LOW-THRUST GEO TRANSFER OFF-LINE NAVIGATION AND CONTROL*

M.N. Krasilshikov, A.V. Fedorov, K.I. Sypalo, and D.A.Kozorez
Moscow Aviation Institute (National Research University), Volokolamskoe shosse 4, Moscow, GSP-3, 125993, phone +7(499)158-45-51, mnkr@mail.ru

Abstract: *Autonomous maintenance allows space system to be economically viable. In respect to a satellite in GEO off-line transfer, its station acquisition and station keeping control design should be implemented considering international regulations. Implementation of the "off-line" idea calls for new control and navigation (C&N) technology for entire system lifetime cycle. Plasma propulsion allows considerable fuel mass reduction along with payload mass in GEO increase. A multichannel GNSS receiver combined with optoelectronic onboard sensors seems to be efficient for highly accurate navigation. New C&N technology integration with GEO satellite onboard control system is under consideration. Inter-orbital journey is most complicated phase of GEO satellite lifecycle. Utilization of low-thrust electric propulsion for GEO transfer leads to long process duration. Accuracy specifications, as well as orbit elements, attitude and actual thrust vector estimation reliability becomes strongly significant. Simplified architecture of integrated navigation system as well as set of various data fusion algorithms is given. The problem under consideration consists in development of a new algorithm for navigation problem solution considering GEO satellite control demands. A GEO satellite lifecycle includes: 1) insertion in orbital position vicinity; 2) orbital position acquisition; 3) station keeping; 4) space disposal. Closed loop orbit control strategy is based on sufficient conditions of optimality in deterministic, stochastic and guaranteeing (min-max) statements. The separation theorem proves independent design for linear white-nosed motion model, and linear measurements model only. The authors approve possibility of separate control and navigation solutions for autonomous case.*

Keywords: *Geostationary satellite, Autonomous navigation, Autonomous control, Plasma propulsion, GEO transfer, Station acquisition, Station keeping.*

1. Introduction

A payload insertion into geostationary orbit (GEO) by Soyuz/Proton launcher equipped with upper stage rocket takes up to 24 hours. Modern "Proton/Breeze" or "Proton/Frigate" bundle can insert up to 3500 kg in GEO. Prolongation of the satellite life cycle up to 15 years shows its mass will be over the launcher limit. Thus, we have either wait for a new powerful launcher, or design a new insertion method. The second option looks more attractive. Insertion method under consideration assumes spacecraft delivery as much as possible close to GEO with respect to launcher lift power. Then the satellite approaches GEO using onboard plasma propulsion unit (PPU). This method considerably increases transfer duration (up to 300 days). However, the economic efficiency of the transfer as compared with the standard procedure is much higher because of the payload mass might be increased considerably. The use of PPU requires current position, velocity, attitude, and actual thrust vector permanent estimation during the entire mission. This paper is devoted to justification of hardware and requirements for it, to the choice

* This study supported by the Russian Federation Ministry for Education and Science, Proj. RFMEFI57414X0100

of the architecture of the onboard integrated navigation system, and to the development of algorithms for autonomous (i.e. using only onboard hardware) solution of navigation problem. Applied control theory treats the problem as control synthesis with respect to incomplete erroneous data. This calls for simultaneous navigation and control algorithms design. Unfortunately, practically reasonable solution cannot be obtained in such a statement. The so-called separation theorem proves independent solution of navigation and control problem in case of white-noised motion model, and liner measurements only. The authors have approved possibility of separate control and navigation solution for various aerospace applications. Here we extend the experience for autonomous C&N case.

2. Navigation Problem Solution

The preliminary analysis [1] shows that strict technical conditions for GEO acquisition require precise satellite position, velocity, attitude, and thrust determination by onboard navigation system. Highly accurate navigation requires use of onboard multichannel GNSS receiver data. At low-thrust transfer to GEO, orbit altitude will be in the range from 21000 to 86000 km [1]. It is known [2-4] that GLONASS satellites visibility in this altitude range allows solve the navigation problem in principle. It is also clear that the autonomous onboard system must have integrated architecture because of processing discrepancy of true and predicted navigation parameters to estimate thrust vector components [5]. Figure 1 shows a simplified diagram of the integrated navigation system.

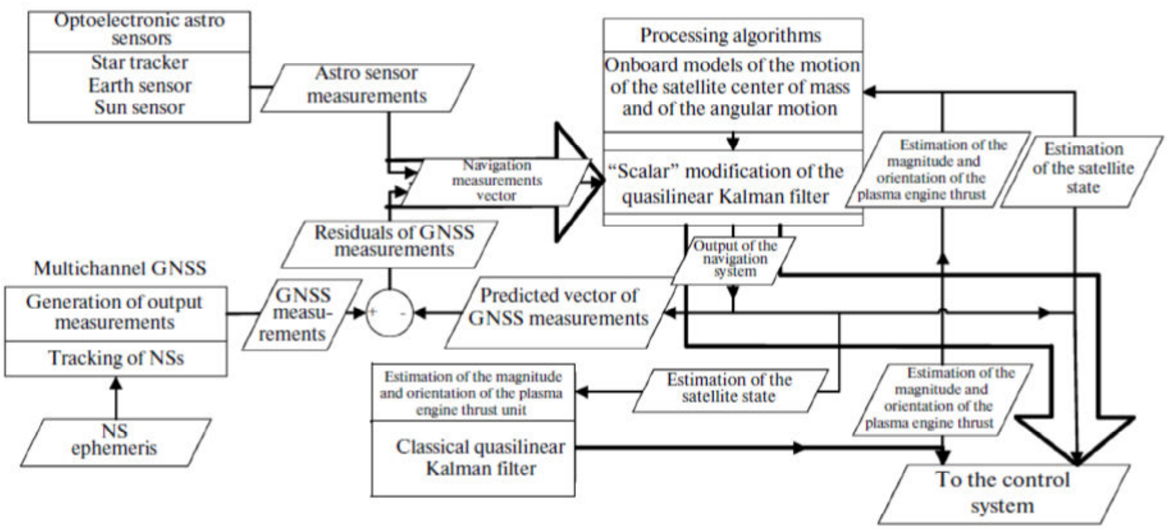

Figure 1. Integrated navigation system simplified diagram

Let us introduce satellite extended state vector with 87 elements:

$$\mathbf{X}_1 = \big(X_{IF} \quad Y_{IF} \quad Z_{IF} \mid V_{X\,IF} \quad V_{Y\,IF} \quad V_{Z\,IF} \mid \vartheta \quad \psi \quad \gamma$$
$$\mid \Delta\cos\alpha_{Sun1} \quad \Delta\cos\alpha_{Sun2} \quad \Delta\cos\alpha_{St1} \quad \Delta\cos\alpha_{St2} \quad \Delta\cos\alpha_{E1} \quad \Delta\cos\alpha_{E2}$$
$$\mid \Delta\vartheta_{ES}^{sys} \quad \Delta\gamma_{ES}^{sys} \quad \Delta\vartheta_{ES}^{ort} \quad \Delta\gamma_{ES}^{ort} \quad \Delta\psi_{SunS}^{sys} \quad \Delta\gamma_{SunS}^{sys} \quad \Delta\psi_{SunS}^{ort} \quad \Delta\gamma_{SunS}^{ort} \qquad , \tag{1}$$
$$\mid \Delta\vartheta_{StS}^{sys} \quad \Delta\psi_{StS}^{sys} \quad \Delta\vartheta_{StS}^{ort} \quad \Delta\psi_{StS}^{ort} \quad \Delta t_{KA} \quad \Delta t_0 \quad \ldots \quad \Delta t_{59}$$
$$\mid \tilde{P} \quad \alpha \quad \beta \quad \big)^{\mathrm{T}}$$

where X_{IF}, Y_{IF}, Z_{IF} , $V_{X\,IF}$, $V_{Y\,IF}$, $V_{Z\,IF}$ are position and velocity components in J2000 inertial frame; ϑ, ψ, γ are the pitch, yaw, and roll in orbital frame; $\Delta\cos\alpha_{Sunk}$, $\Delta\cos\alpha_{Stk}$, $\Delta\cos\alpha_{Ek}$ – direction cosines systematic errors of stellar, Sun, and Earth sensors due to installation misalignment; $k = 1, 2$ is the index of the sensor skew plane; $\left(\Delta\vartheta_E^{sys} \quad \Delta\gamma_E^{sys} \quad \Delta\vartheta_E^{ort} \quad \Delta\gamma_E^{ort}\right)$ are systematic errors of the Earth sensor in the vertical and horizontal planes of the orbital frame due to unorthogonality of sensors axes and zero drift; $\left(\Delta\psi_{Sun}^{sys} \quad \Delta\gamma_{Sun}^{sys} \quad \Delta\psi_{Sun}^{ort} \quad \Delta\gamma_{Sun}^{ort}\right)$ and $\left(\Delta\vartheta_{St}^{sys} \quad \Delta\psi_{St}^{sys} \quad \Delta\vartheta_{St}^{ort} \quad \Delta\psi_{St}^{ort}\right)$ are systematic errors of the Sun and stellar sensors (its definition is similar to that above); $(\Delta t_{KA} \quad \Delta t_0 \quad \ldots \quad \Delta t_{59})$ – GLONASS/GPS satellite onboard timescale shift; $\tilde{P}$ is the thrust magnitude; α and β are thrust attitude angles with respect to inertial frame.

Let us emphasize the propulsion acceleration magnitude comparable to that caused by nature, namely: solar radiation, Earth' oblateness, solar and lunar gravitation, etc. By this, we will estimate thrust vector separately of the satellite inertial position and velocity. Thus, we introduce one more state vector for the thruster:

$$\mathbf{X}_2 = \left(X_{IF} \quad Y_{IF} \quad Z_{IF} \quad V_{X\,IF} \quad V_{Y\,IF} \quad V_{Z\,IF} \quad \tilde{P} \quad \alpha \quad \beta\right)^{\mathrm{T}}. \tag{2}$$

The integrated navigation system involves two procedures to estimate the components of the satellite position, velocity, attitude, and the thrust vector.

The first procedure estimates current state vector $\mathbf{X}_1$ with regard to quite accurate satellite attitude and systematic errors computed from onboard GNSS receiver data using the so-called scalar modification of Kalman's filter [5]. The onboard algorithm forms two satellite trajectories at each step. The first trajectory is from measurements processing. The second one is reference trajectory obtained from integration of satellite equations of motion considering all natural disturbances except for propulsion. The output of this procedure is vector difference between reference trajectory and its estimation.

The second procedure estimates actual magnitude and direction of the thrust by processing the difference vector using the extended Kalman's filter [5] at each step. We put emphasis on that the reference trajectory produced at each step of the procedure is corrected using the estimates of the thrust vector obtained at the preceding step.

