Non-Traditional Robust UKF against Attitude Sensors Faults

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Two non-traditional approaches for nanosatellite attitude estimation are investigated. In the non-traditional approach for attitude filtering the magnetometer and Sun sensor measurements are first used in the Singular Value Decomposition (SVD) algorithm to determine the spacecraft's attitude. Then the attitude estimates from the SVD are fed into the Unscented Kalman Filter (UKF) for getting finer attitude estimates and estimating the attitude rate. In the first case, this algorithm is used without any further modification for sensor faults. In the second case the SVD-aided UKF is adapted against the sensor measurement faults. For adaptation the filter's measurement noise covariance (R) matrix is tuned with a scale factor. The filters - the SVD-aided UKF (SVD/UKF) and the SVD-aided Robust UKF (SVD/RUKF) - are tested in two different fault scenarios: constant continuous bias and measurement noise increment. Fault scenarios are repeated for both sun sensor and magnetometer. The results indicate the non-traditional approach is inherently robust against the sensor faults as vector measurements are first pre-processed in the SVD before running the UKF.

Key Words: unscented Kalman filter, sensor fault, attitude estimation, nanosatellite, non-traditional approach

1. Introduction

Attitude estimation algorithms for small satellites that use magnetometer and sun sensor measurements have been designed in several studies. Kalman filtering is the most widely implemented algorithm for this purpose.[1,2)] The basic idea is to compare the vectors measured in the body frame with the known vectors in the inertial (or any other reference) frame to estimate the attitude of the satellite.

The traditional approach to design a Kalman filter for attitude estimation uses the nonlinear measurement model.[1-4)] The measurements and states are related via nonlinear equations. This may be called as the traditional approach for attitude filtering. On the other hand, in the so called non-traditional approach the attitude angles are first estimated by using the vector measurements and a suitable single-frame attitude estimation method.[5)] Then these estimated attitude angles are fed into a filtering algorithm as the measurements. The measurement model for the filter is linear in this case.[6-8)]

In the non-traditional approach for attitude filtering, magnetometer and sun sensor measurements are used together with the calculated reference directions to estimate the spacecraft's attitude by minimizing the Wahba's loss function.[9)] Surely the attitude cannot be estimated by single-frame methods when the measurement vectors are aligned (parallel) or one of them is missing (e.g. when the satellite is in eclipse and there is no sun sensor measurement). Yet the Kalman filter, which is used at the second stage of the attitude estimation scheme, can adapt its gain in such condition to provide attitude estimates.

Hajiyev and Bahar proposed an integrated attitude determination algorithm for small satellites to estimate the attitude angles and angular velocities by Extended Kalman Filter (EKF) and algebraic method combination.[10)] In this scheme for non-traditional attitude filtering the proposed EKF uses the outputs of the algebraic method. The algebraic method estimates the attitude using two measurement vectors in the body frame and their corresponding reference directions in the orbit frame. The method works with any of the two vector combinations for sun sensor, magnetometer and horizon sensor.

In our previous studies we proposed different non-traditional attitude filtering algorithms. We used the Singular Value Decomposition (SVD) method as the single-frame attitude estimator at the first stage of the algorithm. We investigated the EKF and Unscented Kalman Filter (UKF) as the attitude filter at the second stage of the algorithm.[7, 8, 11, 12)]

In this study, we first construct the SVD-aided UKF (SVD/UKF) algorithm as a non-traditional filtering algorithm for attitude estimation. The SVD/UKF has, inherently, an adaptive structure since it uses the estimation covariance of the SVD algorithm as the measurement noise covariance (R) of the UKF. Secondly, we propose using R-adaptive UKF at the second stage of the SVD/UKF. The algorithm uses the measurement noise scale factor (MNSF), which is calculated by covariance matching, to adapt the measurement noise covariance matrix of the filter. The adaptation is achieved by multiplying the measurement noise covariance matrix with the MNSF and tuning the Kalman gain. Thus the filter becomes robust against the sensor faults. This algorithm is called SVD/RUKF.

