18th Australian International Aerospace Congress Optimized transfers between Earth-Moon invariant manifolds

Laurent Beauregard [1], Emmanuel Blazquez [1], Stéphanie Lizy-Destrez [1]

[1] *Institut Supérieur d'Aéronautique et de l'Espace (ISAE-SUPAERO), 10 Avenue Edouard Belin, 31400 Toulouse*

Abstract

Launching its first module in 2022, the upcoming Lunar Orbital Platform-Gateway (LOP-G) or simply the Gateway will be a space station assembled in cis-lunar space, most likely on a Near Rectilinear Halo Orbit (NRHO). The best transfer strategy to the LOP-G remains an open question, one candidate involves going to an intermediate Halo as a parking orbit. The focus of this work is on the transfer methodology between Halo orbits. Two burns direct transfers can provide simple transfer trajectories, however when longer transfer time is permitted, structures of the natural dynamics of the Earth-Moon system, such as manifolds, can be exploited for transfers with lower cost in velocity increment Δv. In this work, low thrust trajectories, Lambert arcs and new manifold intersection methods will be compared for the lowest cost Δv.

Keywords: Earth-Moon system, Manifolds, Transfer, NRHO, Optimization

1. Introduction

The International Space Exploration Coordination Group (ISEGC), composed of most of the international space agencies, has agreed on deploying a space station in cis-lunar space, called the Lunar Orbital Platform-Gateway (LOP-G), as a gateway to enable deep space exploration [1]. The most likely position of the LOP-G will be a L_2 southern Near Rectilinear Halo Orbit (NRHO) of apoapsis 1500 km above the surface of the Moon. The NRHO is a particular case of the family of Halo orbits which exist in the Circular restricted three body problem (CR3BP). One of the common metric when analysing orbital transfers is the Δv of the maneuver, the change in velocity necessary to perform the transfer. Although some work has been done for the best transfers in the dynamics of cis-lunar space [2] there still is an open question as to the optimal transfer strategy. One method to reach the orbit of the LOP-G is to go unto an intermediate Halo orbit, acting as a parking orbit. In this article, three transfer methods will be compared for the best Δv to travel across Halo orbits. The first method will consider a long series of maneuvers of low Δv to "jump" along the Halo family. The second method will be a classic case of optimized Lambert arcs. The last method will consider invariant structures of the Halo orbits, called unstable and stable manifolds and their intersections. While such structures have been used before [3], this article will present a new methodology to obtain a one-dimensional subset of intersections.

2. Circular Restricted Three Body Problem

The only gravitational bodies considered in this article are the Earth and the Moon; furthermore, the Moon is approximated to move in a circular orbit around the Earth-Moon barycenter. Spacecrafts or stations moving in this environment are considered massless. This framework is called the CR3PB for the Earth Moon system and has been extensively studied. The data for this system can be found in table 1.

Table 1. Data

	Symbol	Value	Units
Earth's gravitational parameter	μ_E	398600.4415	km^3/s^2
Moon's gravitational parameter	μ_M	4902.8005821478	km^3/s^2
Earth-Moon distance	a	384400	km

The frame chosen has the following properties

- The Earth-Moon barycenter is located at the origin.
- The frame rotates in such a way that the Earth's and Moon's position are stationary
- The Moon is located on the +x axis (x > 0, y = 0, z = 0)
- The Earth is located on the –x axis (x < 0, y = 0, z = 0)
- The direction of rotation of the Moon is counterclockwise as seen from above (z-axis)
- The y direction is chosen to create a right handed coordinated system

The unit vectors $\hat{\imath}, \hat{\jmath}, \hat{k}$ correspond to the +x, +y and +z direction respectively. In this framework the angular rotation of the Moon is

$$\vec{\omega} = \sqrt{\frac{\mu_M + \mu_E}{a^3}}\ \hat{k} \tag{1}$$

Due to simple kinematics, the location of the of the Earth $\vec{r}_E$ and Moon $\vec{r}_M$ can be shown to be

$$\vec{r}_E = -\frac{\mu_M}{\mu_E + \mu_M}\hat{\imath} \qquad \vec{r}_M = \frac{\mu_E}{\mu_E + \mu_M}\hat{\imath} \tag{2}$$

