Spinning Spacecraft Attitude Filtering with Spin Parameters: Performance Evaluation with Real Data

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Three-axis attitude estimation for spinning spacecraft is recently of considerable practical interest. In this scope, sequential filtering algorithms are also being studied recently. In this study, we extend our recent research for a nonlinear filtering algorithm for spinning spacecraft attitude estimation. In the filter the attitude of the spacecraft is represented using a set of spin parameters. These parameters consist of the spin-axis orientation unit vector in the inertial frame and the spin phase angle. As the system and measurement models are nonlinear an Unscented Kalman Filter (UKF) is implemented to estimate the spacecraft's attitude. In this paper, we investigate the accuracy of the algorithm by using telemetry data gathered by the CONTOUR spacecraft in 2002. We discuss different methods for satisfying the spherical norm constraint for the spin-axis orientation unit vector terms. The filter works well and produces consistent results with those of the Tanygin-Shuster and TRIAD algorithms. Investigations on the norm constraint condition shows that unit vector normalization must be applied specifically in the presence of measurement biases.

Key Words: spinning spacecraft, attitude filtering, UKF, spin parameters

1. Introduction

The spin-stabilization is one of the concepts that have been used for spacecraft attitude stabilization since the early space era. Recently it has become attractive once more especially for the small satellite missions[1,2)]. The reason is its simplicity and suitability to conduct science missions at a relatively low cost, which are both desirable criteria sought for small satellites.

Nonetheless, attitude estimation for spinning spacecraft is not as simple as the concept itself. Three-axis attitude estimation for a spinning spacecraft is likely to be even more challenging than that for a three-axis stabilized spacecraft. In particular, for recent spinning spacecraft missions the attitude requirements are more stringent. For these missions various filtering algorithms are proposed to enhance the capability of the attitude determination system[3-5)].

Recently a nonlinear attitude filtering algorithm that is designed for spinning spacecraft has been proposed[6)]. Its essence is representing the attitude of spinning spacecraft using a set of spin parameters. These parameters are the components for the spin-axis orientation unit vector in the inertial frame plus the spin phase angle. This representation is advantageous as the spin axis direction do not change rapidly in the inertial frame. Furthermore the phase angle changes at a constant rate in the absence of a torque. An Unscented Kalman Filter (UKF) is used to estimate spacecraft's attitude in terms of these parameters. The algorithm is called the SpinUKF.

The results with the simulated data for JAXA's ERG (the Exploration of Energization and Radiation in Geospace) spacecraft showed that the SpinUKF works well even with large propagation step size. Depending on the simulation conditions the filter has similar or better attitude estimation accuracy than a filter with quaternions in its state vector[6)].

This study applies the SpinUKF algorithm to the in-flight sensor data[7)] collected by NASA's CONTOUR spacecraft. CONTOUR was designed and operated by Johns Hopkins University Applied Physics Laboratory, Laurel, MD, USA. It was launched in July 2002 and injected into an elliptical Earth-phasing orbit that lasted about 6 weeks. During this period the spacecraft was stabilized at nominal spin rates of either 20 or 60 rpm and used a combined Sun-Earth sensor for the attitude measurements.

In this paper, we evaluate the performance of the SpinUKF with the real CONTOUR flight data. Moreover we investigate two different methods for satisfying the spherical norm constraint for the spin-axis unit vector. These are the brute force normalization and Lagrange multiplier methods. We compare the attitude estimation results when SpinUKF incorporates one of the normalization methods with those of a filter without norm constraint.