In this paper we compare the attitude estimation results of SVD/UKF and SVD/RUKF algorithms in case of a sensor fault. For this purpose, two different sensor fault scenarios are investigated for both the magnetometers and sun sensor. In the first scenario, an additional constant continuous bias is considered for the sensor measurements. Secondly, a scenario, where the sensor noise increases, is investigated.

The structure of this paper is as follows. In Section 2, mathematical models of the magnetometer and sun sensor are given. Section 3 introduces the SVD algorithm. The SVD/UKF and SVD/RUKF algorithms are explained in detail in Section 4. The performance of the filters are investigated for different measurement fault scenarios in Section 5 using the simulated data for a hypothetical nanosatellite. Finally, Section 6 gives a brief summary of the obtained results and concludes the paper.

2. Sensor Models

There are magnetometer and sun sensor onboard the nanosatellite as the attitude sensors. The IGRF-12 model is used for modelling the Earth's magnetic field and calculating the reference magnetic field vector, $\boldsymbol{B}_o$, in the orbital reference frame.[13] The magnetometers measure the magnetic field vector in the body frame, $\boldsymbol{B}_b$, as

$$\boldsymbol{B}_b(k) = A(k)\boldsymbol{B}_o(k) + \boldsymbol{v}_b(k) \tag{1}$$

$\boldsymbol{v}_b(k)$ is the Gaussian white magnetometer measurement noise and $A(k)$ is the attitude matrix that represent the transformation from orbital reference frame to the body frame.

The sun direction vector in the orbital frame is modeled using the Julian date and the orbital parameters as input.[14] The measurement model for the sun sensor is

$$\boldsymbol{S}_b(k) = A(k)\boldsymbol{S}_o(k) + \boldsymbol{v}_s(k)\,, \tag{2}$$

where $\boldsymbol{S}_b(k)$ is the measured sun direction vector in the body frame, $\boldsymbol{S}_o(k)$ is the sun direction vector calculated in the orbital frame and $\boldsymbol{v}_s(k)$ is the Gaussian white sun sensor measurement noise.

3. Single-Frame Method Based on Vector Measurements: SVD

In the presence of two or more vector measurements the optimal attitude of the spacecraft can be found by minimizing the Wahba's loss function[9] that is given as

$$L(A) = \frac{1}{2}\sum_i a_i \,|\, \boldsymbol{b}_i - A\boldsymbol{r}_i \,|^2\,. \tag{3}$$

Here $\boldsymbol{b}_i$ is the set of unit vector measurements in the body frame, $\boldsymbol{r}_i$ is the set of reference unit vectors in the orbital frame and a_i is the nonnegative weight for each measurement. If the loss function is rewritten as

$$L(A) = \lambda_0 - \mathrm{tr}\left(AZ^T\right), \tag{4}$$

$$Z = \sum a_i \boldsymbol{b}_i \boldsymbol{r}_i^T\,;\ \ \lambda_0 = \sum a_i\,, \tag{5}$$

The minimization problem becomes same as maximizing the $\mathrm{tr}\left(AZ^T\right)$. In this study, SVD method is chosen to minimize the loss function.[15] The Z matrix has singular value decomposition as

$$Z = USV^T = U\mathrm{diag}\left|S_{11}\ S_{22}\ S_{33}\right|V^T\,. \tag{6}$$

The matrices U and V are orthogonal left and right matrices respectively, and S_{11}, S_{22} and S_{33} are the primary singular values. The spacecraft's attitude is found as

$$A_{opt} = U\mathrm{diag}[1 \quad 1 \quad \det(U)\det(V)]V^T\,. \tag{7}$$

Then the attitude angles are found using the components of the estimated attitude matrix A_{opt}.