The equations of motion of a third massless body, whose position and velocity are $(\vec{r}, \vec{v})$ respectively, are given by

$$\frac{d}{dt}\vec{r} = \vec{v} \tag{3}$$

$$\frac{d}{dt}\vec{v} = -\mu_E \frac{(\vec{r} - \vec{r}_E)}{|\vec{r} - \vec{r}_E|^3} - \mu_M \frac{(\vec{r} - \vec{r}_M)}{|\vec{r} - \vec{r}_M|^3} - \vec{\omega} \times (\vec{\omega} \times \vec{r}) - 2\,\vec{\omega} \times \vec{v} \tag{4}$$

3. Theory of Halo orbit

Several types of closed orbit of the CR3PB exists [4]. The ones that are of interest in this study are called Halo orbits, they exists as bifurcations of the planar Lyapunov orbit associated with the Lagrangian points L_1, L_2 or L_3. Different parameterization of these orbits are possible, the most common parameterization comes from applying the Poincaré-Lindstedt method to the linearized equation of motion and by varying the amplitude of the out-plane motion A_z until the in plane and out of plane frequency of the motion are equal [4]. While this method is perfectly valid, it suffers from the drawback of being a one way function from A_z to the orbital states $\big(r(t), v(t)\big)$. In other words, given the orbital states of the Halo orbit $\big(r(t), v(t)\big)$, it is very difficult to obtain the A_z which produced this orbit. In this article the parameterization used is the out of plane component of the velocity V_z at the x-y plane $(z = 0)$ crossing. Every Halo orbit is uniquely characterized by this variable. This has the advantage of being obtainable from the state variables $\big(r(t), v(t)\big)$ alone in a straightforward way. Another common parameterization is the smallest distance from the orbit to the Moon's surface, called the periapsis P_e. For reference, figure 1 shows the relation between A_z and V_z. Figure 2 shows the relationship between P_e and V_z.

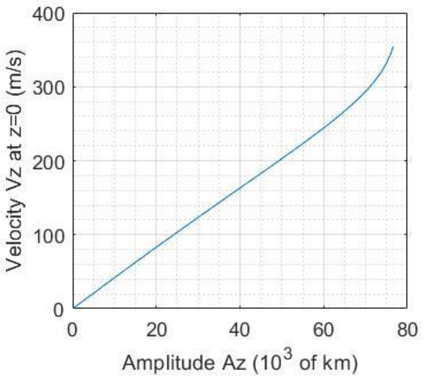

Figure 1. Relation between the amplitude A_z and V_z

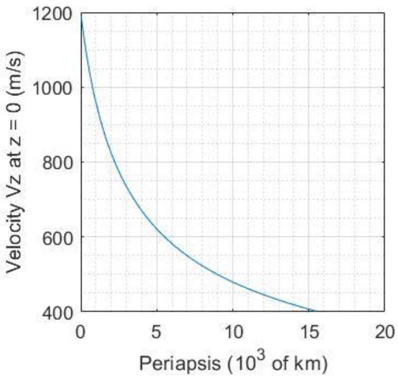

Figure 2. Relation between the periapsis P_e and V_z

It was found that up to an $\mathbf{A_z}$ of roughly 60,000 km, an approximately linear relation is satisfied given by equation 4.

$$V_z = \frac{4\ m/s}{1000\ km}\ \mathrm{A_z} \tag{4}$$

An approximate relationship between P_e and V_z was found to given by equation 5.

$$V_z = \frac{1683}{\sqrt{P_e + 2.25}} \tag{5}$$

Where P_e is in units of 1000 km and V_z is in m/s.

4. Energy of orbits

A well-known constant of motion for the CR3PB is the energy (a.k.a Jacobi constant). Its expression is given by

$$J = \frac{1}{2}v^2 - \frac{\mu_E}{|\vec{r} - \vec{r}_E|} - \frac{\mu_M}{|\vec{r} - \vec{r}_M|} - \frac{1}{2}\left(\vec{\omega} \times \vec{r}\right)^2 \tag{6}$$

For the L_2 family, the energy increases from a minimum near their bifurcation with the planar Lyapunov orbit and reaches a maximum around $V_z = 450\ m/s$, it then falls back down. This behavior is shown in figure 3.

