Systematic Study of the Dynamics about and between the Libration Points of the Sun-Earth-Moon system

Bastien Le Bihan,[1*] Josep J. Masdemont,[2] Gerard Gómez,[3] and Stephanie Lizy-Destrez[1]
[1]*ISAE-Supaéro, France;*
[2]*IEEC & Departament de Matemàtiques, Universitat Politècnica de Catalunya, Spain;*
[3]*IEEC & Departament de Matemàtiques i Informàtica, Universitat de Barcelona, Spain*
bastien.le-bihan@isae.fr

Keyword : N-body problems, Sun-Earth-Moon system, Quasi-Bicircular Model, Invariant Manifolds, Natural Trajectories

For the last three decades, the dynamics about the libration points of the Sun-Earth (SEL_i) and Earth-Moon (EML_i) systems have been increasingly studied and used as the backbone of numerous space missions, both in terms of transfer trajectory determination and nominal orbit design. Famous successful applications include the ARTEMIS probe in the Earth-Moon system as well as the SOHO, DSCOVR and Gaia spacecrafts at $SEL_{1,2}$, to mention just a few.

Besides these practical examples in each system, the dynamics of both problems can be combined to pro-duce efficient transfers in the extended Sun-Earth-Moon (SEM) system. Typically, the SEM system can be seen as two coupled Circular Restricted Three Body Problems (CRTBP), the Sun-Earth and Earth-Moon systems, with their associated libration points. The invariant manifolds of the orbits about EML_2 and $SEL_{1,2}$ provide dynamical channels that can be suitably combined to produce low-energy trajectories. This so-called *coupled* CRTBP *approximation* has been previously used to compute various types of connections, including: low-energy Earth-Moon transfers (1), Earth-to-EML_2 trajectories (2), and natural $SEL_{1,2}$-to-EML_2 transfers (3). The coupled approximation relies on the fact that the dynamics associated with the EM and SEM subsystems are partially preserved in the four-body context. However, for every computed trajectory, this option requires both a specific Sun-Earth-Moon configuration and an arbitrary connection between the CRTBPs, which prevents the use of this model as a basis for a systematic tool.

In this paper, free $SEL_{1,2}$-to-EML_2 transfers are consistently obtained for the first time in a single, coherent model of the Sun-Earth-Moon system called the Quasi-Bicircular model (4). The set of staging orbits and its associated hyperbolic manifolds are obtained semi-analytically at both ends of the transfer, using the parameterization method (5, 6). A systematic search for connection can then be performed in the parameterization space: initial conditions on the center-unstable manifold at EML_2 are propagated forward in time and projected on the center manifold at $SEL_{1,2}$. A transfer is found each time that the distance of projection is close to zero. These solutions are refined using a differential correction scheme in the parameterization space, which can be coupled with a continuation procedure to easily obtain families of natural transfers. Finally, the resulting trajectories are refined to JPL ephemerides.

References

[1] W. S. Koon, M. W. Lo, J. E. Marsden, and Shane D. Ross, Shoot the moon, *AAS/AIAASpace Flight Mechanics Meeting*, 2000, pp. 1017–1030.
[2] J. S. Parker. Low-energy Ballistic Lunar Transfers. *PhD thesis*, University of Colorado, 2007.
[3] E. Canalias and J. J. Masdemont. Computing natural transfers between Sun-Earth and Earth-Moon Lis-sajous libration point orbits. *Acta Astronautica*, **63** (2008), pp. 238–248.
[4] M. A. Andreu. The Quasi-Bicircular Problem. *PhD thesis*, Universitat de Barcelona, 1998.
[5] A. Haro, M. Canadell, J. L. Figueras *et al.*, The parameterization method for invariant manifolds: from rigorous results to effective computations, *Springer*, **195 (**2016).
[6] B. Le Bihan, J. J. Masdemont, G. Gómez, and S. Lizy-Destrez. Invariant manifolds of a non-autonomous quasi-bicircular problem computed via the parameterization method. *In preparation*.