Rotation angle error covariance matrix is calculated as

$$P_{SVD} = U\mathrm{diag}[(s_2+s_3)^{-1} \quad (s_3+s_1)^{-1} \quad (s_1+s_2)^{-1}]U^T\,. \tag{8}$$

where $s_1 = S_{11}$, $s_2 = S_{22}$ and $s_3 = \det(U)\det(V)\,S_{33}$.

4. Non-Traditional Approach for Attitude Filtering using the UKF

The attitude angles that are estimated by the SVD are used as the measurements for the UKF. The SVD and UKF algorithms are integrated to get finer estimates for the attitude angle and estimate the angular velocities.

Two different algorithms are built. The first one, SVD/UKF, uses the rotation angle error covariance matrix of SVD algorithm (P_{SVD}) as the measurement error covariance matrix (R) in the UKF. The second algorithm, SVD/RUKF, uses an adaptive UKF instead of a regular one. In this version the R matrix for the UKF is determined by an adaptive rule.

4.1. SVD/UKF

UKF algorithm is derived for nonlinear system and linear measurement models. The model is expressed as

$$\boldsymbol{x}(k+1) = \boldsymbol{f}(\boldsymbol{x}(k),k) + \boldsymbol{v}_p(k)\,, \tag{9}$$

$$\boldsymbol{y}(k) = H\boldsymbol{x}(k) + \boldsymbol{v}_m(k)\,. \tag{10}$$

Here, $\boldsymbol{x}(k)$ is the state vector containing the quaternions and angular rates of the nanosatellite,

$$\boldsymbol{x} = \left[\boldsymbol{q}^T \quad \boldsymbol{\omega}_{BI}^T\right]^T, \tag{11}$$

$\boldsymbol{y}(k)$ is the measurement vector and H is the measurement matrix. In Eqs. (9, 10) $\boldsymbol{v}_p(k)$ and $\boldsymbol{v}_m(k)$ are the process and measurement error noises, which are assumed to be Gaussian white noises with the covariance matrices of $Q(k)$ and $R(k)$, respectively.

The initial step of the UKF algorithm is determining the $2n+1$ sigma points with a mean of $\hat{\boldsymbol{x}}(k|k)$ and a covariance of $P(k|k)$. For a n dimensional state vector, these sigma points are obtained by[16]

$$x_0(k|k) = \hat{x}(k|k), \tag{12a}$$

$$x_\gamma(k|k) = \hat{x}(k|k) + \left(\sqrt{(n+\kappa)P(k|k)}\right)_\gamma, \tag{12b}$$

$$x_{\gamma+n}(k|k) = \hat{x}(k|k) - \left(\sqrt{(n+\kappa)P(k|k)}\right)_\gamma, \tag{12c}$$

where, $x_0(k|k)$, $x_\gamma(k|k)$ and $x_{\gamma+n}(k|k)$ are sigma points, and κ is the scaling parameter which is used for fine tuning. $\left(\sqrt{(n+\kappa)P(k|k)}\right)_\gamma$ corresponds to the γ^{th} column of the indicated matrix for $\gamma = 1 \ldots n$.

The next step of the UKF procedure is evaluating the transformed set of sigma points for each of the points by,

$$x_l(k+1|k) = f\left[x_l(k|k), k\right]. \qquad l = 0 \ldots 2n \tag{13}$$

Thereafter, these transformed values are used for calculating the predicted mean, $\hat{x}(k+1|k)$, and covariance, $P(k+1|k)$

$$\hat{x}(k+1|k) = \sum_{l=0}^{2n} \lambda_l x_l(k+1|k), \tag{14}$$

$$P(k+1|k) = \left\{ \sum_{l=0}^{2n} \lambda_l \left[x_l(k+1|k) - \hat{x}(k+1|k)\right] \left[x_l(k+1|k) - \hat{x}(k+1|k)\right]^T \right\} + Q(k). \tag{15}$$