2. Sensor Measurement Models

A combined Earth-Sun sensor is used for attitude measurements onboard the CONTOUR spacecraft[7)]. The measurement model for unit-vector measurements is simply,

$$\boldsymbol{S}_b = A_i^b \boldsymbol{S}_i + \boldsymbol{v} \tag{1}$$

where, $\boldsymbol{S}_b$ is the measurement vector in the body frame, $\boldsymbol{S}_i$ is the reference vector in the inertial frame, A_i^b is the attitude matrix that transforms a vector from inertial to body frame and $\boldsymbol{v}$ is the measurement noise. With reference to the geometry of the measurements (Fig.1) the unit vector measurements in the body frame can be formed as

$$S_b^{sun} = \begin{bmatrix} \sin\upsilon\cos\delta \\ \sin\upsilon\sin\delta \\ \cos\upsilon \end{bmatrix} , \tag{2a}$$

$$S_b^{earth} = \begin{bmatrix} \sin\beta\cos(\delta+\alpha) \\ \sin\beta\sin(\delta+\alpha) \\ \cos\beta \end{bmatrix} , \tag{2b}$$

using the sensor outputs for Sun-sensor and Earth-sensor measurements, respectively.

The construction of the Sun-sensor unit-vector measurements is relatively easy. A typical V-slit Sun-sensor for a spinning satellite provides the Sun crossing times and the Sun aspect angle (SAA), υ, which is the angle between the spin axis (i.e., body Z_b axis in Fig. 1) and the Sun direction $\boldsymbol{S}$. When considering the rotation angle shift of the Sun-sensor vertical slit plane relative to the body X_b axis, i.e. angle δ, we can calculate the unit vector measurement at the time of Sun crossing as in Eq.(2a).

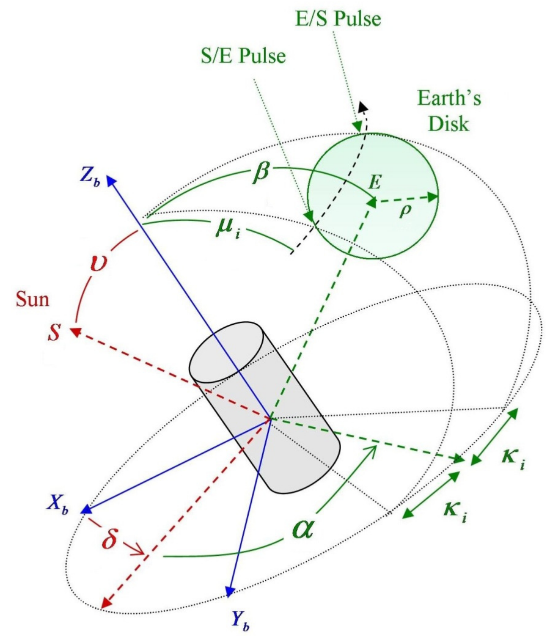

Figure 1. Geometry for Sun and Earth-sensor measurements.

The Earth-sensor has two pencil-beams oriented at angles μ_i ($i=1, 2$) with respect to the spin axis. Each pencil-beam measures the crossing times for Space/Earth (S/E) and Earth/Space (E/S) infra-red boundaries. With the knowledge of the spin rate, these crossing pulses can be transformed into the half-chord angles κ_i ($i=1, 2$), which are the fundamental measurements produced by the Earth-sensor. By using these measurements and as well the apparent Earth radius angle, ρ, we find the relationship for the Earth Aspect Angle (EAA), β, from the spherical geometry (Fig.1):

$$\cos\mu_i\cos\beta + \sin\mu_i\cos\kappa_i\sin\beta = \cos\rho \qquad i=1,2 \quad . \tag{3}$$

Here, the instantaneous value for the apparent Earth radius angle, ρ, can be calculated as $\rho(t)=\sin^{-1}(R_{IR}/r(t))$, where r is the orbital radius and R_{IR} is the Infrared (IR) Earth radius. Note that the nominal value of R_{IR} is ~40km above the Earth radius[8]. The actual value of R_{IR} is unknown and varies over time and location.

