

## PRECISE DETERMINATION OF MEAN ORBITAL ELEMENTS FOR LEO MONITORING AND MAINTENANCE

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### ABSTRACT

For formation monitoring and maintenance as well as for orbit maintenance of single satellites, the so-called mean orbital elements are generally used. These are free from short-period perturbations and reveal the long-term behaviour of the satellite system. For LEO satellites, altitude manoeuvres are the most important and frequent ones. However, the osculating semi-major axis varying by  $\pm 10$  km cannot be used for  $\Delta v$  calculation. Instead, the mean semi-major axis is applied, and has to be determined very precisely. For example, in the case of the GRACE satellites, the required change in the mean semi-major axis of the manoeuvring satellite is normally about 15-30 meters. In this case, for accurate manoeuvre planning, the mean semi-major axis has to be resolved to the accuracy of 2-3 meters.

Numerical averaging of osculating elements for generation of mean elements has a number of uncertainties and computational disadvantages as compared to analytical algorithms. However, most analytical theories do not provide sufficient accuracy. This work aims at designing an algorithm for retrieving stable mean orbital elements to the required accuracy which can be used operationally at GSOC. For this, a combination of two existing theories from [1], [2] and [3] has been designed and evaluated.

The analytical theory of Eckstein-Ustinov, [1] and [2], was applied in a modified form to account for the short-period perturbations due to the Earth's oblateness, the so-called  $J_2$  term. The major advantage of this theory as compared to other analytical theories such as e.g. the famous satellite theory of Brouwer, is that the short-period variations are developed to the second order, including the influence of  $J_2^2$  on the semi-major axis. The mean elements of the modified Eckstein-Ustinov's theory were calculated iteratively. The first-order perturbations due to other spherical harmonics were calculated with the perturbation theory of Kaula, [3], on the basis of the mean elements of the Eckstein-Ustinov's theory.

For validation, the algorithm was first applied to the ephemeris of TOPEX/Poseidon propagated with gravity field of degree and order  $60 \times 60$  and no consideration of air drag. The uncertainties in the resulting mean orbital elements were compared to those presented in [4] for the same satellite, obtained with a similar algorithm where Kaula's theory was combined with a theory of Konopliv from [5]. It was observed, that the uncertainty in the mean semi-major axis obtained with our algorithm corresponds to that in [4]. At the same time, the uncertainties in other two

important parameters, the eccentricity and the argument of perigee, are both by factor 4 lower than the uncertainties shown in [4]. The suggested combination of the Eckstein-Ustinov's and Kaula's analytical theories was further tested on the ephemeris of GRACE and TerraSAR-X. The results show that the mean semi-major axis can be estimated to the accuracy of 2-3 meters (see Figure 1). The variation in the mean eccentricity is by more than two orders of magnitude smaller than the variation of the osculating eccentricity. Apart from the secular variation, the mean arguments of perigee of the GRACE and the TerraSAR-X satellites have the uncertainty of mere a few tenths of a degree.

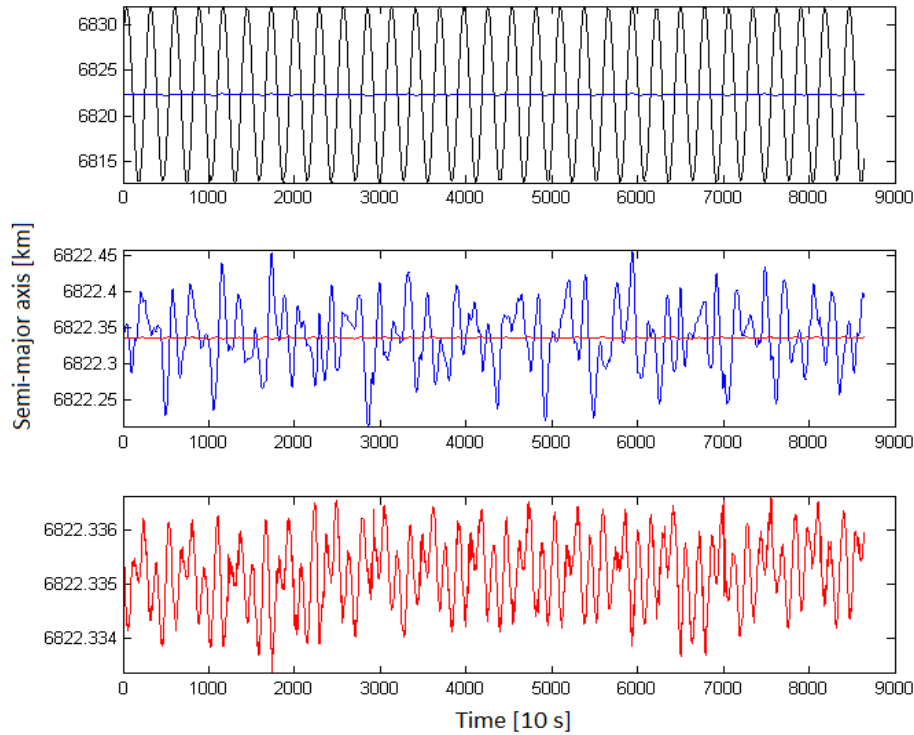


Figure 1: Black: osculating semi-major axis of GRACE, blue:  $J_2$ -mean semi-major axis, red: final mean semi-major axis; on the x-axis: 24h, sampling interval: 10 seconds.

The introduced algorithm, therefore, demonstrates encouraging results, shows a better performance than a comparable algorithm from [4], and is operationally-applicable in terms of computational expenses.

## References

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