This property of several orbits having the same energy will be exploited later when considering transfers from different Halos.

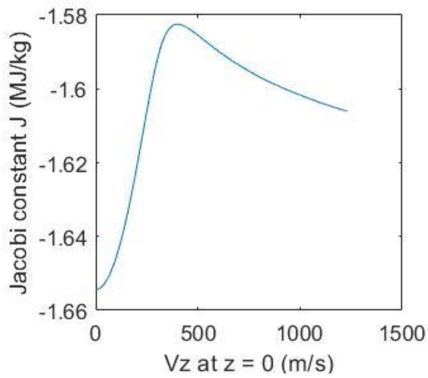

Figure 3. Jacobi constants for the L_2 Southern Halo orbit

5. Low thrust approach

Concerning orbital transfers between orbits, two limiting cases are often considered, the case of infinite thrust with instantaneous maneuvers or the case of infinitesimal thrust with infinitely long maneuvers. Each limiting case has an associated Δv to it. For the case of two body orbital dynamics with central body gravitational parameter μ, the optimal transfer between two circular co-planar orbit of radius a_1 and a_2 is given by the Hohmann transfer. The Δv cost of the Hohmann transfer is denoted by Δv_{Hoh} whereas the low thrust trajectory has cost denoted Δv_{Low}.

Their difference can expanded to the lowest nonzero order in $a_2 - a_1$ and can be shown to be

$$\Delta v_{Low} - \Delta v_{Hoh} \sim \frac{1}{32}\sqrt{\frac{\mu}{a_1}}\frac{(a_2 - a_1)^3}{a_1^3} \tag{7}$$

The conclusion is that, in terms of Δv, a low thrust transfer can be well approximated by a Hohmann transfer, and vice versa, for orbits that are close. One can therefore approximate the Δv cost of a low thrust trajectory by a series of impulsive transfers going along a continuous family of closed trajectory from the initial orbit to the final orbit. This is precisely the methodology used in the article to estimate the low thrust Δv.

Given a sampling of the L_2 Halo family, which is finely spaced enough to approximate low thrust cost, the algorithm described above can used to establish a baseline for the Δv between any two Halo orbits. It does so by "jumping" in between every Halo orbits of the family connecting the starting and end orbit. The results obtained for this procedure are summarized in figure 4 and 5.

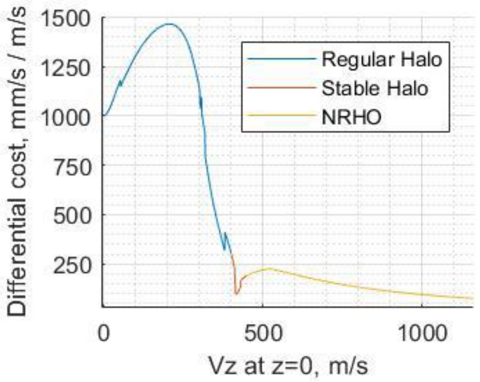

Figure 4. Differential cost to travel between neighboring orbit

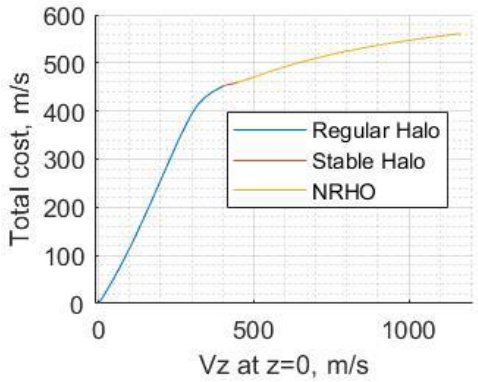

Figure 5. Cumulative cost to travel from $V_z = 0$ to any V_z

Figure 4 shows the differential cost $\frac{d\Delta V}{dV_z}$ of traveling between two close orbits. Figure 5 shows the cumulative cost to travel between orbits with $V_z = 0$ to $V_z = V_f$. The whole Halo family can be covered with roughly 560 m/s. In particular, the NRHO family only requires at most 100 m/s to cover.