In fact, the scan paths of each of the two IR pencil-beams over the Earth are different due to different mounting angles (μ_1 and μ_2). As a result, their S/E and E/S crossings are at *different* locations on the Earth's IR rim. Therefore, the two ρ values observed by the IR sensors will in general differ under seasonal, diurnal, and local variations in the IR radiation intensities. Yet it is very hard to model all of these effects realistically so we assume, a priori, that the Earth's IR radius is perfectly uniform so we may use the same ρ value for both IR beams.

We may use different methods to derive the EAA when there are two individual pencil-beams. To start with, we may rewrite the Eq.(3) in the form[9]:

$$b_i\cos\chi_i\cos\beta + b_i\sin\chi_i\sin\beta = \cos\rho \qquad i=1,2 , \tag{4}$$

where the auxiliary functions are defined as:

$$b_i = \sqrt{1-\left(\sin\mu_i\sin\kappa_i\right)^2} ;$$

$$\chi_i = \arctan(\tan\mu_i\cos\kappa_i) \qquad i=1,2 . \tag{5a,b}$$

Thus, each pencil-beam gives its own EAA solution, β_i:

$$\beta_i^{\pm} = \chi_i \pm \arccos\{\cos(\rho)/b_i\} \qquad i=1,2 . \tag{6}$$

One method to construct a single EAA value to use in the estimator is to take the average of the two different β_i values produced by each of the two pencil-beams as $\beta_{ave}=(\beta_1^{\pm}+\beta_2^{\pm})/2$, after the sign ambiguity is resolved. The second method is to calculate an optimal (in a minimum variance sense) β_{opt} value as a weighted combination of the two individual β_i angles. This method is based on the sensitivity analysis for each of the two β_i solutions with respect to the half-chord angle measurements, κ_i [9].

Lastly, a single solution for β may be calculated from the two distinct relationship equations (Eq.3) for the two pencil beams without requiring the individual β_i solutions (Eqs.4-6). Since we assume here that the ρ value is identical for both pencil-beam scans we obtain the single β_{sin} solution as,

$$\beta_{sin} = \arctan\left(\frac{\cos\mu_1 - \cos\mu_2}{\sin\mu_2\cos\kappa_2 - \sin\mu_1\cos\kappa_1}\right). \tag{7}$$

In this study, we use the "single beta" β_{sin} solution of Eq.(7) to build unit-vector measurements from the Earth-sensor outputs.

Fig. 2 gives the derived EAA from the chord measurements and includes β_1 and β_2 values for each pencil-beam, which are calculated with Eqs.(5,6). The usable good data, which is least corrupted by sensor biases, is collected during the scans over Earth's mid-latitude region that starts approximately at 36.6h since the perigee pass and lasts for about 1h[9]. The interval before the mid-latitude region should be avoided because it is near the singularity in the chord-length measurements for the second pencil-beam and the same should be done for the first pencil-beam after the mid-latitude region.

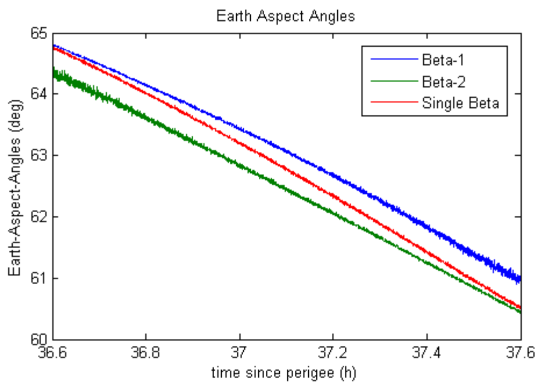

Figure 2. Earth aspect angles derived from chord measurements.

To build the unit-vector measurements for the Earth-sensor (Eq.2b) the Sun-Earth dihedral angle, α, is also needed. Calculation of this angle from two distinct α_i measurements of the pencil-beams is straightforward as these two measurements can be equally weighted as,

$$\alpha = (\alpha_1 + \alpha_2)/2 \,. \tag{8}$$

3. Attitude Filtering with Spin Parameters

3.1. Spin Parameters

The so called spin parameters are defined as the components of the spin axis unit vector, which is

$$\mathbf{1}_i^{spin} = \begin{bmatrix} x & y & z \end{bmatrix}^T , \tag{9}$$

and the spin phase angle γ (Fig.3)[6]. To use the spin parameters in the attitude filter, we need to have the attitude matrix and spacecraft kinematics in terms of these parameters.