The weights are defined as

$$\lambda_0 = \frac{\kappa}{n+\kappa}; \qquad \lambda_l = \frac{1}{2(n+\kappa)} \quad l = 1 \ldots 2n. \tag{16}$$

The measurement model is linear. Thus the predicted observation vector is,

$$\hat{y}(k+1|k) = H(k+1)\hat{x}(k+1|k). \tag{17}$$

The observation covariance matrix is determined as,

$$P_{yy}(k+1|k) = H(k+1)P(k+1|k)H^T(k+1). \tag{18}$$

The cross correlation matrix can be obtained as,

$$P_{xy}(k+1|k) = P(k+1|k)H^T(k+1). \tag{19}$$

Next the update stage of the algorithm comes. The innovation vector for the filter is

$$\varepsilon(k+1) = y(k+1) - \hat{y}(k+1|k), \tag{20}$$

The innovation covariance is,

$$P_{vv}(k+1|k) = P_{yy}(k+1|k) + R(k+1). \tag{21}$$

Here $R(k+1)$ is the measurement noise covariance matrix, which is taken as the covariance of SVD estimates, $R(k+1) = P_{svd}(k+1)$, as mentioned.

The Kalman gain is computed as,

$$K(k+1) = P_{xy}(k+1|k)P_{vv}^{-1}(k+1|k). \tag{22}$$

At last, the state vector and covariance matrix are updated as,

$$\hat{x}(k+1|k+1) = \hat{x}(k+1|k) + K(k+1)\varepsilon(k+1), \tag{23a}$$

$$P(k+1|k+1) = P(k+1|k) - K(k+1)P_{vv}(k+1|k)K^T(k+1). \tag{23b}$$

4.2. SVD/RUKF

Specifically in case of measurement faults, we need to adapt the filter to make it robust against the sensor faults. Otherwise the faults deteriorate the estimation accuracy. In the non-traditional approach for attitude filtering, the adaptation is also needed when we do not use the estimation covariance of the single-frame method and instead prefer having a self-tuning filtering algorithm.

Here, the adaptation rule is applied as a change in the covariance matrix of the innovation sequence as

$$P_{vv}(k+1|k) = P_{yy}(k+1|k) + S(k)R(k+1), \tag{24}$$

where $S(k)$ is the MNSF that is calculated with the analysis of the innovation sequence, $\varepsilon(k+1)$. In the robust case, the filter gain becomes

$$K(k+1) = P_{xy}(k+1|k)\left[P_{yy}(k+1|k) + S(k)R(k+1)\right]^{-1}. \tag{25}$$

The gain matrix is changed when the condition of

$$tr\left\{\varepsilon(k+1)\varepsilon^T(k+1)\right\} \geq tr\left\{P_{yy}(k+1|k) + R(k+1)\right\} \tag{26}$$

is satisfied.

When the predicted observation vector $\hat{y}(k+1|k)$ is reasonably different from measurement vector, $y(k+1)$, the real filtration error exceeds the theoretical one. The Kalman gain must be adapted hereafter. To calculate the MNSF the following equation is used:

$$\mathrm{tr}\left[\varepsilon(k+1)\varepsilon^T(k+1)\right] = \mathrm{tr}\left[P_{yy}(k+1|k) + S(k)R(k+1)\right]. \tag{27}$$

Eq. (27) can be rewritten as,

$$\mathrm{tr}\left[\varepsilon(k+1)\varepsilon^T(k+1)\right] = \mathrm{tr}\left[P_{yy}(k+1|k)\right] + S(k)\mathrm{tr}\left[R(k+1)\right]. \tag{28}$$

It is known that

$$\mathrm{tr}\left[\boldsymbol{\varepsilon}(k+1)\boldsymbol{\varepsilon}^T(k+1)\right]=\boldsymbol{\varepsilon}^T(k+1)\boldsymbol{\varepsilon}(k+1). \tag{29}$$