6. Lambert arcs

Given a dynamical system, such as the CR3PB, two points in space, r_1 & r_2, and two times, t_1 & t_2, the Lambert problem consists in finding a curve $r(t)$ that satisfies the dynamics of the system, such that $r(t_1) = r_1$ & $r(t_2) = r_2$, this is also equivalent to finding the velocity $v_1 = \dot{r}(t_1)$ that produces the trajectory above. In the case of two body orbital mechanics this problem has been fully solved. For the CR3PB, there is no known analytical solution, and one must resort to a numerical approach. One method is based on the fact that considering small enough Time Of Flight (ToF), one possible trajectory can be approximated by a line connecting the initial point r_1 and final point r_2, with initial velocity approximated by $\vec{v}_1 = \frac{r_2 - r_1}{ToF}$. A Newton's method can then be used to correct $\vec{v}_1$ until $\vec{r}(ToF) = \vec{r}_2$ is obtained. The next step involves incrementally increasing the ToF, using the previous $\vec{v}_1$ as a guess and correcting the trajectory at each step, until the desired ToF is obtained (or one with least Δv is obtained).

7. Manifold method

7.1 Manifold generation

The stability of a given Halo orbit can be analyzed by the eigenvalues of the differential of the Poincaré map [5]. The magnitude and phase of these eigenvalues for the L_2 family are shown in figure 6.

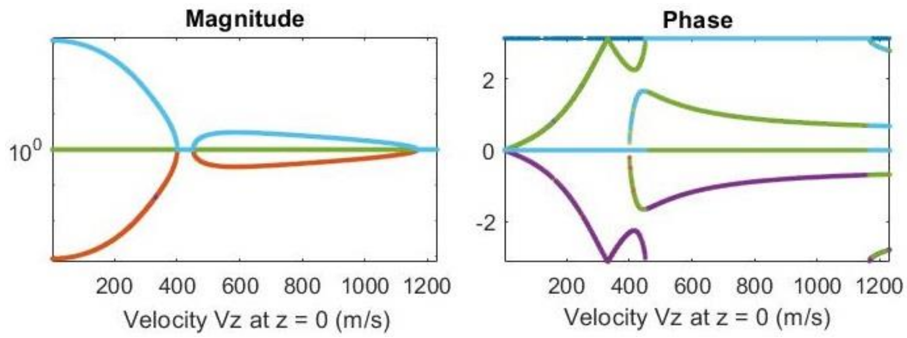

Figure 6. Magnitude and phase of the eigenvalues of the differential of the Poincaré map

Three different regions of stability are observed, regular Halos from $V_z = 0 \rightarrow \sim 400\ m/s$ are unstable, neutral Halos from $V_z = \sim 400\ m/s \rightarrow \sim 450\ m/s$ are stable, and lastly, NRHOs from $V_z = \sim 450\ m/s \rightarrow \sim 1150\ m/s$ are again unstable.

Given an unstable direction $\vec{e}_\lambda$ with eigenvalue $|\lambda| > 1$, perturbations ϵ along this direction grow exponentially with each period, with ϵ_0 the initial perturbation. After N revolutions

$$\epsilon = \lambda^N \epsilon_0 \quad (8)$$

The set of trajectories generated by all unstable directions is called the unstable manifold of the orbit. Being unstable, these trajectories can be achieved with a small Δv. Analogously, the stable manifold can be generated with all stable directions $|\lambda| < 1$, these converge to the Halo orbit with each revolution. When time is propagated backward, the unstable and stable trajectories invert behavior and the trajectories on the unstable manifold converge to the Halo and the stable manifold diverges from the Halo. Points on these manifolds can be described by two parameters (α, t), a perturbation size α and a time of flight t.

7.2 Manifold Intersection

One method for a spacecraft to travel from Halo A to Halo B using their manifolds is to insert into the unstable manifold of A, wait for some (to be solved for) amount of time, then perform a maneuver to insert into the stable manifold of B and thus converging to the final orbit. The main challenge of this method is the determination of the intersection point between the two manifolds; this problem is equivalent to solving for the combination (α_u, t_u) and (α_s, t_s) which has the same positions.

$$\vec{r}_u = \vec{r}_s \quad (9)$$

Because one is searching the intersection of two surfaces in space, it is expected that the intersection is generally a one dimensional object. The method employed in this article involves computing several arcs of the manifolds, typically on the order of 100, at regularly spaced intervals in time. From this grid in (α, t), a triangulation is performed as shown in figure 7.