Figure 2 shows simplified diagram of the algorithm where $\mathbf{X}_1^*$ and $\mathbf{X}_1^0$ denotes the estimate of the state vector $\mathbf{X}_1$ and its reference value on board the satellite, respectively, $\mathbf{P}^*$ is thrust vector estimate.

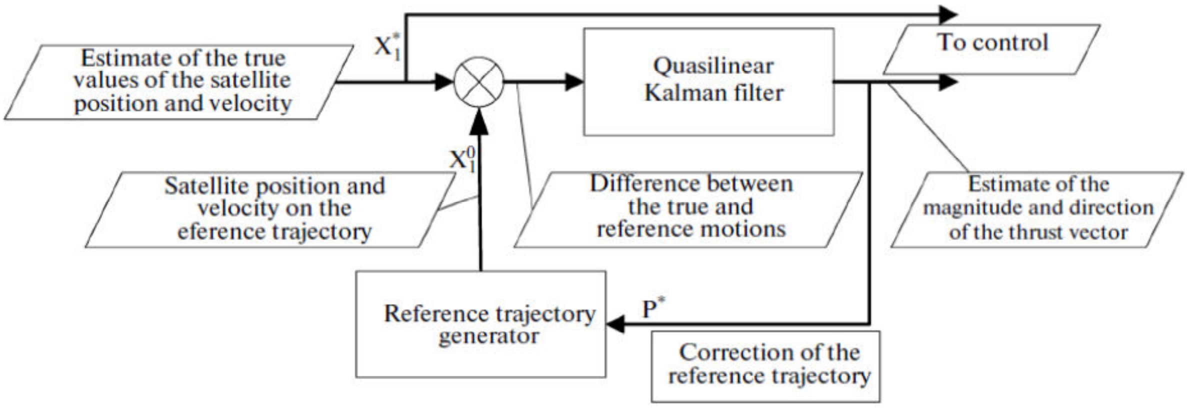

Figure 2. Simplified diagram of the state vector and PPU parameters estimation

The set of uncontrollable factors to consider by navigation problem solution in order to provide required estimation accuracy includes the following items:

- Earth oblateness with complete set of spherical harmonics according to the International Earth Rotation Service (IERS) standard bulletin [6];
- Solar and lunar gravity according to [5];
- Atmospheric drag for the low parts of the satellite trajectory with regard to the model of atmosphere density according to the Russian State Standard GOST 25645.115-84 “Density Model for the Ballistic Flights of Artificial Earth Satellites”;
- Solar radiation pressure;
- Oceanic and Earth body tides according to IERS materials [6].

To examine visibility of navigation satellites during payload transfer to GEO, which determine principal possibility of autonomous mission, we have plotted the number of available navigation satellites. Navigation satellite availability stands for its signal appears useful for navigation purposes. Figure 3 shows three or more GLONASS navigation satellites available during the mission. The number of available navigation satellites decreases when the altitude of transfer orbit increases.

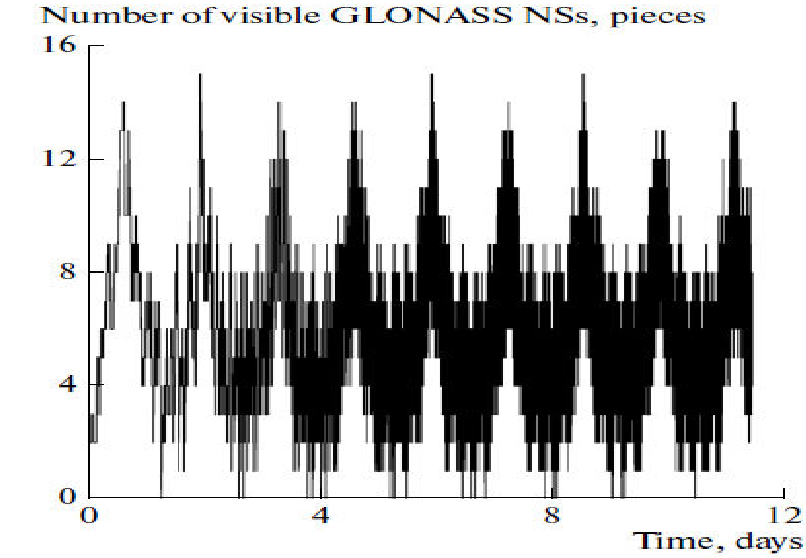

Figure 3. Number of available GLONASS satellites on 12 days interval

Transfer orbit might have the eccentricity as large as 0.5. Figures 4 and 5 shows sample orbit projections onto the Earth-related frame planes during 12 days transfer.

