Nonlinear Uncertainty Propagation of Orbital Mechanics subject to Stochastic Error in Atmospheric Mass Density

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Orbital uncertainty propagation plays a crucial role in space situational awareness (SSA). Many methods for uncertainty propagation, e.g., Monte Carlo simulations (MCS), Gaussian closure, state transition tensors, and polynomial chaos expansion, have been proposed for the deterministic dynamical systems [1]. However, the stochastic noise in the dynamical system turns the uncertainty propagation into a stochastic problem, which makes these methods become inadequate. For example, the noise in atmospheric mass density (AMD) is one of the most dominant errors in the dynamics of low Earth orbit (LEO) satellites.

The dynamical system for LEO satellites perturbed by stochastic noise of AMD can be expressed by the following Itô stochastic differential equation (SDE) [1]:

$$d\boldsymbol{x} = \boldsymbol{f}(\boldsymbol{x}, t)\, dt + \boldsymbol{g}(\boldsymbol{x}, t)\, dW(t), \boldsymbol{x} \in \mathbb{R}^6 \quad (1)$$

where $\boldsymbol{x}(t)$ is the state vector containing position and velocity; $\boldsymbol{f}(\boldsymbol{x}, t)$ is the deterministic forces of the dynamics, $\boldsymbol{g}(\boldsymbol{x}, t)$ is the matrix characterizing the stochastic forces; $W(t)$ is a Wiener process (Brownian motion) with a correlation function of $\mathbf{E}\{dW(t) \cdot dW(s)\} = Qdt \cdot \delta(t - s)$, and δ is the Dirac function.

Uncertainty propagation for stochastic systems as expressed by Equation (1) still remains an unsolved task because of the computational burden and the curse of dimensionality [2]. In this study, the uncertainty of state vector is propagated by the temporal evolution of the state probability density function (PDF) $p(\boldsymbol{x}, t)$, which is governed by the Fokker-Planck equation (FPE):

$$\frac{\partial p(\boldsymbol{x}, t)}{\partial t} = -\sum_{i=1}^{6} \frac{\partial}{\partial x_i} [p(\boldsymbol{x}, t) \boldsymbol{f}(\boldsymbol{x}, t)]_i + \frac{1}{2} \sum_{i=1}^{6} \sum_{j=1}^{6} \frac{\partial^2}{\partial x_i \partial x_j} [p(\boldsymbol{x}, t) \boldsymbol{g}(\boldsymbol{x}, t) Q \boldsymbol{g}^T(\boldsymbol{x}, t)]_{ij} \quad (2)$$

This study investigates the sensitivity of orbital uncertainty to the stochastic error in AMD along with orbit propagation (OP) for LEO satellites by analytically mapping the uncertainty of AMD into the state uncertainty of the satellites. The solution is obtained by solving the FPE (Equation (2)) using the Adomian decomposition (AD) method. The time-varying state PDF is approximated in the form of Adomian polynomials with a fast convergence and without the need for linearisation [3, 4]. The two-body orbit problem model with non-conservative atmospheric drag is used to evaluate the new AD-based solution with MCS results. Sensitivity analysis of the error associated with AMD in orbital uncertainty propagation can provide suggestive guidance to both SSA applications and AMD modelling of LEO space objects.

References

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