@agi.com](mailto:woodburn@agi.com)*

<sup>(2)</sup> *Analytical Graphics, Inc., 220 Valley Creek Blvd., Exton, PA 19340, USA, +16109818030, [stanygin@agi.com](mailto:stanygin@agi.com)*

**Keywords:** *covariance, coordinates, linearity, orbit, uncertainty*

## ABSTRACT

Knowledge of the uncertainty in the trajectories of spacecraft is becoming increasingly important for use in spacecraft operations and Space Situational Awareness (SSA). Applications include automated tracking data validation, determination of statistical consistency between orbit trajectories and conjunction assessment. While the actual orbit error distribution is represented by an unknown probability density function, it is common practice to assume that errors have a zero mean and are Gaussian distributed which allows for the set of all possible trajectories to be represented by a nominal trajectory and an associated orbit error covariance matrix.

The nominal trajectory and associated error covariance matrix must be realized in a set of coordinates. Conversion of the trajectory between different sets of coordinates is generally achieved through a non-linear transformation. We assume that non-linear transformations are exact and invertible such that the trajectory representation is equivalent in all coordinates. Conversion of covariance information between different sets of coordinates can be achieved in a linear transformation using the Jacobian between the two sets of coordinates or through a non-linear technique such as the unscented transform. Coordinate independence, such as that of the trajectory representation, does not generally exist for the covariance.

The ability of the covariance, as expressed in a particular set of coordinates, to accurately represent the actual probability density function associated with the orbit errors varies with the selection of coordinates and determines the validity of computations which require that covariance as an input. When orbit uncertainties are small, any set of coordinates can typically be used with equal validity and the problem is considered to be linear. As the orbit uncertainty increases, judicious coordinate selection can sometimes greatly expand the valid domain for uncertainty based computations. For example, the comparison of trajectory differences to the orbit error covariance using orbital elements produces much more satisfactory results to performing the comparisons in Cartesian coordinates. On the other hand, the use of covariance information to provide probability based gating for observation acceptance during orbit determination requires information in a particular set of coordinates. In this case, the orbit error covariance is transformed into measurement space to enable the required accept/reject decision. The success of this procedure will depend on how well the covariance transformed to measurement space captures the actual orbit error distribution.

In this study, we examine coordinate effects on some common operational uses of orbit uncertainty information such as trajectory comparisons, data editing in the orbit determination

process and conjunction assessment. Our analyses will include numerical and visual results which will promote intuitive understanding of the effects of coordinate selection on the studied problem set. In cases where the natural coordinate selection leads to poor representation of the orbit uncertainty, we will explore alternate formulations in an attempt to minimize the effects of non-linearity. We will also identify conditions when the coordinate selection for the orbit error covariance prior to transformation to problem specific coordinates is important.

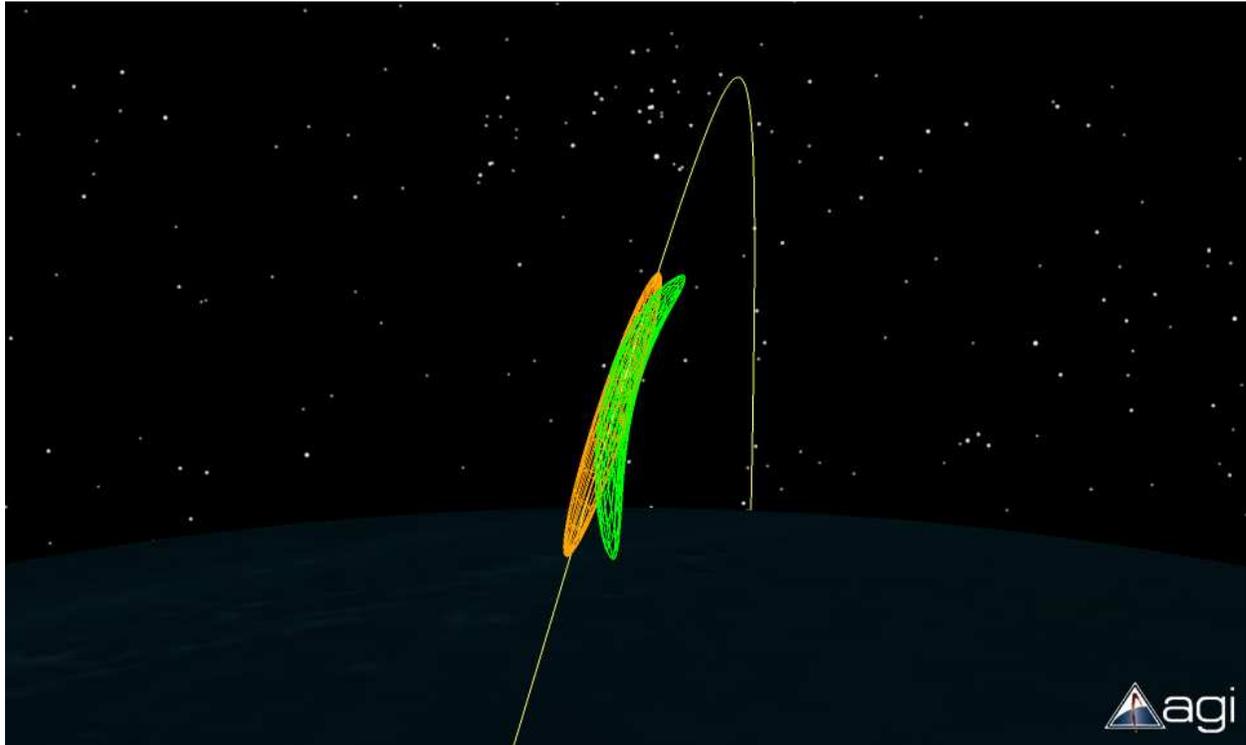


Figure 1: Orbit position error covariance in orbit and measurement space coordinates