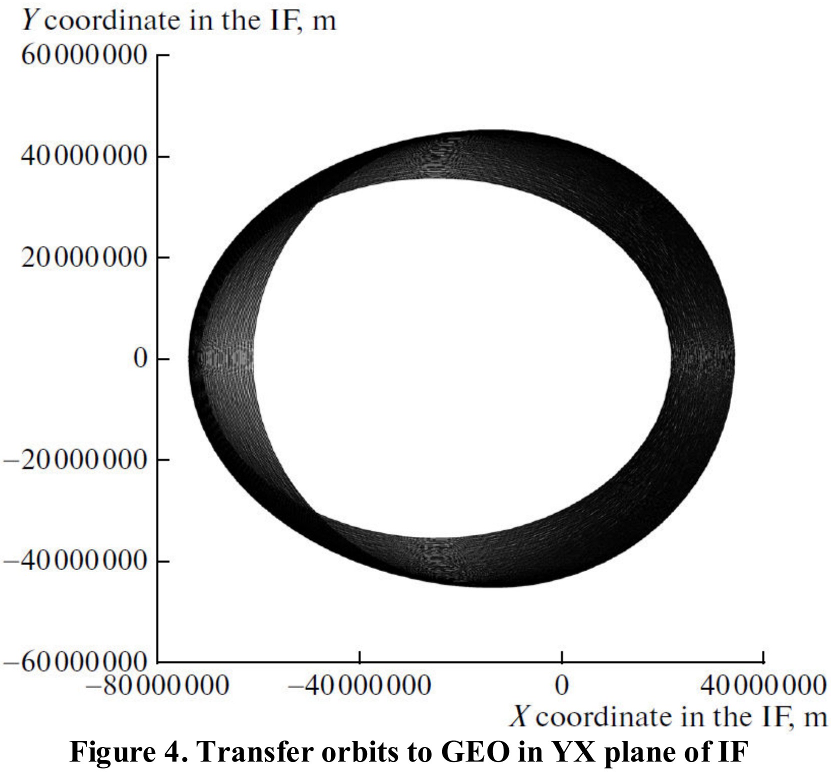

Figure 4. Transfer orbits to GEO in YX plane of IF

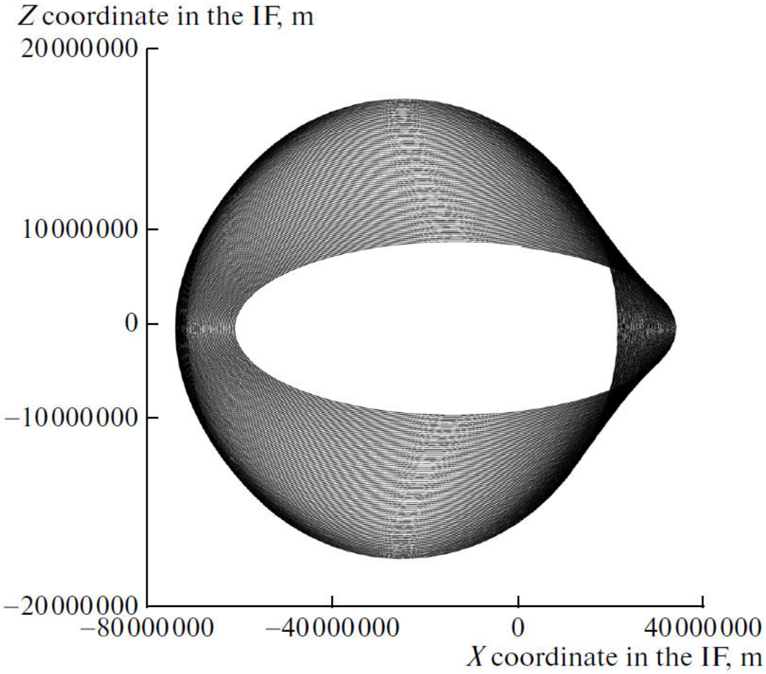

Figure 5. Transfer orbits to GEO in ZX plane of IF

Periodic availability of up to ten navigation satellites takes place at the perigee of transfer orbit, that is, when its altitude becomes as low as 20000 km. In other cases, the number of available navigation satellites decreases with altitude grows, and it can reach zero at apogee. Thus, we see that four navigation satellites are typically available for GEO, but sometimes that number increases to six. Figure 6 shows both GPS and GLONASS satellites availability during the mission. This results allow conclude the navigation solution accuracy of the integrated autonomous navigation system during the GEO transfer using PPU is not worse than 3 m (standard deviation) in coordinates and not greater than 0.3 m/s (standard deviation) in velocity components [3, 4].

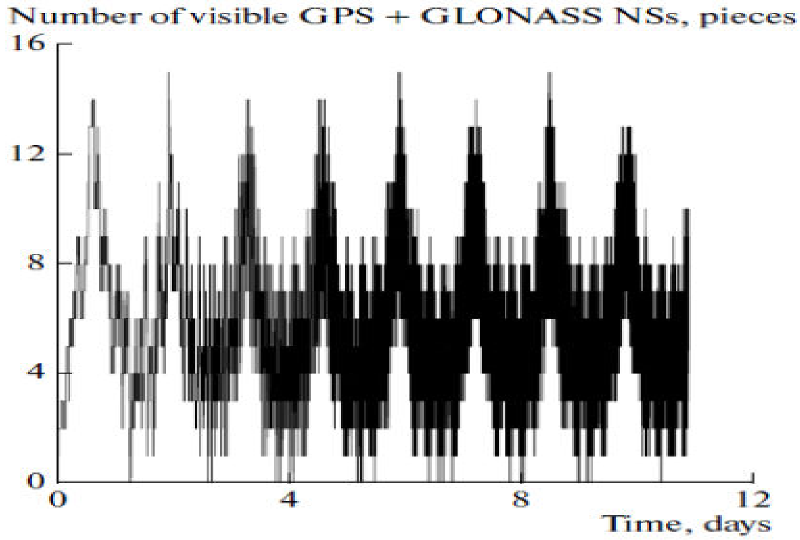

Figure 6. Number of available GPS + GLONASS satellites on 2 days interval

Table 1 summarizes results of autonomous navigation simulation at the GEO satellite attitude precision less than three angular minutes (standard deviation) [4].