Thus

$$\boldsymbol{\varepsilon}^T(k+1)\boldsymbol{\varepsilon}(k+1)=\mathrm{tr}\left[P_{yy}(k+1|k)\right]+S(k)\,\mathrm{tr}\left[R(k+1)\right]. \tag{30}$$

As a result, the MNSF can be calculated as,

$$S(k)=\frac{\boldsymbol{\varepsilon}^T(k+1)\boldsymbol{\varepsilon}(k+1)-\mathrm{tr}\left[P_{yy}(k+1|k)\right]}{\mathrm{tr}\left[R(k+1)\right]}. \tag{31a}$$

or equally,

$$=\frac{\boldsymbol{\varepsilon}^T(k+1)\boldsymbol{\varepsilon}(k+1)-\mathrm{tr}\left[H(k+1)P(k+1/k)H^T(k+1)\right]}{\mathrm{tr}\left[R(k+1)\right]} \tag{31b}$$

The gain of the RUKF in the SVD/RUKF scheme is adapted with the MNSF at every k.

5. Simulation Results and Analysis

The orbit of the nanosatellite considered in this paper is assumed to be a Low Earth Orbit (LEO) with a small eccentricity of $e=6.4\times10^{-5}$, an inclination of $i=74^\circ$ and approximate altitude of 612 km. Algorithm runs for 6000 s and for 1 Hz measurement frequency for the sensors. The UKF and RUKF are also propagated with a sampling time of $\Delta t = 1\,\mathrm{s}$.

For the magnetometer measurements, the sensor noise is characterized by zero mean Gaussian white noise with a standard deviation of $\sigma_m = 300\ \mathrm{nT}$. The standard deviation for the sun sensor noise is taken as $\sigma_s = 0.002$ (for unit vector measurements). Two different sensor faults are considered for both magnetometer and sun sensor: continuous bias and noise increment. The measurement faults occur between 4500th and 5500th seconds.

Four different sensor fault scenarios are formulated as

$$\left.\begin{aligned}\boldsymbol{B}_b(k)&=A(k)\boldsymbol{B}_o(k)+\boldsymbol{v}_b(k)+0.2\sigma_m\begin{bmatrix}1&1&1\end{bmatrix}^T\\ \boldsymbol{S}_b(k)&=A(k)\boldsymbol{S}_o(k)+\boldsymbol{v}_s(k)+0.2\sigma_s\begin{bmatrix}1&1&1\end{bmatrix}^T\\ \boldsymbol{B}_b(k)&=A(k)\boldsymbol{B}_o(k)+3\boldsymbol{v}_b(k)\\ \boldsymbol{S}_b(k)&=A(k)\boldsymbol{S}_o(k)+3\boldsymbol{v}_s(k)\end{aligned}\right\}$$

$$\text{for } 4500\le k\le 5500. \tag{32}$$

In Figs. 1-3, SVD/UKF estimations are presented for quaternion vector components. In Fig. 1, magnetometer has the bias type of fault as $0.2\sigma_m\begin{bmatrix}1&1&1\end{bmatrix}^T$ in 4500-5500s interval. As it can be seen from the figure, SVD/UKF has the ability to adapt itself in case of fault. Yet the bias in the attitude estimations is not completely removed.

In Fig. 2 (a), the quaternion estimations of SVD/UKF can be seen together with the actual attitude of the satellite in case of measurement noise increment in the magnetometer measurements. As can be seen the effects of the measurement fault on the SVD/UKF attitude estimations are not severe. The quantity of the degradation in the attitude estimates is better seen for the estimation errors in Fig.2(b). Clearly, despite the fault, the estimation accuracy is still better than 0.1deg.

