# Relative Spiral Trajectories for Low-Thrust Formation Flying

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This work introduces a novel approach to formation flying by extending shape-based continuous-thrust trajectory design methods to the relative motion of two spacecraft. The expanding capabilities of high-$I_{sp}$ electric propulsion systems and multi-satellite constellations pose challenges for mission planners which are hereby addressed with a geometrically intuitive, semi-analytical solution to the low-thrust problem. Under a prescribed thrust profile, the equations of motion for a single spacecraft admit analytical solutions that describe a family of generalized logarithmic spirals [1]. For the first time an analogous approach is applied to the relative motion of two spacecraft, an unperturbed chief and a continuously-thrusting deputy. Intelligent construction of the deputy's thrust profile transforms the equations of relative motion into a form that is solved analytically, yielding a family of relative spiral trajectories characterized by the thrust parameter $\xi_\delta$. Fig. 1 samples the diversity of this family, which includes conic sections as well as generalized logarithmic spirals. Closed-form expressions are derived for the trajectory shape and time-of-flight. The analysis invokes the following assumptions: the chief spacecraft follows an unperturbed circular orbit, the separation $\delta r_0$ between chief and deputy is small compared to the radius of the chief's orbit, and the relative motion occurs within the chief's orbital plane. These assumptions apply to many scenarios of interest, and the particular case of a robotic spacecraft servicing a satellite in GEO is chosen as a practical example. A novel patched-spirals trajectory design and optimization method is developed and applied to this sample mission for direct cost comparison of low-thrust and impulsive-thrust architectures. Continuous low-thrust maneuvers impart a higher delta-v burden due to kinematic inefficiency, but the utilization of high-$I_{sp}$ low-thrust propulsion systems may reduce propellant mass overall. For many scenarios, especially in the increasingly popular nano- and pico-satellite domain, impulsive maneuvers are not an option and the technique described provides a natural mission planning tool. In cases involving cooperative satellites, the technique can be readily extended to balance fuel-consumption. Finally, the prescribed thrust profile requires knowledge of the relative state and is therefore subject to estimation errors. The analysis concludes with a discussion of the solution sensitivity to uncertainties in state variables.

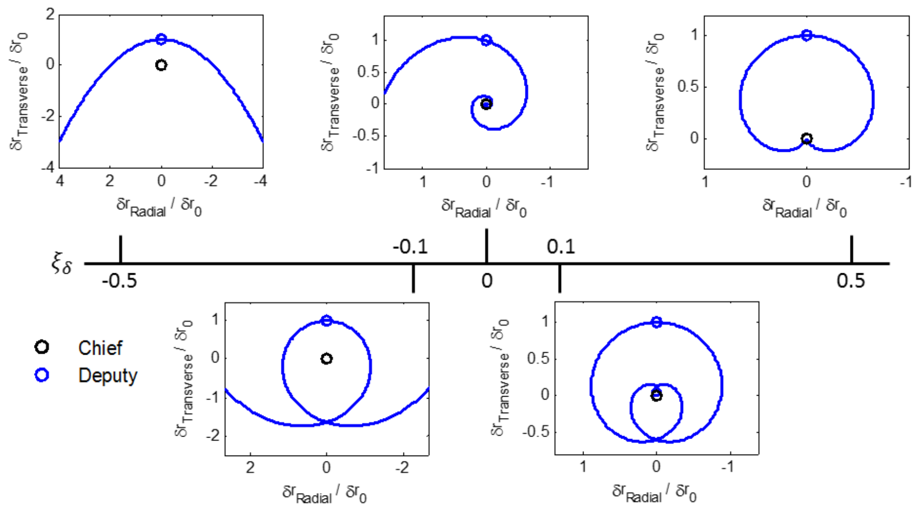


Fig. 1. Family of relative spiral trajectories.

## References

[1] J. Roa and J. Peláez, "Introducing a degree of freedom in the family of generalized logarithmic spirals," *American Astronautical Society*, 16-317 (2016)