Table 1. Simulation results

Satellite state vector estimation precision	Errors in the estimates of the thrust vector components	
	Thrust magnitude	Thrust pitch and yaw
Without errors	1% of the rated level after 1000 s	Several angular minutes after 2000 s
Position of the center of mass 3 m (standard deviation) velocity components 0.3 m/s (standard deviation)	1% of the rated level after 20000 s	Several angular minutes after 15000 s
Position of the center of mass 7 m (standard deviation) velocity components 1 m/s (standard deviation)	1% of the rated level after 25000 s	Estimate accuracy is inacceptable
Position of the center of mass 10 m (standard deviation)	Estimation algorithm unstable	

Accuracy of thrust vector estimation depends on state vector precision. State vector precision depends on navigation satellites availability. The satellite of interest 3 meters positioning precision and 0.3 m/s (standard deviation) velocity precision takes place in case of at least three navigation satellites available while its attitude precision less than $3'$ (standard deviation). This makes it possible to have thrust magnitude accurate to 1% of its rated value, and thrust attitude accurate to several angular minutes after 15000 s. Such navigation characteristics satisfy quasi-optimal control demands for a satellite transfer to a GEO using PPU [1].

3. Autonomous Control Synthesis

In framework of autonomous satellite mission, we have a typical stochastic control optimization with respect to incomplete erroneous data. Stochastic control theory proves separation of navigation from control synthesis only when the controlled object and sensors has linear models disturbed by Gaussian additive white noise. The authors have approved possibility of separate control and navigation problem solution for various aerospace applications. By the control design, we act on the premise that navigation system estimates state vector (1) which, in its turn, is the co-called vector of sufficient coordinates [10]. This stands for the vector (1) has, at least, two a posterior moments: mathematical expectation and covariance matrix. By this, we able solve control problem with respect to complete data separately of the navigation.

The GEO position acquisition, station keeping, and space disposal are typical orbit dynamical operations in course of the satellite lifecycle. Each operation is a satellite transfer from initial state into a movable terminal region. For sake of convenience, we treat it as satellite transfer between two points in space. Such a problem has various applications, for instance: station acquisition, station keeping, station reposition, space disposal, space rendezvous, etc. Anyway, we assume existence of a satellite state vector $\mathbf{Z}(t_k)$ at any instant t_k. The components of vector $Z(t_k)$ are Kepler elements $a, e, u, \omega, \Omega, i$, where a is the semi major axis; e is the eccentricity; u is argument of the latitude; ω is argument of the perigee; Ω is right assent ion of ascending node (RAAN); i is the inclination. Orbital elements directly follow from vector (1).
The transfer terminal requirements make up a spatial box with borders

$$T \in [T^{*} \pm \Delta T_m];\ e \le e_m;\ \lambda \in [\lambda^{*} \pm \Delta\lambda_m];\ \ i \in [i^{*} \pm \Delta i_m], \quad (3)$$

where T is period of revolution; e is orbit eccentricity; λ is longitude. Hereinafter T^*, λ^*, i^* are the nominal values, ΔT_m, e_m, and Δi_m are admissible deviations. Satellite control objective is to put a satellite into terminal position box considering control limitations.
Autonomous control brain ware should generate the PPD on/off timeline with respect to deterministic, stochastic, and guaranteed approach on-board the satellite. The use of a number of approaches is important measure for control algorithm adaptation to navigation system output. For instance, at negligible random errors the deterministic approach should be preferred since it is most optimistic one. The stochastic approach allows for upon the average optimal solution when navigation system estimates control actions random errors. The guaranteeing approach is most pessimistic one; however, it allows for optimal solution with respect to the worst control errors.

Availability of control and navigation algorithms developed for in the three approaches framework allows create an autonomous integrated C&N system with flexible closed-loop PPD control algorithms that use the satellite state vector forecast as well as random and uncontrolled errors estimations in feedback loop.

The autonomous control problem formalization calls for deriving a mathematical model for the satellite in-orbit dynamical actions, optimality criterion, and technical constraints account method. By the model of motion construction, we assume the process consists of elementary control actions, or orbital corrections. Each correction has coast (PPD "off") part, and powered (PPD "off") part. Let duration of the parts be t_k and τ_k respectively.
Autonomous control design bases on linearization of equations of motion in a neighborhood of a nominal orbit. Use of linear models is only the way to obtain constructive solution for off-line satellite control. The linear model has discrete appearance [10]

$$\mathbf{X}_{k+1} = \mathbf{A}_k \mathbf{X}_k + \mathbf{B}_k \left[\mathbf{u}_k \left(1 + \mu_k\right) + \boldsymbol{\eta}_k \right] + \mathbf{D}_k + \xi_k, \; k = \overline{1, N}\,, \tag{4}$$

where $\mathbf{X}_k$ is the state vector at the start of correction; N is corrections number; $\mathbf{u}_k$ is control vector; μ_k is control vector magnitude error; η_k is control vector attitude error; $\mathbf{D}_k$ is systematic (non-random) disturbance due to Earth oblateness, Sun and Moon gravity, etc.; $\mathbf{A}_k$ is transfer matrix; $\mathbf{B}_k$ is control efficiency matrix.

