A NEW INTERMEDIARY FOR GRAVITY-GRADIENT ATTITUDE DYNAMICS. THE TRIAXIAL SPACECRAFT

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ABSTRACT

The roto-translatory dynamics of the full two-body problem continues to be one of the challenging problems of Astrodynamics [1]. The proposed models usually rely on simplifying assumptions. When one of the bodies is an artificial satellite, the mass ratio allows to use a restricted model where the main body behaves like a central gravitational field. Moreover, another constraint, valid in many missions, is to assume almost circular orbits, i.e. where elliptic effects are treated as higher order terms. But even with these simplifications the roto-translatory problem is far from being integrable; hence it is commonly approached by perturbative techniques [2]. In addition, depending on the scope of these roto-translatory studies, ranging from autonomous navigation algorithms and control strategies to long-term dynamics surveys, requirements for those perturbative schemes may be quite different, being critical the choice made for the zero order model on which the rest of the perturbation process hinges. From the first studies in attitude dynamics, special configurations were found to be solutions and *critical inclinations,* among the several planes involved, were reported out of the averaged models, both in triaxial and axial-symmetric cases. Common to almost all of these studies is the assumption of fast rotation.

In this paper the attitude dynamics of a generic triaxial spacecraft in a central gravitational field moving in a rotating frame is discussed in Hamiltonian formalism. Continuing our work on intermediary models [3] we propose here a new one. Our approach relies on two main aspects: (i) we abandon the free rigid-body model as the unperturbed system; (ii) we consider the full set of initial conditions from slow to fast rotations and any inclination among the instantaneous angular momentum plane and the spacial and body frames. The McCullagh approximation of the potential is assumed. Following Poincaré and Arnold we split the Hamiltonian $H = H_0 + H_1$ where H_0 is the *intermediary* Hamiltonian defining a non-degenerate integrable 1-DOF system, which includes the free rigid-body as a particular case. When Andoyer variables are used the Hamiltonian takes the form $H_0 = H_0\,(-, -, \nu, \Phi, M, N; A_i, n)$, where A_i are the principal moments of inertia and n is the mean orbital motion. This model has received some attention in the past, but in averaged variables and only partially. Here, in the original variables, the main features of this system are studied in detail. In fact, contrary to previous claims, there are regions in the domain of motion where this model behaves rather different than the free rigid-body. In the case of slow motion we identify conditions under which pitchfork bifurcations of the classic unstable trajectories occur, scenario of great interest in relation to stabilization purposes. We find equilibria around which we have oscillations that, to our knowledge, have not been reported previously;

we give the range of values of the momenta for their existence. Moreover several relative equilibria (or frozen rotations at critical inclinations) are identified where the integration reduces to elementary functions. The comparison with the full model is presented showing that, in the absence of resonances, only short periodic oscillations separate the intermediary from the complete gravity-gradient model Hamiltonian H.

Finally a complete reduction of the intermediary is carried out by using the Hamilton-Jacobi equation, which gives the action-angle variables defined by the model, being them a generalization of the Sadov's classic set of variables [4]. A further analysis is underway at present in order to look for the best scheme, namely successive approximation or Lie transforms, with the aim of including the rest of the periodic perturbation that leads to a compact set of first order perturbed solutions of the problem. The approach here presented may be easily extended by including also the oblateness of the central body. This, in turn, will incorporate to the model some of the features of the radial intermediaries [5], which are the analogous intermediaries in orbital dynamics to the one considered here.

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