Information Processing at Unstable Equilibrium Points in the Restricted Three Body Problem

Dylan R. Boone, dylan.boone@jpl.nasa.gov *

This work investigates the properties of phase space in the vicinity of an equilibrium point in the Restricted Three Body Problem and examines the time evolution of an equilibrium point orbiter's position-velocity covariance matrix. A mathematical development of information accumulation in the orbit determination process is given using the eigenstructure decomposition of the State Transition Matrix. This decomposition involves matrices of right eigenvectors, eigenvalues, and left eigenvectors. The computed stable and unstable manifolds of an unstable fixed point, of which the Lagrange equilibrium points in the Restricted Three Body Problem are an example, are shown to affect the accumulation of information in the orbit determination process. Information is preferentially accumulated along the left unstable manifold direction for measurements mapped to epoch and along the left stable manifold direction for measurements mapped to the current state. This asymmetry in information mapping is due to the orthogonality of left and right eigenvectors. For epoch state filtering, the left unstable direction is best known and the right stable direction is least known. For a current state filter, the left stable direction is best known and the right unstable direction is least known. Analytical examples show this effect by decomposing the State Transition Matrix into its matrix exponential form.

A numerical simulation of trajectories in the vicinity of an equilibrium point shows that the covariance matrix collapses along preferred directions in phase space based on the properties of the State Transition Matrix. Trajectories in the Restricted Three Body Problem are drawn from a spherical covariance matrix about an equilibrium point in a Monte Carlo simulation and projected onto the plane of intersection of the stable and unstable manifolds of the equilibrium point. The Jupiter-Europa L2 equilibrium point is taken as a numerical example. The concept of manifold coordinates reveals what components of the dispersed orbit initial conditions are aligned with the stable, unstable, and center manifolds associated with the equilibrium point, respectively. Simulations using a current state and an epoch state square-root information filter are presented and compared.

The possibility of having deficient measurement directions in phase space is also explored. For measurements composed of left or right eigenvectors, epoch and current state mapping is examined in more detail to determine if there exists a direction in phase space where no information is available. A full rank information matrix is constructed from a series of the same measurement aligned with a right eigenvector. However, a series of measurements missing a left eigenvector are shown to have a deficient measurement direction along the right eigenvector paired with the omitted left eigenvector. These results show a fundamental structure to information and measurement partial mapping in terms of the stability properties of an equilibrium point. The covariance matrix of an orbiter in the vicinity of an equilibrium point collapses in preferential directions in phase space based on whether a current state or epoch state filter is used. This effect can be utilized for planning tracking schedules of a spacecraft and conducting covariance analysis of desired parameters to be estimated in the orbit determination process.

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