The equation (4) is linearization of non-linear source model written in evenly rotated around the inertial J2000 frame origin. Rotation rate corresponds to nominal period of revolution of the orbit. The main plane of rotating frame coincides with the nominal orbit plane. The benefit of such a frame is the satellite in-orbital plane motion splits from inclination and RAAN evolution. In fact, the model (4) contains two vectorial equations with respect to the state vector

$$\boldsymbol{X}_k = \left(x_1 \;\; x_2 \;\; x_3 \;\; x_4 \;\; \Delta \mathrm{i} \;\; \Delta\Omega \;\; \Delta\omega \right)^{\mathrm{T}} \tag{5}$$

where x_1 is angular deviation from station longitude; x_2 is angular drift; x_3 and x_4 are the eccentricity vector components; Δi, $\Delta\Omega$, and $\Delta\omega$ are inclination, RAAN, and perigee argument deviation from their nominal values, respectively. Initial values of vector (5) components follows from **Z** vector components estimated by the navigation system.
The control vector components are characteristic velocity projections onto the orbital frame axes. Its magnitude equals to the characteristic velocity of correction. Duration τ_k of the powered part depends on correction characteristic velocity as

$$\tau_k = k_\tau \left\| \mathbf{u}_k \right\|, \tag{6}$$

where k_τ is proportionality factor. The transfer matrix as well as the control efficiency matrix depends on the nominal orbit parameters, and correction parts duration t_k and τ_k. Due to this the model (3) appears to be non-linear with respect to control vector $\mathbf{u}_k$ whereas it is linear with respect to the state vector. In other words, we have $\mathbf{A_k}$=$\mathbf{A_k}(t_k, \tau_k)$, and $\mathbf{B_k}$=$\mathbf{B_k}(t_k, \tau_k)$. Now let us introduce generalized control vector for system (4):

$$\mathbf{U} = \left(\mathbf{u} \mid \mathbf{t} \mid N \right), \tag{7}$$

with the range of values:

$$\hat{U} = \left\{ \boldsymbol{U} \mid \ t_k^l \le t_k \le t_k^h, \quad k_\tau |u_k| \le \tau_k^h, \ \ k = \overline{1,N} \ \right\} , \tag{8}$$

where $\mathbf{u} = \left\{ \mathbf{u}_k , k = \overline{1,N} \right\}$ is the powered part control sequence; $\mathbf{t} = \left\{ t_k , k = \overline{1,N} \right\}$ is coast part control sequence; t_k^l и t_k^h are lower and upper bounds for duration of the k^{th} correction coast part; τ_k^h is upper bound for duration of the k^{th} correction powered part.

Terminal requirements (3) to position acquisition might be written as $\mathbf{X}_{N+1} \in S_{N+1}$ where

$$S_{N+1} = \left\{ \mathbf{X} \middle| g_j \left(\mathbf{X}_{N+1} \right) \le 0, \ j \in \left[T, e, \alpha, i \right] \right\} \tag{9}$$

where

$$g_T\left(x_{N+1}\right) = \left| T_{N+1} - T^* \right| - \Delta T_m ,$$
$$g_e\left(x_{N+1}\right) = e_{N+1} - e_m ,$$
$$g_\alpha\left(x_{N+1}\right) = \left| \alpha_{N+1} - \alpha^* \right| - \Delta\alpha_m ,$$
$$g_i\left(x_{N+1}\right) = \left| i_{N+1} - i^* \right| - \Delta i_m ,$$

Characteristic velocity ΔV_Σ and transfer duration t_Σ are typical criteria for any space manoeuvre. Transfer duration and characteristic velocity easy follow from control items:

$$t_\Sigma = \sum_{k=1}^{N} \left(t_k + \tau_k \right) = \sum_{k=1}^{N} \left(t_k + k_{\tau k} \left\| \boldsymbol{u}_k \right\| \right), \tag{10}$$

$$\Delta V_\Sigma = \sum_{k=1}^{N} \left\| \mathbf{u}_k \right\| . \tag{11}$$

Before to discuss how to do with random and uncontrolled factors, let us introduce a set of technical tasks with regard to autonomous in-orbit satellite control.

Task1 is to place a satellite in orbital position considering transfer duration and characteristic velocity limits:

$$\mathbf{U} = \arg \left\{ t_\Sigma \le t_\Sigma^*, \quad \Delta V_\Sigma \le \Delta V_\Sigma^* \ \mid \ \mathrm{U} \in \hat{\mathrm{U}} \right\} .$$

Task 2 is to place a satellite in orbital position at minimum of characteristic velocity considering transfer duration limit

$$\mathbf{U} = \arg \min_{\mathbf{U} \in \hat{U}} \left\{ \ \Delta V_\Sigma \ \mid t_\Sigma \le t_\Sigma^* \ \right\} .$$

Task 3 is to place a satellite in orbital position at minimum of transfer duration, i.e.

$$\mathbf{U} = \arg\min_{\mathbf{U}\in\hat{U}} \left\{ t_\Sigma \mid \Delta V_\Sigma \le \Delta V_\Sigma^* \right\}.$$

Task 4 is to place a satellite in orbital position at minimum of characteristic velocity subject to fixed transfer duration:

$$\mathbf{U} = \arg\min_{\mathbf{U}\in\hat{U}} \left\{ \Delta V_\Sigma \mid t_\Sigma = t_\Sigma^* \right\}.$$

Let us admit the station keeping coincides with task 4 subject to initial state vector already belongs to the terminal box, i.e. $x_1 \in X_{N+1}$.

All the tasks above are reduced to generalized initial optimization problem with respect to the following optimality criterion

$$\boldsymbol{F}(U) = a_t t_\Sigma + a_v \Delta V_\Sigma + \boldsymbol{\Phi}(U), \tag{12}$$

where $\Phi(U)$ is a penalty for constraints (8) and (9) violation; a_t and a_v are weighting coefficients. Thus, satellite transfer initial problem consists in minimization of scalar function (12) with respect to control vector (7). The problem is deterministic. It might be solved using non-linear programming numerical methods. In our opinion, this is not good idea because of the function (12) appears multiextremal. Due to this the result of solution depends on strongly depends on control vector initial guess. In addition, it is hard for solution with respect to random and uncontrolled control errors. Practical experience shows control numerical generation requires operator supervision, i.e. it is senseless for autonomous case.