To derive the attitude matrix in terms of spin parameters we start by expressing the transformation from inertial frame to spacecraft body frame by a sequence of three Euler angles α, β and γ in the order 3-2-3[10]. Note that, since we consider the motion of a spinning spacecraft here, the Euler angles, α, β and γ, represent the *precession*, *nutation* and *spin* (or *phase*) angles, respectively[11]. In this case, the body axis, Z_b, of the spacecraft is the design spin axis. For this sequence the attitude matrix that transforms a vector from inertial to body frame is given as

$$A_i^b = \begin{bmatrix} c(\alpha)c(\beta)c(\gamma) - s(\alpha)s(\gamma) & s(\alpha)c(\beta)c(\gamma) + c(\alpha)s(\gamma) & -s(\beta)c(\gamma) \\ -c(\alpha)c(\beta)s(\gamma) - s(\alpha)c(\gamma) & -s(\alpha)c(\beta)s(\gamma) + c(\alpha)c(\gamma) & s(\beta)s(\gamma) \\ c(\alpha)s(\beta) & s(\alpha)s(\beta) & c(\beta) \end{bmatrix} , \tag{10}$$

where $c(\cdot)$ and $s(\cdot)$ are $\cos(\cdot)$ and $\sin(\cdot)$ functions, respectively.

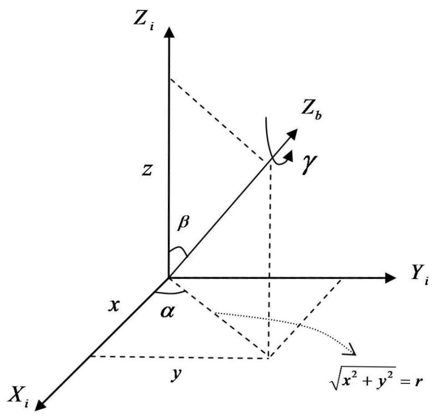

Figure 3. Spin axis in the inertial frame and the spin parameters.

From Eq.(10) we notice that the body Z_b axis can be represented in the inertial frame by the unit spin axis vector, $\mathbf{1}_i^{spin}$, as

$$\mathbf{1}_i^{spin} = \begin{bmatrix} c(\alpha)s(\beta) & s(\alpha)s(\beta) & c(\beta) \end{bmatrix}^T . \tag{11}$$

Then, it is straightforward from Eqs.(9) and (11) that trigonometric functions of the Euler angles relate to the components of the unit vector along the spin axis through the following equations (see Fig.3):

$$c(\beta) = z \,, \tag{12a}$$

$$s(\beta) = \sqrt{x^2 + y^2} = r \,, \tag{12b}$$

$$c(\alpha) = \frac{x}{\sqrt{x^2 + y^2}} = \frac{x}{r} \,, \tag{12c}$$

$$s(\alpha) = \frac{y}{\sqrt{x^2 + y^2}} = \frac{y}{r} \,. \tag{12d}$$

These four equations constitute the basis for expressing the attitude matrix in terms of the spin axis parameters. Obviously, the direction of the spin axis is determined only by α and β and is not dependent on the spin phase angle γ. In fact, a rotation matrix that takes $\mathbf{1}_i^{spin}$ to $\mathbf{1}_b^{spin}$ can be defined by just these two angles. On the other hand, to determine the full three-axis attitude of the spacecraft and the attitude matrix, we need to include the phase angle. By substituting the trigonometric relations of Eq.(12) in Eq.(10) while leaving the terms that include the phase angle we obtain