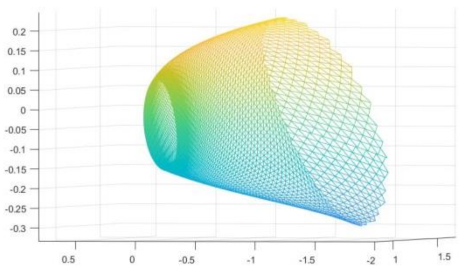

Figure 7. Triangulation of a manifold

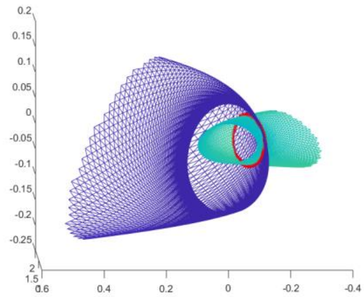

Figure 8. Manifold intersection

Intersections between two triangulated surfaces can be computed efficiently with a Möller triangle-triangle intersection algorithm [6]. Such intersections are shown in figure 8. From these intersections, the velocities can be obtained by a linear regression applied on the intersecting triangles followed by a differential correction to obtain more accurate intersections. From this set of intersections, the one with least Δv is chosen.

8. Results

Fourteen different cases were considered in this article, covering a wide range of qualitatively different scenarios, the three transfer methods were applied. The ToF applies to the Lambert transfer.

Table 2. Comparison of the three transfer methods

	V_z (m/s)		A_z (A) or P_e (P) (10^3 km)		Low thrust (m/s)	Manifold (m/s)	Lambert (m/s)	ToF (hours)	Comments
Case	Dep	Arr	Dep	Arr					
1	42	83	10.04A	19.98A	48.92	49.76	48.55	125.03	Close Halo to Halo
2	233	285	57.24A	68.50A	72.13	69.32	72.56	128.82	Close Halo to Halo
3	42	285	10.04A	68.50A	331.02	312.28	340.72	119.6	Far Halo to Halo
4	285	42	68.50A	10.04A	331.02	312.44	356.64	138.8	Opposite of #3
5	549	635	7.03P	4.71P	17.09	18.42	17.05	121.84	Close NRHO-NRHO
6	793	885	2.32P	1.49P	11.44	12.33	11.48	91.74	Close NRHO-NRHO
7	549	885	7.03P	1.49P	53.41	61.87	54	105.18	Far NRHO-NRHO
8	885	549	1.49P	7.03P	53.41	59.1	52.84	111.55	Opposite of #7
9	424	885	13.64P	1.49P	79.15	N/A	83.39	134.71	Neutral Halo to NRHO
10	885	424	1.49P	13.64P	79.15	N/A	81.51	130.63	Opposite of #9
11	316	587	72.21A	5.88P	80.3	33.75	44.98	340.16	Halo to NRHO same energy
12	587	316	5.88P	72.21A	80.3	33.78	54.89	293.56	Opposite of #11
13	275	883	66.5A	1.51P	173.05	81.71	110.11	281.95	Halo to NRHO same energy
14	883	275	1.51P	66.5A	173.05	80.3	146.2	209.11	Opposite of #13

It is clear that when the departing and arriving orbits are close in space, the three methods give essentially the same Δv, differing by less than 10%. However for orbits that differ significantly, the manifold method can provide improvement over Lambert arcs. The instances where the manifold method dominates are for transfers between orbits with the same energy #11-12-13-14, reducing the cost between 25% and 45% as compared to Lambert arcs.

9. Conclusion

In this article, three methods of transfer between L_2 Halo orbits have been considered; a low thrust approximation, a two burn optimized Lambert arc and a manifold intersection method. The low thrust approximation provides a benchmark of the Δv required to compare other methods. For orbits that are close, there is little difference in the outcome of the three methods. However, it has been shown that for far orbits, the manifold method can reduce significantly the cost Δv, by up to 45%. Further studies should increase the degree of freedoms available to the algorithm by also considering the central Eigenspace.

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