# PARTICLE SWARM OPTIMIZATION OF 2-MANUVER, IMPULSIVE TRANSFERS FROM LEO TO LAGRANGE POINT ORBITS VIA SHOOTING

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## ABSTRACT

The study of Lagrange Point Orbits (LPO) in the Earth-Moon system is gaining popularity within the astrodynamics community. Farquahr[(1)] noted that halo orbits could be used for communication relays as early as the late 1960's while Heppenhiemier[(2)] envisioned early space colonization of Lagrange points in 1978. Recent studies have focused on human operations, observation and surveillance, and telerobotics. The ARTEMIS mission even orbited the Earth-Moon $L_1$ and $L_2$ Lagrange points to study Earth's magnetotail in great detail. Despite the interest in utilizing LPO's only a modest amount of research exists in optimizing transfers from geocentric orbits to LPO's. These transfers come in two varieties: low-thrust and high-thrust. Unfortunately, the global optimization of LPO transfers is extremely challenging due to the complexity of three body dynamics. Nevertheless, attempts of global optimization have been made for the low-thrust case by Abraham et al.[(3)] utilizing manifold theory and evolutionary algorithms.

The present work is intended to extend Abraham's prior work to the realm of high-thrust transfers. Specifically, this study focuses on the optimization of two-maneuver transfers from Low Earth Orbit (LEO) to a desired LPO via an evolutionary algorithm known as Particle Swarm Optimization (PSO). This study will outline a method to merge direct shooting with PSO in a manner that can quickly sample the global search space and identify near optimal transfer conditions. This hybrid PSO/Shooting approach attempts to utilize the strength of each parent technique and minimize their respective weaknesses. This is accomplished by optimizing individual “manifold insertion points” using the gradient-based, single shooting technique which is very fast and accurate. In addition to the gradient-based algorithm an evolutionary PSO algorithm was used to identify the best “manifold insertion point” based on an objective function that uses the results of the shooting algorithm as its input. The PSO method does not require gradient information and is very adept at working with non-convex objective functions with a large number of extrema; especially the “local” PSO version. Furthermore, the PSO algorithm does not require a priori knowledge of the shape of the objective function. This is highly advantageous when an analytic expression of the objective function is unknown and sapling the function requires intense computational resources.

An example problem is investigated where an optimal transfer trajectory between a 400 km Low Earth Orbit (LEO) and an Earth-Moon, $L_1$, northern halo orbit is achieved. This involves the exploration of the halo orbit's invariant stable manifold using Dynamical Systems Theory (DST) and the PSO algorithm. The spacecraft is allowed two burns to complete it's journey; one to escape LEO and a second to insert itself into the stable manifold. After manifold insertion the spacecraft will ballistically coast to the target LPO without any further propulsive maneuvers. In accordance with the results found by Alessi et al.[4] the optimal manifold insertion point for a slow transfer was found to be roughly an apogee condition. This is reassuring since Alessi et al. used an entirely different optimization approach to come to this conclusion. Surprisingly, however, it was discovered that "fast" approaches into manifold regions very near the target LPO offer better performance in terms of both $\Delta V$ and time of flight considerations. This is in stark contrast with the more traditional "slow" approaches studied in published literature (Alessi; Renk2010). Indeed, since these fast approaches are superior to slow approaches they may be better candidates for human spaceflight missions; especially considering the large discrepancy in time of flight.

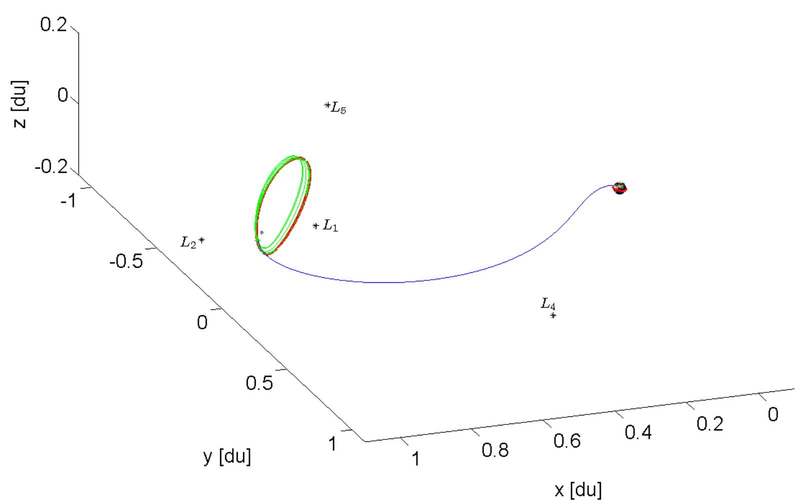


**Figure 1: Fast transfer from a 400 km LEO (red) to the target LPO (red) via a cislunar coast (blue) following a cislunar injection burn and a stable manifold coast (green) following a manifold injection burn.**


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