$A_i^b(x,y,z,\gamma) =$

$$\begin{bmatrix} \dfrac{xz\,\mathrm{c}(\gamma) - y\mathrm{s}(\gamma)}{r} & \dfrac{yz\,\mathrm{c}(\gamma) + x\mathrm{s}(\gamma)}{r} & -r\mathrm{c}(\gamma) \\ \dfrac{-xz\mathrm{s}(\gamma) - yc(\gamma)}{r} & \dfrac{-yz\mathrm{s}(\gamma) + x\mathrm{c}(\gamma)}{r} & r\mathrm{s}(\gamma) \\ x & y & z \end{bmatrix}. \quad (13)$$

Therefore we have the attitude matrix in terms of the spin parameters. We call this attitude representation for the spin spacecraft the xyz-γ representation.

To derive the kinematics equations for the spin parameters we use the well-known differential equation for the attitude matrix in Eq.(13) (or direction cosine matrix –DCM) [12]

$$\dot{A}_i^b = -[\boldsymbol{\omega}\times]A_i^b. \quad (14)$$

After a series of calculations - refer to Ref.6) for details - we obtain the kinematics equation for the spin spacecraft in terms of the spin parameters,

$$\dot{x} = \left(\frac{xz\mathrm{s}(\gamma) + yc(\gamma)}{r}\right)\omega_x + \left(\frac{xz\,\mathrm{c}(\gamma) - y\mathrm{s}(\gamma)}{r}\right)\omega_y, \quad (15a)$$

$$\dot{y} = \left(\frac{yz\mathrm{s}(\gamma) - x\mathrm{c}(\gamma)}{r}\right)\omega_x + \left(\frac{yz\,\mathrm{c}(\gamma) + x\mathrm{s}(\gamma)}{r}\right)\omega_y, \quad (15b)$$

$$\dot{z} = -r\mathrm{s}(\gamma)\omega_x - r\mathrm{c}(\gamma)\omega_y. \quad (15c)$$

$$\dot{\gamma} = \omega_z - \frac{z}{r}\left(\mathrm{s}(\gamma)\omega_y + \mathrm{c}(\gamma)\omega_x\right). \quad (15d)$$

3.2. Unscented Kalman Filtering with Spin Parameters

The UKF is a nonlinear filtering method. Its accuracy for three-axis attitude estimation has been proven extensively in literature[13,14]. It does not need any linearization and is valid to higher order estimation of the Taylor series expansion than the Extended Kalman Filter (EKF). Compared with the EKF's first-order accuracy, the estimation accuracy of the UKF is improved to the third-order for Gaussian data and at least second-order for non-Gaussian data[15].

Specifically for spin spacecraft attitude estimation the UKF is attractive as it is capable of dealing with longer measurement interruptions than the EKF[16]. This is advantageous for slow-spinning spacecraft attitude estimation in the absence of magnetometer measurements. For instance, Sun and Earth sensors produce only a single measurement per spin period.

The state variables for the UKF are the spin parameters and the vector of body angular rates with respect to the inertial frame,

$$\boldsymbol{X} = \begin{bmatrix} x & y & z & \gamma & \boldsymbol{\omega} \end{bmatrix}^T. \quad (16)$$

The UKF is derived for discrete-time nonlinear equations, so the system model is given by;

$$\boldsymbol{X}_{k+1} = \boldsymbol{f}(\boldsymbol{X}_k, k) + \boldsymbol{w}_k, \quad (17a)$$

$$\boldsymbol{Y}_k = \boldsymbol{h}(\boldsymbol{X}_k, k) + \boldsymbol{v}_k. \quad (17b)$$

Here, $\boldsymbol{X}_k$ is the state vector and $\boldsymbol{Y}_k$ is the measurement vector. Moreover $\boldsymbol{w}_k$ and $\boldsymbol{v}_k$ are the process and measurement error noises, which are assumed to be Gaussian white noise processes with covariances $Q(k)$ and $R(k)$, respectively. The filtering equations for the UKF can be found in Ref. 15).