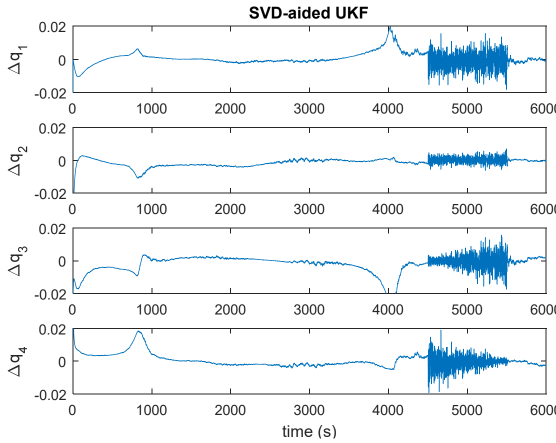

Fig. 1. SVD/UKF quaternion estimation errors in case of bias type of fault in magnetometer measurements.

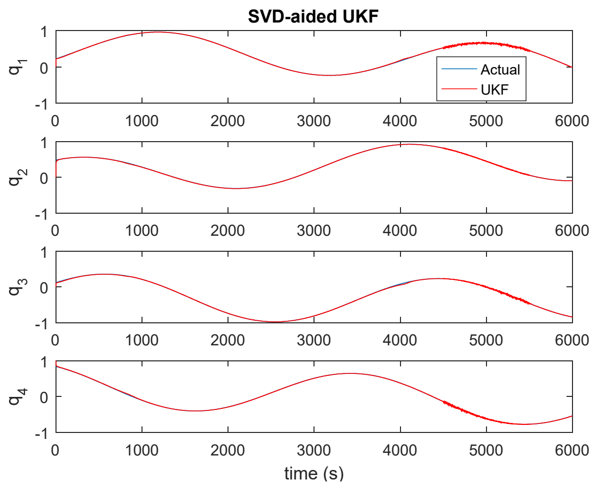

Fig. 2. (a) SVD/UKF quaternion estimations in case of noise increment type of fault in magnetometer measurements.

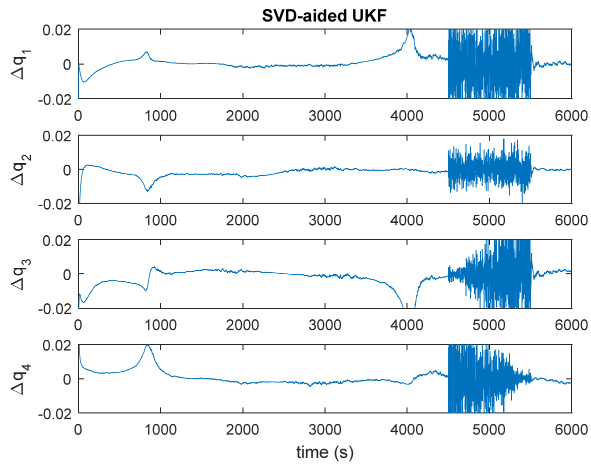

Fig. 2. (b) SVD/UKF quaternion estimation errors in case of noise increment type of fault in magnetometer measurements.

In Fig. 3, the quaternion estimation errors for the SVD/UKF are given when the measurement noise increases for the sun sensor. We see that the filter is capable of decreasing the effects of the fault irrelevantly from its source (from which sensor the fault is originated).

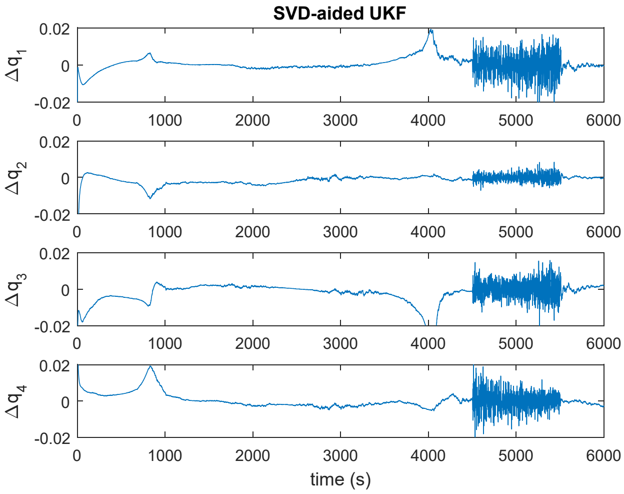

Fig. 3. SVD/ UKF quaternion estimation errors in case of measurement noise increment type of fault in sun sensor measurements.