In framework of autonomous satellite in-orbit control engineering the so-called combined optimization method [10] is developed. The method assumes division of the control vector (7) into synthesized and programmed components. Sufficient optimality condition for control of a dynamic system is to determine the synthesized component (SC). Necessary optimality condition for control of a dynamic system is to determine the programmed component (PC). The PC search envelopes the SC evaluation. In other words, the SC evaluation routine serves the PC search routine with criterion value. By this, the SC routine computes criterion according to deterministic, stochastic, or guaranteeing approach, optionally.

Let the powered part control sequence $\mathbf{u} = \left\{\mathbf{u}_k, k = \overline{1,N}\right\}$ is the SC; whereas PC is coast control sequence $\mathbf{t} = \left\{t_k, k = \overline{1,N}\right\}$. The reason of such a separation is due to linearity of the model (4) with respect to both the state vector **X** and control vector **u**. Quadratic approximation of optimality criterion allows the use of Bellman's backward dynamical programming recursive procedure for linear control feedback gains computation. Let the quadratic loss be

$$J = \sum_{k=1}^{N} \left(\mathbf{X}_k^T \mathbf{Q}_k \mathbf{X}_k + \mathbf{u}_k^T \mathbf{W}_k \mathbf{u}_k \right) + \mathbf{X}_{N+1}^T \mathbf{K} \mathbf{X}_{N+1} , \tag{13}$$

where $\mathbf{Q}_k$, $\mathbf{W}_k$, and $\mathbf{K}$ are positively definite symmetric matrix. Subject to approach the criterion for control synthesis will be:

- initial loss for deterministic approach;
- mathematical expectation of initial loss for stochastic approach;
- maximum of initial loss with respect to uncontrolled errors for guaranteeing approach.

Bellman's optimality condition is the dynamical programming method with respect to the future loss function $R_k\left(\mathbf{X}_k\right)$. At stochastic approach, it takes the following form

$$\begin{aligned} &R_k\left(\mathbf{X}_k\right) = \min_{\mathbf{u}_k} \left\{ \mathbf{X}_k^T \mathbf{Q}_k \mathbf{X}_k + \mathbf{u}_k^T \mathbf{W}_k \mathbf{u}_k + M\left[R_{k+1}\left(\mathbf{X}_{k+1}\right) / \mathbf{X}_k, \mathbf{u}_k \right] \right\} \\ &R_{N+1}\left(\mathbf{X}_{N+1}\right) = \mathbf{X}_{N+1}^T \mathbf{K} \mathbf{X}_{N+1} \quad , \quad k = \overline{N,1} ; \end{aligned} \tag{14}$$

where $M\left[R_{k+1}\left(\mathbf{X}_{k+1}\right) / \mathbf{X}_k, \mathbf{u}_k \right]$ is conditional mathematical expectation. The future loss function $R_k\left(\mathbf{X}_k\right)$ at any step k has quadratic appearance

$$R_k\left(\mathbf{X}_k\right) = \mathbf{X}_k^T \mathbf{K}_k \mathbf{X}_k + 2\mathbf{G}_k^T \mathbf{X}_k + c_k . \tag{15}$$

Optimal closed loop control is linear function with respect to the state vector $\mathbf{X}_k$:

$$\mathbf{u}_k = -\mathbf{L}_k \mathbf{X}_k - \mathbf{d}_k \tag{16}$$

where

$$\begin{aligned} &\mathbf{L}_k = \mathbf{\Gamma}_k^{-1} \mathbf{B}_k^T \mathbf{K}_{k+1} \mathbf{A}_k ; \\ &\mathbf{d}_k = \mathbf{\Gamma}_k^{-1} \mathbf{B}_k^T \left(\mathbf{K}_{k+1} \mathbf{D}_k + \mathbf{G}_{k+1} \right) ; \\ &\mathbf{\Gamma}_k = \mathbf{W}_k + \mathbf{B}_k^T \mathbf{K}_{k+1} \mathbf{B}_k \left(1 + \sigma_{\mu k}^2 \right) ; \\ &\mathbf{K}_k = \mathbf{Q}_k + \mathbf{A}_k^T \mathbf{K}_{k+1} \mathbf{A}_k - \mathbf{L}_k^T \mathbf{\Gamma}_k \mathbf{L}_k , \qquad \mathbf{K}_{N+1} = \mathbf{K} ; \\ &\mathbf{G}_k = \mathbf{A}_k^T \left(\mathbf{G}_{k+1} + \mathbf{K}_{k+1} \mathbf{D}_k \right) - \mathbf{L}_k^T \mathbf{\Gamma}_k \mathbf{d}_k , \qquad \mathbf{G}_{N+1} = 0 ; \\ &c_k = c_{k+1} + \mathrm{Sp}\left(\mathbf{V}_k \mathbf{K}_{k+1} \right) + \mathbf{D}_k^T \mathbf{K}_{k+1} \mathbf{D}_k + 2\mathbf{G}_{k+1}^T \mathbf{D}_k - \mathbf{d}_k^T \mathbf{\Gamma}_k \mathbf{d}_k , \qquad c_{N+1} = 0 ; \end{aligned} \tag{17}$$

$\sigma_{\mu k}$ is control vector magnitude error rooted mean square;
$\mathbf{V}_k$ is control attitude error covariance matrix.