We estimate the inertial attitude of the spacecraft. The process is propagated using the discrete-time versions of Eq.(15) and the Euler's dynamics equation which is required in the absence of gyros,

$$\dot{\boldsymbol{\omega}} = J^{-1}\left[\boldsymbol{N} - \boldsymbol{\omega}\times\left(J\boldsymbol{\omega}\right)\right]. \quad (18)$$

Here, J is the inertia matrix of the spacecraft and $\boldsymbol{N}$ is the torque vector, which is the sum of the external disturbance torques such as those induced by solar radiation pressure and control torques, if there are any. We shall note that, in this study, the filter is not given the external disturbance torque values and there is no control torque input for the specific period when the sensor measurements are collected.

The measurement model for the UKF is as given in Eq.1.

3.3. Norm Constraint

The spin axis direction, which is represented by a unit vector in the inertial frame, is a part of the spin parameters representation. Clearly this estimated unit vector, $\hat{\mathbf{1}}_i^{spin}$, must satisfy the spherical norm constraint,

$$\left\|\hat{\mathbf{1}}_i^{spin}\right\| = 1. \quad (19)$$

In Ref.6) we show that if the norm constraint is initially satisfied, the solution for the kinematics (Eqs.15a-15c) will theoretically satisfy the constraint all the time. Notwithstanding, in practice, due to measurement biases, spin-axis tilt and unmodeled external disturbance torques the norm constraint for the estimated spin axis direction terms might be violated.

In this study, we suggest two methods for satisfying the norm constraint for the spin-axis direction terms: Brute force normalization and Lagrange multiplier method.

3.3.1. Brute Force Normalization

Brute force normalization is simply unitizing the estimated spin-axis direction after each recursive step,

$$\bar{\hat{\mathbf{1}}}_i^{spin} = \hat{\mathbf{1}}_i^{spin} / \left\|\hat{\mathbf{1}}_i^{spin}\right\|. \quad (20)$$

Although this is a first-order approximation, usually its accuracy is sufficient as the SpinUKF estimates are expected to be close to the correct estimate of the spin-axis orientation vector.

3.3.2. Lagrange Multiplier Method

Another method for satisfying the norm constraint is to introduce a Langrage multiplier in the filtering equations[17]. In Ref. 17) the method is introduced for a linear Kalman filter. Here, we derive the equations for the Lagrange multiplier method when it is implemented as a part of the UKF. The optimal Lagrange multiplier is defined as

$$\lambda_k = \frac{-1}{\bar{\varepsilon}_k} + \frac{\left\|\boldsymbol{\varepsilon}_k^T P_{yy,k}^{-1} P_{xy,k}^T + \left(\hat{\boldsymbol{X}}_k^-\right)^T\right\|}{\bar{\varepsilon}_k\sqrt{l}}, \quad (21)$$

and the UKF gain is modified as

$$K_k^* = \left(P_{xy,k}^T - \lambda_k \hat{x}_{k/k-1} \boldsymbol{\varepsilon}_k^T\right)\left(P_{yy,k} + \lambda_k \boldsymbol{\varepsilon}_k \boldsymbol{\varepsilon}_k^T\right)^{-1}, \quad (22)$$

to satisfy the norm constraint, whose defined value is $l = 1$. Here λ_k is the Lagrange multiplier, $\boldsymbol{\varepsilon}_k$ is the innovation vector, $\bar{\varepsilon}_k$ is the normalized innovation as $\bar{\varepsilon}_k = \boldsymbol{\varepsilon}_k^T P_{yy,k}^{-1} \boldsymbol{\varepsilon}_k$, $P_{yy,k}$ is the innovation covariance, $P_{xy,k}$ is the cross correlation matrix and $\hat{\boldsymbol{X}}_k^-$ is the predicted state vector at discrete time step of *k*. The asterisk shows that the gain is modified and different from the unconstrained UKF gain K_k.

