Low-Thrust Trajectory Optimization with Dualized Collocation

Nathan L. Parrish, [1*] Daniel J. Scheeres[1]
[1] *University of Colorado, Boulder*
nathan.parrish@colorado.edu

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The greater fuel efficiency of electric propulsion (EP) systems as compared to chemical systems makes them appealing for many missions. However, the lower thrust of EP systems requires the engines to be turned on for longer durations. While chemical propulsion trajectories can be approximated by impulsive burns and ballistic coasts, EP trajectories are characterized by a continuously varying thrust law.

Optimal control can be divided into direct methods and indirect methods. Direct methods transcribe the optimal control problem into a nonlinear programming problem. Indirect methods make use of Pontryagin's principle to derive necessary conditions for optimality. In this work, direct and indirect methods for optimal control are brought together to efficiently and accurately solve the trajectory optimization problem. The optimal control problem is solved in two stages: Stage 1: Hermite-Simpson collocation is used to directly solve and optimize the trajectory at low fidelity. Then, in Stage 2: the same collocation method is used to solve the "dualized" problem with states and costates to more efficiently find the local optimum solution.

Two problems are used as benchmarks for the methods used. The first problem uses purely Keplerian dynamics for a heliocentric Earth-Mars rendezvous. The second problem uses the Earth-Moon circular restricted three body problem (CRTBP) dynamics to transfer from a distant retrograde orbit about the Moon to an L_2 halo orbit. Example solutions to these two benchmark problems are shown below.

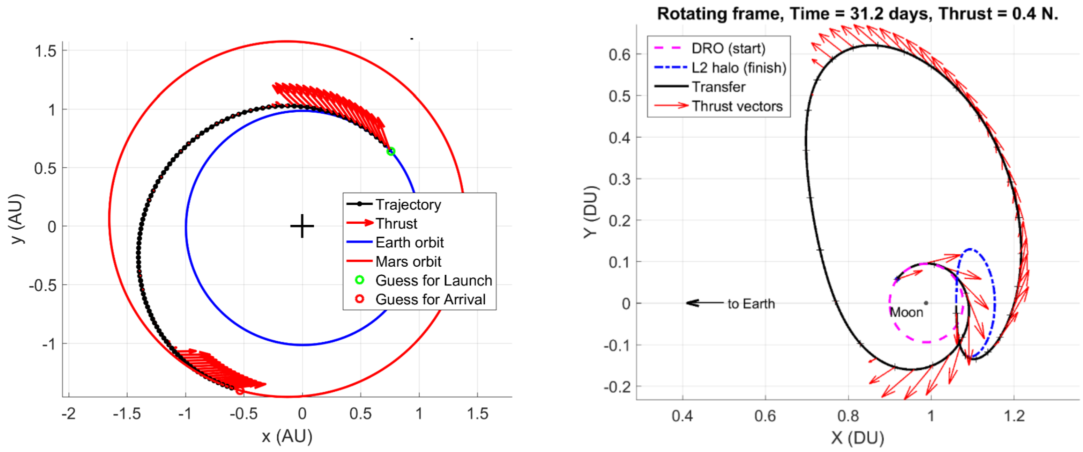

Figure 1. Examples solutions for the two example problems used.

In Stage 1 of solving the problem, collocation transcribes the optimal control problem into an NLP (nonlinear programming) problem, solved with the SQP (sequential quadratic programming) method as implemented in the MATLAB Optimization Toolbox. The optimization variables for this stage are: state $\vec{x}$ and control $\vec{u}$ at each collocation point, initial time t_0 and final time t_f. While it is possible to find a solution entirely with this approach, it is inefficient. In the context of this paper, the purpose of Stage 1 is to approximate the costates at each collocation point. These costate estimates come for free as a by-product of solving the NLP problem.

In Stage 2 of solving the problem, we use the solution from Stage 1 as an initial guess. Hermite-Simpson collocation is used here again. However, instead of parameterizing the trajectory with states and controls, we use states $\vec{x}$ and costates $\vec{\lambda}$ at each collocation point, initial time t_0 and final time t_f. Control at each collocation point is computed as a function of states and costates.