Note that attitude error affects the future loss value only. Control structure (16) for both deterministic and guaranteeing case remains unchanged except $\sigma_{\mu k}$ and $\mathbf{V}_k$ is null for deterministic approach; whereas we should substitute μ_m^2 instead of $\sigma_{\mu k}$ in guaranteeing case.

This allows consider the solution (16), (17) be the universal platform for autonomous control synthesis. However, it is applicable only when powered control does not depend on powered part duration according to (6). In other words, to launch recursive dynamical programming procedure we need unknown powered control parts. Due to this, the sufficient optimality conditions implementation requires iterative method. Input data for iteration is corrections number, coast part sequence $\mathbf{t} = \left\{t_k, k = \overline{1, N}\right\}$, and initial guess for powered part sequence $\mathbf{u} = \left\{\mathbf{u}_k, k = \overline{1, N}\right\}$. From powered sequence follows powered part duration $\left\{\tau_k, k = \overline{1, N}\right\}$ according to (6); so using backward process (17), the control feedback gains **L** and vectors **d** are computed. Next step is satellite transfer simulation using non-linear J2000 equations of motion taking into account Earth's oblateness (up to 10 harmonics), Solar and Lunar effects. By this, coast part sequence **t** determines thruster ON instants, whereas thruster OFF instants are computed according to (16), (6) using simulated state vector and reference orbit parameters. As the result of the simulation, we have new powered sequence as well as criterion and its summands. The process repeats from feedback gains update until the algorithm converges with respect to criterion value.

The linear control synthesis using the process above, do not meet strong Bellman's optimization principle. Due to this fact, the control appears quasioptimal.

Station acquisition, repositioning, and space disposal control algorithm implements recursive procedure (17) at corrections number not greater than 10. The station keeping in 15-year satellite lifecycle requires thousands of the PPU ON-OFF cycles. By this, we can treat the station keeping as a steady-state process. This stands for feedback gains in (16) do not depend on time, so procedure (17) runs once (i.e. N=1) at preset corrections interval.

4. Conclusions

The proposed integrated autonomous navigation system makes it possible to perform navigational support on board the satellite in entire interval of its GEO transfer, using plasma propulsion unit, for station acquisition and station keeping with specification tolerance.

The accuracy of solving the navigation problem obtained by simulation is 3 m in terms of coordinates, 0.3 m/s in terms of the velocity components, 1% of the rated actual magnitude of the engine thrust and several angular minutes of the thrust direction provided that the satellite attitude angles are estimated not worse than with 3 angular minutes accuracy.

Geostationary satellite station acquisition, reposition, station keeping, and space disposal control bases on combined optimization method. The method assumes the control vector consists of programmed and synthesized items. The programmed items are PPD "ON" time instants and corrections number to obtain using necessary condition of optimality. The synthesized items are characteristic velocity increments conditioning the PPU "OFF" time instants to obtain using sufficient conditions of optimality.

Deterministic, stochastic, and guaranteeing orbit control algorithms obtained using Bellman's dynamic programming procedure combined with iteration process allows control dynamic operations in GEO using integrated navigation system data.

5. References

[1] Petukhov V. G. "Quasioptimal control with feedback for multiorbit low-thrust transfer between noncoplanar elliptic and circular orbits," Cosm. Res., 49, 121--130 (2011).

[2] Kozorez D. A., Krasilshchikov M. N., Kruzhkov D. M., and Sypalo K. I. "Integrated navigation system for a space vehicle on a geostationary or highly elliptic orbit operating in the presence of active jam," J. Comput. Syst. Sci. Int. 52, 468–479 (2013).

[3] Kruzhkov D. M. Accuracy characteristics of the onboard integrated navigation system of an autonomous space vehicle," Elektron. Trudy MAI, No. 57 (2012).

[4] Kozorez D. A., Krasilshchikov M. N., and Kruzhkov D. M. "Integrated autonomous system for navigation and actual thrust evaluation by payload insertion into geostationary orbit with electric propulsion system assistance," in 9th ESA Int. Conference on Spacecraft Guidance, Navigation, and Control Systems, Port, Portugal, 2014.

[5] Modern Information Technologies in Problems of Navigation and Guidance of Unmanned Maneuverable Flying Vehicles, Ed. by M. N. Krasilshchikov and G. G. Serebryakov (Fizmatlit, Moscow, 2009) [in Russian].

[6] IERS Conventions (2010): IERS Technical Note No. 36 (Bureau International des Poids et Mesure, US Naval Observatory, Frankfurt am Main, 2010).

[7] Arbinger C., Enderle W., Fraiture L., and Wagner O. "New algorithm (STAR) for cycle ambiguity rsolution within GPS based attitude determination" in Proc. of the 4th ESA Int. Conf. on Spacecraft Guidance, Navigation and Control Systems, ESTEC, Noordwijk, 1999.

[8] GLONASS Interface Control Document, Version 5.0 (Moscow, 2002).

[9] Bulynin Yu. L., Grechkoseev A. K., and Grechkoseeva D. D. "Results of ballistic and navigation support of geostationary satellites" in Proc. of the 15th Int. Conf. on System Analysis, Control, and Navigation, Eupatoriya, Ukraine, 2010. (Informatsionnye Sputnikovye Systema, 2010).

[10] Stochastic Dynamics and Flight Vehicle Control Optimization, Ed. by M. N. Krasilshchikov and V. V. Malyshev (Alliance, Moscow, 2013) [in Russian].