The state estimation covariance is given by the Joseph formula:

$$P_k^{+*} =$$
$$P_k^- - K_k^* P_{xy,k}^T - P_{xy,k}\left(K_k^*\right)^T + K_k^*\left[P_{xy,k}^T\left(P_k^-\right)^{-1} P_{xy,k} + R_k\right]\left(K_k^*\right)^T . \quad (23)$$

Eqs.(21-23) are given for the norm constrained UKF in general case. In our problem only a part of the state, which is for spin-axis direction unit vector terms, is subject to the constraint as in Eq.(19). Thus the state vector, $\hat{\boldsymbol{X}}_k^-$, cross correlation matrix, $P_{xy,k}$, UKF gain, K_k^*, and state estimation covariance P_k^{+*} must be partitioned. In this case the Lagrange multiplier is calculated using only $\hat{\mathbf{1}}_i^{spin}$ instead of full state vector $\hat{\boldsymbol{X}}$ and the corresponding first three rows of P_{xy}. The Kalman gain and state estimation covariance are calculated independently for the spin-axis direction terms and the rest of the states. A similar procedure is described in detail for linear Kalman filter in Ref.17) (Section III).

4. Evaluation with Real Data

Table 1 gives the estimated spin-axis attitude by different algorithms for CONTOUR in terms of the right ascension (RA) and declination (DE) angles. The attitude is estimated over the Earth's mid-latitude region. T-S is the batch spin-axis estimation method proposed by Tanygin and Shuster in Ref.18). Variations of the SpinUKF include the unconstrained filter, "SpinUKF (UC)", the filter with brute force normalization for the spin-axis direction terms, "SpinUKF (BF)", and the filter that incorporates Lagrange multiplier method for satisfying the norm constraint, "SpinUKF (LM)". The results for the SpinUKF variants and also the TRIAD are mean values of the spin-axis attitude estimates over the period.

The attitude estimation results in Table 1 show that the SpinUKF produces consistent results with those of the T-S and TRIAD algorithms. The SpinUKF estimation results are 0.023° and 0.098° in arc-length distance away from the values estimated with the TRIAD and T-S algorithms, respectively.

Measures for satisfying the norm constraint for spin-axis direction unit vector affect the estimation results, but only in a small quantity when the attitude is estimated over Earth's mid-latitude region. The estimation results for an algorithm with norm constraint differ only 3 arcsec from those of the unconstrained algorithm. We also see this in Fig.4, which presents spin-axis attitude estimation results for SpinUKF (UC) and SpinUKF (LM) over the mid-latitude region.

Effects of using different methods for norm constraint (e.g. brute force normalization or Lagrange multiplier method) on the attitude estimation accuracy is negligible in this case. Both the SpinUKF (BF) and SpinUKF (LM) produce identical spin-axis attitude estimation results.

Table 1. Spin-axis attitude estimation results for CONTOUR.

Algorithm	**RA & DEC (deg)**	
TRIAD	258.61779	29.25748
T-S	258.66524	29.34231
SpinUKF (UC)	258.59383	29.26604
SpinUKF (BF)	258.59470	29.26567
SpinUKF (LM)	258.59470	29.26567

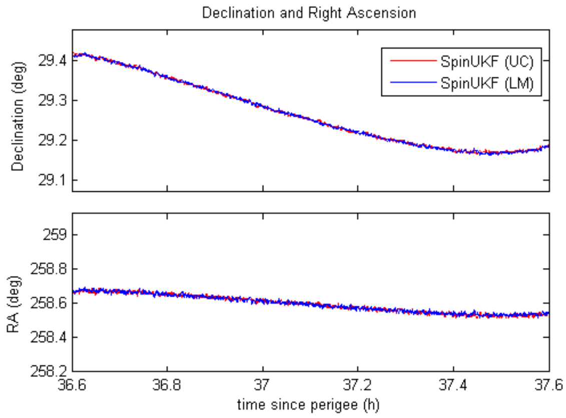

Figure 4. Spin-axis attitude estimation by SpinUKF (UC) and SpinUKF (LM) over the Earth's mid-latitude region.