In Figs. 4-6, the estimation results for the SVD/RUKF algorithm are given. In Fig. 4, magnetometer has the bias type of fault as $0.2\sigma_m\begin{bmatrix}1 & 1 & 1\end{bmatrix}^T$ in 4500-5500s interval. The SVD/RUKF can adapt itself but cannot completely remove the bias on the estimations, similarly with the SVD/UKF. The results are also similar in other fault scenarios. In all cases, although the effect of the sensor fault on the attitude estimation accuracy is minimized, it is not completely removed. Nonetheless the attitude estimation accuracy, which is higher than 0.1deg, is sufficient regarding the expectations for the nanosatellite mission.

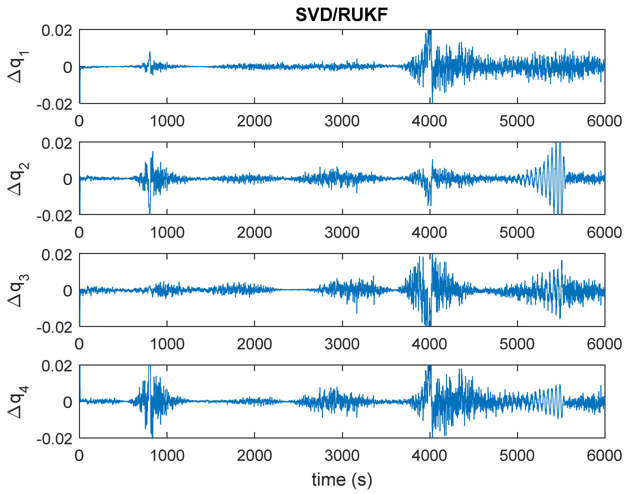

Fig. 4. SVD/RUKF quaternion estimation errors in case of bias type of fault in magnetometer measurements.

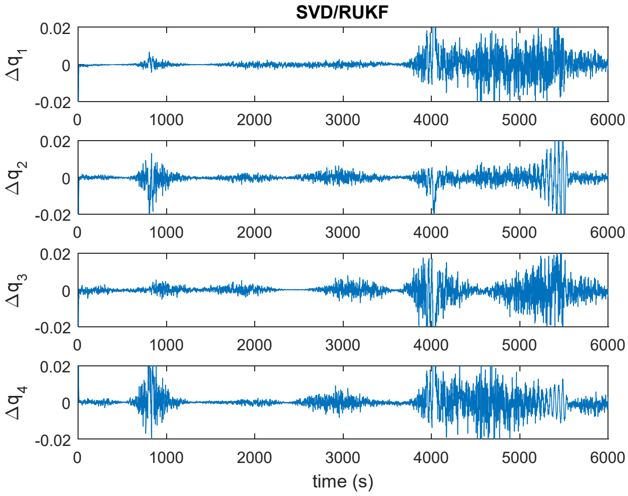

Fig. 5. SVD/RUKF quaternion estimations in case of noise increment type of fault in magnetometer measurements.

