

## Matching of Patched-Hyperbolas for Lunar Gravity-Assist Trajectory Design

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In the patched-conic approximation model, a multiple gravity-assist trajectory is divided into various planet-to-planet (or moon-to-moon) legs connected by conic arcs. To find valid trajectories for gravity assist, we search for incoming and outgoing legs which match their hyperbolic excessive velocities. The swingby body is often assumed to have a zero sphere of influence and the effect of its gravity is regarded as an instantaneous change to the trajectory. However in the high fidelity model, we have to convert the patched conic orbits into a continuous, N-body integrated trajectory. In our test case, we found out that just by matching the C3 of the incoming and outgoing legs maybe not be sufficient or it might require a high additional maneuver. To save time in the conversion and to reduce the added  $\Delta V$  penalty cost, the purpose of this paper is to find a way of selecting good matching trajectories for lunar gravity assists. Three approaches are applied to evaluate the two sets of incoming and outgoing legs: 1) Computing the  $\Delta V$  transfer cost through Lambert fit between an outgoing leg and an opposite virtual hyperbola leg; 2) Computing the total  $\Delta V$  transfer cost through Lambert fit between an incoming leg and an opposite virtual hyperbola leg, and between an outgoing leg and an opposite virtual hyperbola leg, and 3) Computing the total  $\Delta V$  transfer cost through an orbit to tie between an incoming leg and a virtual hyperbola leg of the same side, and between an outgoing leg and a virtual hyperbola leg of the same side. The method for making legs is to propagate the perigee of an incoming and an outgoing orbit (a virtual hyperbolic orbit) to backward and forward for N-days. We use these criteria to select promising candidates for optimization. Before we optimize the whole trajectory, we take an intermediate step to solve a N-body, two-point boundary value problem when the spacecraft is near the flyby body. Table1 shows the  $\Delta V$  costs for 3 approximated 2-body criteria and the optimized solutions in N-body model. Our results show that these criteria can give some guidelines to find good candidate initial guesses for optimization. In the full paper, we compare the results found by these three criteria with the optimized solutions and apply it to the future JAXA missions EQUULEUS, DESTINY<sup>+</sup> that use lunar gravity assists.

Table1. Comparing  $\Delta V$  between three criteria (2-body) and optimized solutions (N-body)

Case	2-body			N-body
	Criteria1[km/s]	Criteria2[km/s]	Criteria3[km/s]	Optimized[km/s]
1	1.97E-01	1.51E-01	2.50E-01	1.85E-02
2	2.46E-01	1.79E-01	2.89E-01	1.91E-02
3	2.03E-01	1.97E-01	2.90E-01	3.51E-02

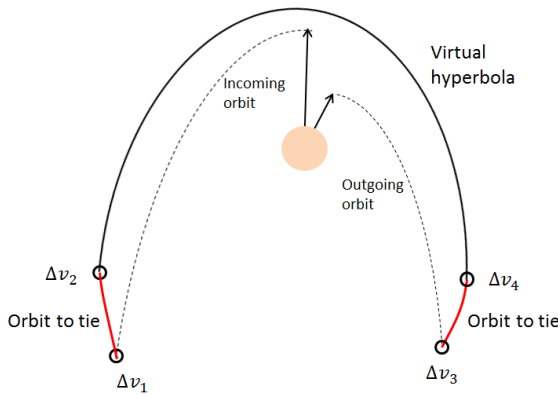


Fig. 1. Schematic of the matching problem in lunar-centered view

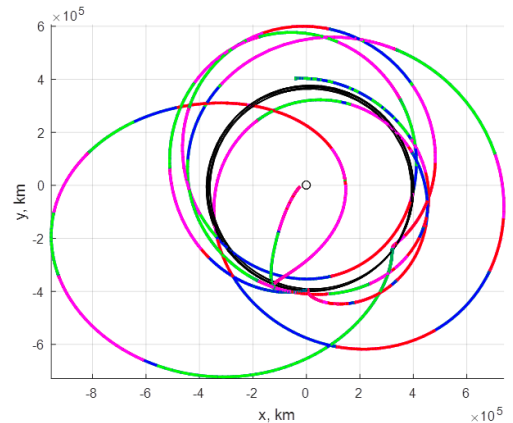


Fig. 2. Optimized solution in Geocentric view