Although having the norm constraint does not improve the spin-axis attitude estimation results over the Earth's mid-latitude region greatly, it is necessary when the measurements are biased. For the Earth-sensor we know that the measurements deteriorate away from the Earth's mid-latitude region because of sensor performance degradations for short Earth-scan intervals. Furthermore, the measurement sensitivity decreases and the IR biases increase. In fact, this data should not be used for attitude-estimation purposes since the sensor has been designed and calibrated to ensure specified performances over the Earth's mid-latitude region only.

Fig. 5 presents the residuals that each EAA produces. It is simply the difference between the derived EAA measurements and the β_p value which is predicted by using the attitude estimate obtained with the SpinUKF (LM). It can be seen that the residuals for "single beta" β_{sin} solution increase outside the mid-latitude region.

Fig. 6 gives the norm error, which is $1 - \left\|\hat{\mathbf{1}}_i^{spin}\right\|$, for spin-axis attitude estimations of SpinUKF (UC) and the variations of the calculated Lagrange multiplier for SpinUKF (LM). It confirms that the norm condition for the filter is satisfied as long as the sensor data is good, even if we do not apply a specific normalization method. It is necessary to apply unit-vector normalization to the filter spin-axis attitude estimates in the presence of biases in the measurements

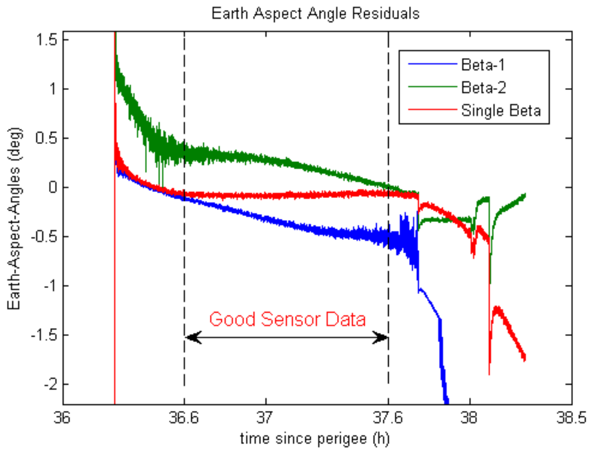

Figure 5. Residuals of the Earth aspect angle

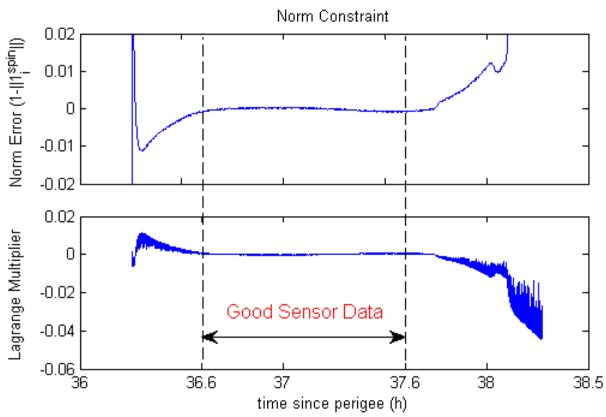

Figure 6. Norm error for SpinUKF (UC) and Lagrange multiplier for SpinUKF (LM).

5. Conclusions

The performance of the SpinUKF, an Unscented Kalman Filter for spinning spacecraft attitude estimation, is evaluated with the help of in-flight data collected by the spinning spacecraft CONTOUR. Different methods for satisfying the spherical norm constraint for the spin axis coordinates are investigated. The results show that the SpinUKF produces consistent attitude estimates with the existing methods such as TRIAD in the specific interval where the operational attitude measurement data is gathered. It is necessary to aid the filter with a norm constraint provision method for spin-axis attitude estimates in the presence of biases in the measurements

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