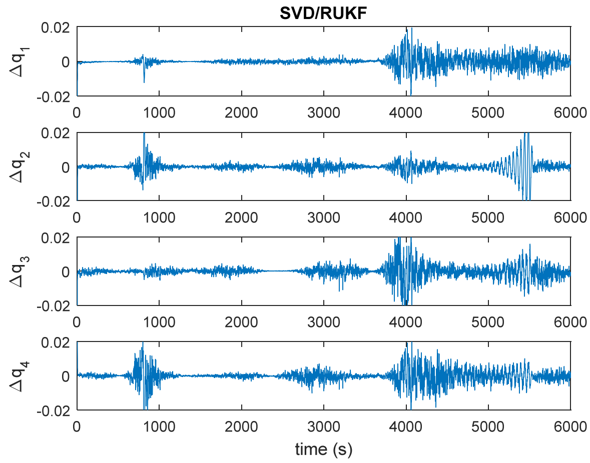

Fig. 6. SVD/RUKF quaternion estimations in case of noise increment type of fault in sun sensor measurements.

In Figs. 4-6 we see that from time to time the attitude estimations of the SVD/RUKF may deteriorate even when there is no sensor fault. This is a disadvantage of the SVD/RUKF algorithm.

In Tables 1 and 2 we present the root mean square (RMS) errors for the algorithms in different scenarios. The RMS errors are calculated in 4500-5500s interval for sensor fault scenarios. In the so called normal mode there is no sensor fault and the RMS error sampled in an appropriate interval.

In Tables 1 and 2 we see that in the normal mode the estimation error of the SVD/RUKF algorithm is slightly higher. This result matches with the observed trend in the Figs.4-6 which tells us the SVD/RUKF estimations may be noisy even when there is no fault. On the other hand, the RMS errors in case of a measurement fault do not have a very clear pattern. The estimation errors for both the SVD/UKF and SVD/RUKF in case of a sensor fault is similar. Therefore we may say that using the SVD/UKF is more advantageous in general, mainly because of its slightly higher accuracy in the "normal mode" (when there is no sensor fault). Surely SVD/UKF is also advantageous in terms of simplicity as we do not need to calculate the adaptive factor for this filter.

Table 1. RMS errors of SVD/UKF in different scenarios.

RMSE	SVD/ UKF				
	Normal Mode	Magnetometer Fault		Sun Sensor Fault	
		Bias	Noise Increment	Bias	Noise Increment
q_1	0.0026	0.0050	0.0140	0.0047	0.0060
q_2	0.0012	0.0019	0.0053	0.0018	0.0020
q_3	0.0017	0.0040	0.0110	0.0037	0.0047
q_4	0.0022	0.0041	0.0110	0.0040	0.0050

Table 2. RMS errors of SVD/RUKF in different scenarios.

RMSE	SVD/RUKF				
	Normal Mode	Magnetometer Fault		Sun Sensor Fault	
		Bias	Noise Increment	Bias	Noise Increment
q_1	0.0027	0.0028	0.0076	0.0027	0.0033
q_2	0.0040	0.0060	0.0068	0.0055	0.0059
q_3	0.0022	0.0033	0.0066	0.0030	0.0036
q_4	0.0030	0.0034	0.0070	0.0031	0.0036

6. Conclusion

In this study, we first construct the Singular Value Decomposition (SVD) aided Unscented Kalman Filter (UKF) algorithm as a non-traditional filtering algorithm for attitude estimation. The algorithm, which is called SVD/UKF, has inherently an adaptive structure since it uses the estimation covariance of the SVD algorithm as the measurement noise covariance (R) of the UKF. Secondly, we propose using R-adaptive UKF at the second stage of the SVD/UKF. The algorithm uses the measurement noise scale factor (MNSF), which is calculated by covariance matching, to adapt the measurement noise covariance matrix of the filter. The adaptation is achieved by multiplying the measurement noise covariance matrix with the MNSF and tuning the Kalman gain. Thus the filter becomes robust against the sensor faults. This algorithm is called SVD/RUKF.

Simulation results show that both of the presented algorithms are robust against measurement faults. Although the fault cannot be removed completely, its effect on the attitude estimation accuracy is minimized. Further investigations show that, specifically in the fault-free normal mode, it is more advantageous to use the SVD/UKF algorithm.

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